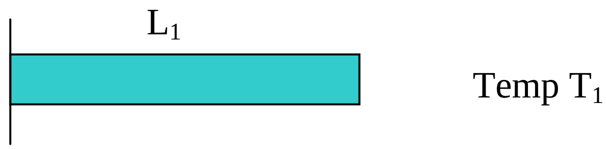
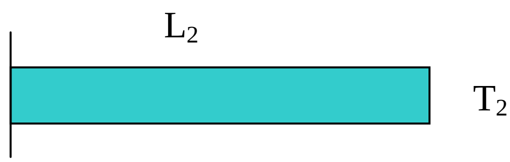
18. Thermal effects

Materials expand or contract with changing temperature.



Change the temperature



$$L_2 - L_1 = \alpha L \left(T_2 - T_1 \right)$$

$$\Delta L = \alpha L \Delta T$$

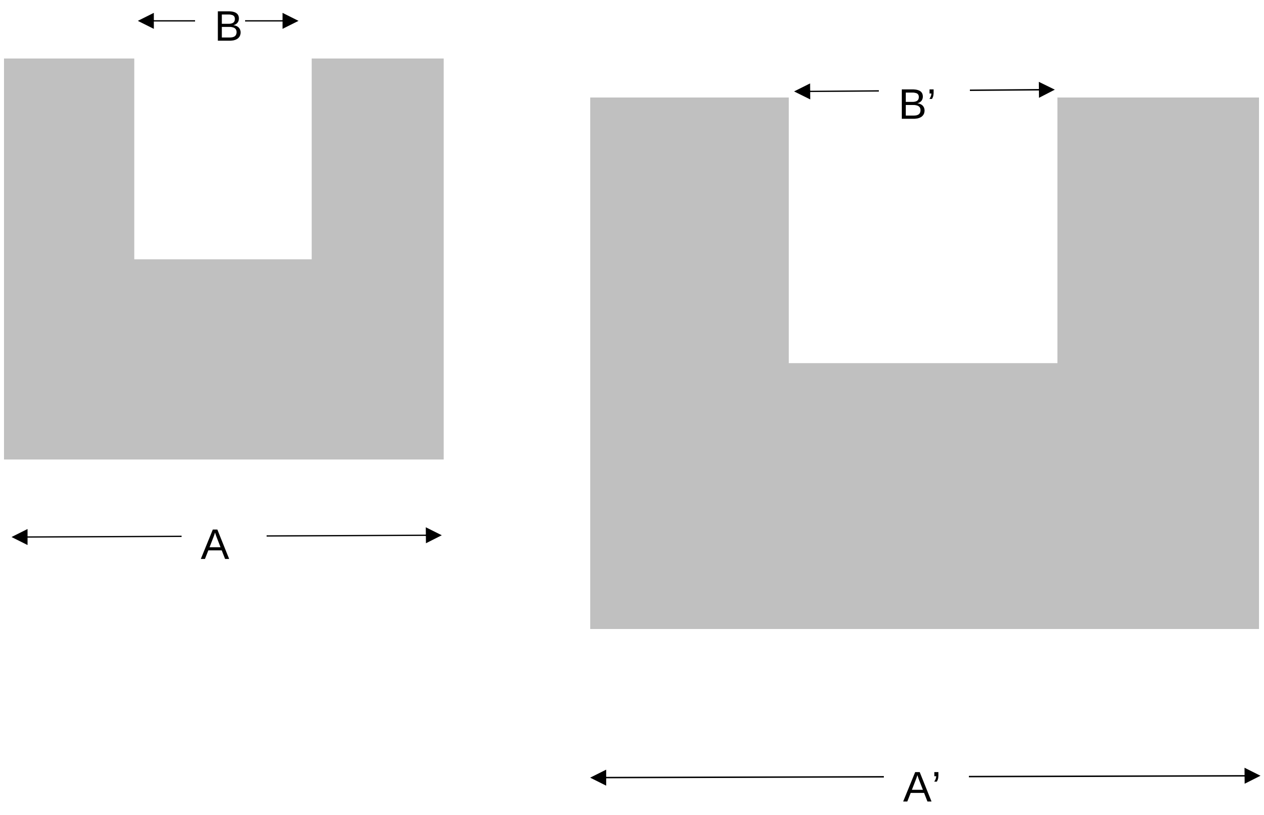
$$\varepsilon \triangleq \frac{\Delta L}{L} = \alpha \Delta T$$

thermal strain

α is the Coefficient of Thermal Expansion CTE

Aluminum	~23 ppm/°C
Optical Glass	~3 – 10 ppm/°C

Isotropic materials, temperature changes cause ALL dimensions to scale proportionally:



$$\Delta A = A\alpha\Delta T$$

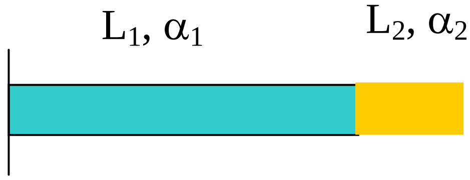
$$\Delta B = B\alpha\Delta T$$

Inside dimensions scale the same as outside dimensions

Low CTE materials:

Borosilicate glass (Pyrex)	~3 ppm/°C
Fused silica	0.6 ppm/°C
Invar	~1 ppm/°C
Super Invar	~0.3 ppm/°C
Zerodur (Schott)	0
ULE (Corning)	0
CFRP	(can be tuned to 0)

Athermalizing -- Combining two materials:



$$L = L_1 + L_2$$

$$\Delta L = \Delta L_1 + \Delta L_2$$

$$= \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T$$

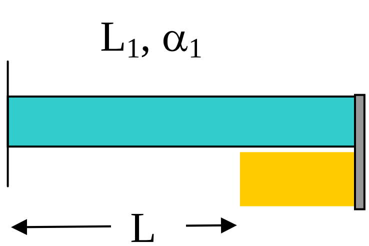
$$= \left(\alpha_1 L_1 + \alpha_2 L_2\right) \Delta T$$

To athermalize over a distance L, use two materials so

$$\alpha_1 L_1 + \alpha_2 L_2 = 0$$

$$L_1 + L_2 = L$$

Using materials with $\alpha > 0$, requires $L < 0$



$$\alpha_1 L_1 - \alpha_2 L_2 = 0$$

$$L_1 - L_2 = L$$

<u>Thermal stress</u>

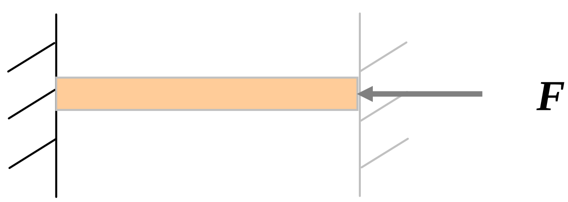
Use superposition

Calculate stress due to temperature change

1. Determine expansion, as if unconstrained

$$\Delta L = L\alpha\Delta T$$

2. Add reaction force that provides constraint by pushing back ΔL

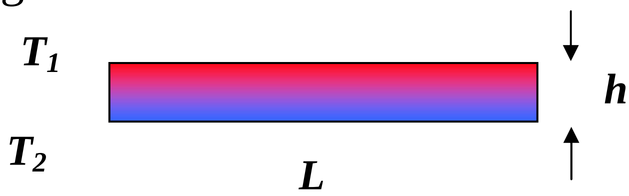


$$\Delta L = \frac{FL}{EA}$$

$$\sigma = \frac{F}{A} = E\alpha\Delta T$$

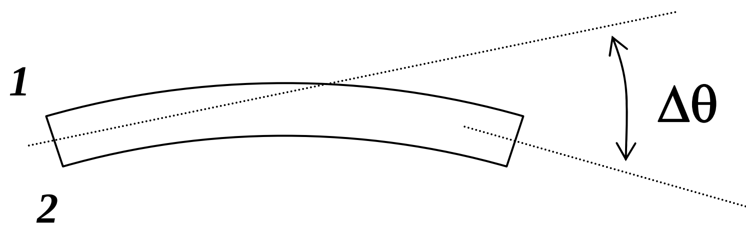
Solve more general problems the same way.

Temperature gradients cause distortions



Gradient = $\frac{\partial T}{\partial y} = \frac{T_1 - T_2}{h}$

This will cause the beam to bend in an arc, in the same way that the applied moment did.



The arc length along surface 1 is longer than the arc length along surface 2 by the amount

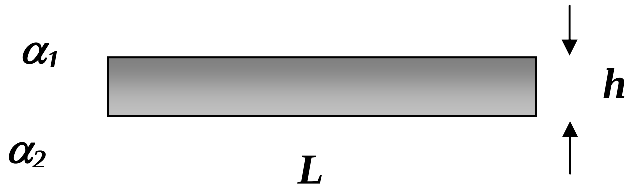
$$L_1 - L_2 = L\alpha\left(T_1 - T_2\right)$$

By geometry: $$\Delta\theta = \frac{L_1 - L_2}{h}$$

or $$\Delta\theta = \frac{\alpha L \Delta T}{h}$$

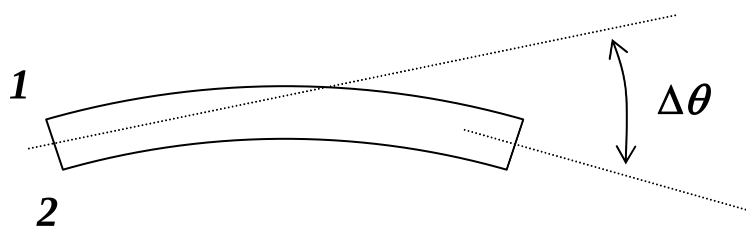
More generally, $$\Delta\theta = \alpha L \frac{\partial T}{\partial y}$$

Materials with inhomogeneous CTE, coupled with a bulk temperate change behave the same way as above:



CTE Gradient = $\dfrac{\partial \alpha}{\partial y} = \dfrac{\alpha_1 - \alpha_2}{h}$

This will cause the beam to bend in an arc, in the same way that the thermal gradient did.



If he temperature is changed uniformly, the arc length along surface 1 is longer than the arc length along surface 2 by the amount

$$L_1 - L_2 = L\Delta T\left(\alpha_1 - \alpha_2\right)$$

By analogy :

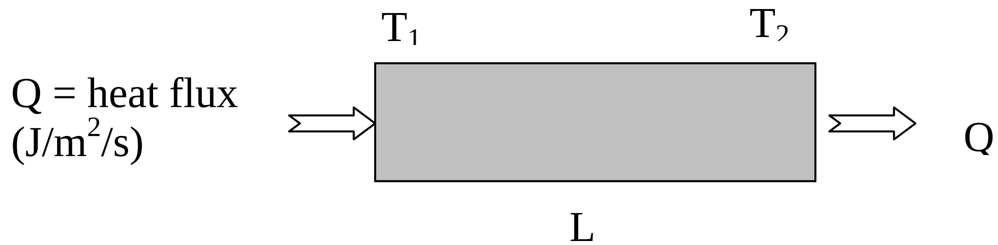
$$\Delta\theta = L\frac{\Delta\alpha}{h}\Delta T$$

and

$$\Delta\theta = L\frac{\partial\alpha}{\partial y}\Delta T$$

Heat flow causes thermal gradients

For steady state: $Q_{in} = Q_{out}$



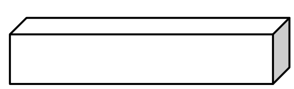
Define thermal conductivity λ :

$$Q = \lambda \frac{(T_1 - T_2)}{L}$$

	λ
Glass	1.1 W/(m K)
Aluminum	170 W/(m K)
Copper	390 W/(m K)
Stainless steel	16 W/(m K)

Heat flow

$$H = Q \bullet A = \frac{A\lambda\Delta T}{L}$$



Apply 1 W through 10 cm long bar of Al, A = 1 cm^2

$$\Delta T = \frac{HL}{A\lambda} = \frac{(1W)(0.1m)}{(0.0001m^2)\left(170\, W/_{m \cdot K}\right)} = 6°K = 10°F$$

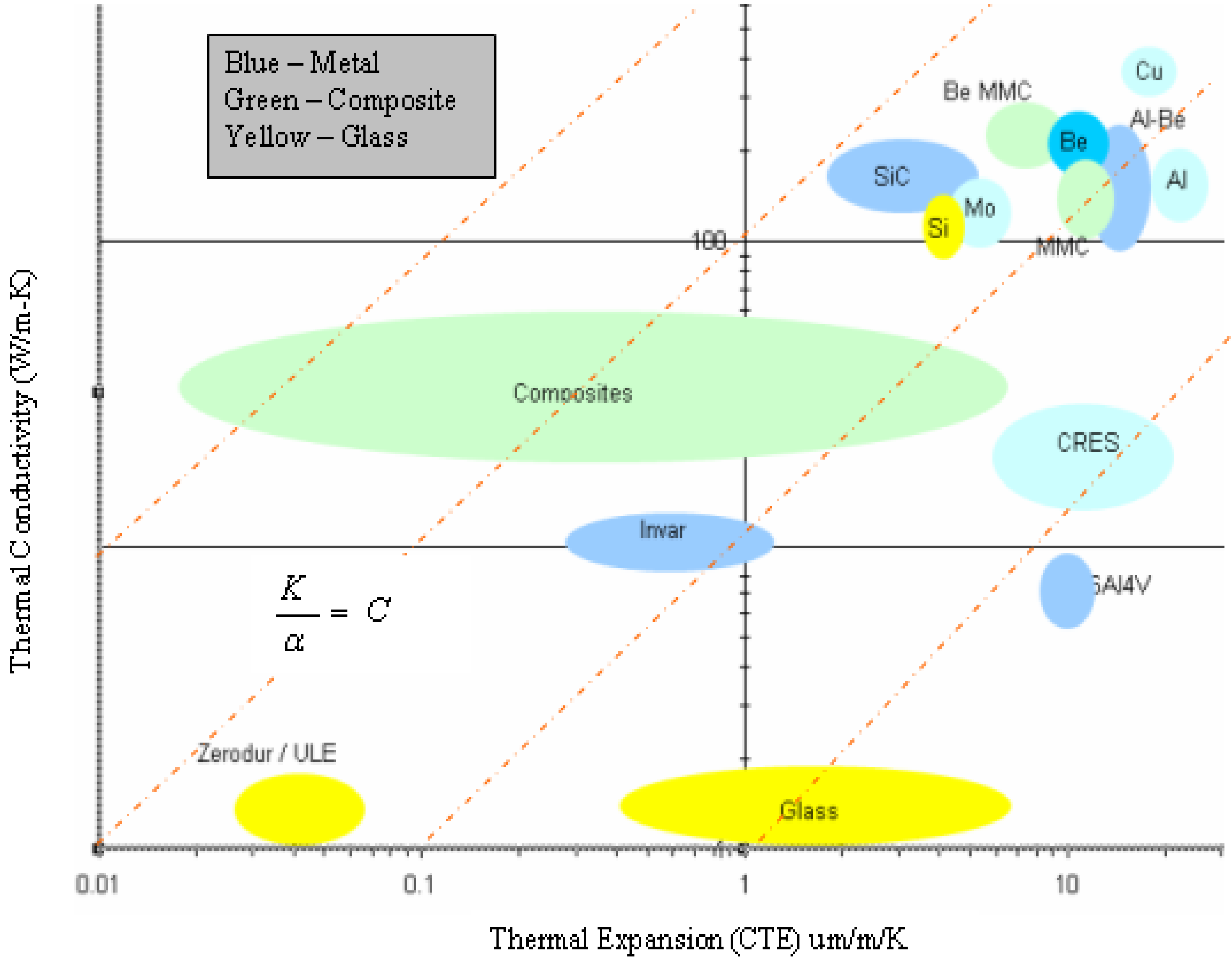
Steady state thermal distortion

Distortion due to temperature gradient ∇T is always proportional to $\alpha \nabla T$

For a constant heat source, with power *H*, the thermal gradient is proportional to $\nabla T \propto \frac{H}{\lambda}$

So the distortion will be proportional to $\frac{\alpha}{\lambda}$

This provides a **figure of merit** to compare sensitivity to steady state heat loading



- Lines of constant thermal stability are shown
- Performance improves toward upper left corner
- SiC has the largest thermal stability excluding some composites
  - Zerodur and ULE have very high thermal stability due to their extremely low CTE
  - Aluminum Beryllium and SiC perform very well
  - Aluminum and CRES are very poor for thermal stability

- A constant thickness plane or shallow spherically curved mirror is distorted by an axial steady state thermal gradient. The distortion is a change in the radius of curvature and is given by:

$$\frac{1}{R_0} - \frac{1}{R} = \left(\frac{\alpha}{k}\right) q$$

Where:

R_o	Is the original radius of curvature
R	Is the new radius of curvature
α	Is the thermal coefficient of expansion
k	Is the thermal conductivity
q	Is the heat flux per unit area

Bibliography References: 2.1.2, 2.1.3, 2.9.1, 2.9.2

- THERMAL DISTORTION

- An important thermal property of an optical material is its ability to resist distortion induced by thermal gradients.

- A window is distorted by an axial gradient. If the thermal gradient is due to a heat flux per unit area (q) through the glass, then the distortion for a plane parallel, circular window is of a spherical curvature of the surfaces. The window becomes a weak meniscus lens of curvature given by:

$$\left(\frac{1}{f'}\right) = \frac{n-1}{n}\left(\frac{\alpha}{k}\right)^2 h\, q^2$$

Where:

l/f'	Is the power
n	Is the index of refraction of the glass
α	Is the thermal coefficient of expansion
k	Is the thermal conductivity
h	Is the window thickness
q	Is the heat flux per unit area absorbed by the window.

- The parameter α/k is called the *thermal distortion* index of the material.

- **NOTE:** This assumes a steady state thermal flux.
- **NOTE:** The distortion is independent of the window diameter.

Bibliography References: 2.8.1, 2.1.2, 2.1.3

Transient heating

- Transient heat flux is when the temperature distribution changes with time
- Thermal diffusivity (D) is the ratio of thermal conductivity to heat capacity

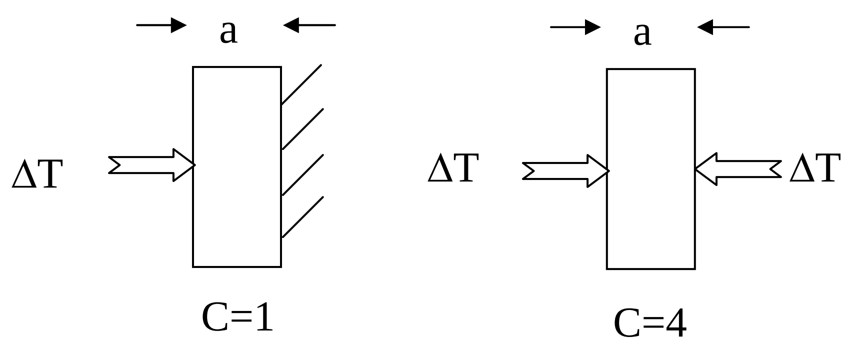
$$D = \frac{\lambda}{\rho \cdot C_P}$$

$$\frac{\partial T}{\partial t} = D \cdot \nabla^2 T$$

Larger thermal diffusivity means quicker response to temperature changes.

The thermal time constant τ governs the rate at which the transient response decays exponentially $(1-e^{-t/\tau})$

$$\tau \cong \frac{a^2}{CD}$$



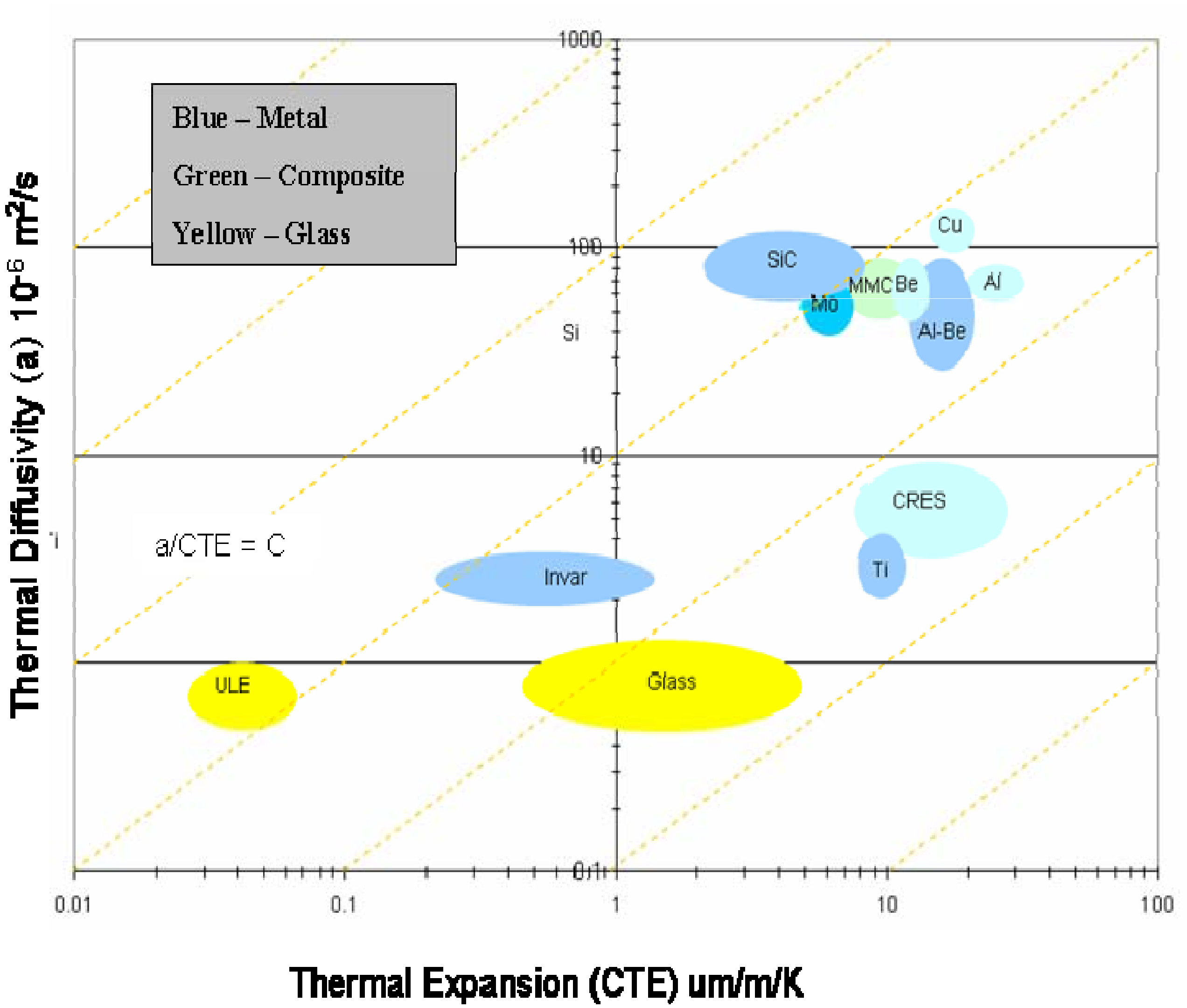
After a given time from a transient heat impulse, the temperature gradient will be proportional to 1/D.

Again, the thermal distortion is proportional to α ΔT, so the transient distortion will be proportional to

$$\frac{\alpha}{D}$$

This provides a merit function for transient thermal stability

Thermal stability under transient heating



- Lines of constant thermal stability are shown
- Performance improves toward upper left corner
- SiC has the largest transient thermal stability
- ULE has high stability due to its extremely low CTE
  - Aluminum Beryllium and SiC perform very well
  - Glass, Titanium and CRES are very poor

Thermal time constant:

For BK7 glass:
λ = 1.1 W/(m – K) = 0.011 J/s /(cm – K)
ρ = 2.5 g/cm^3
c_p = 0.86 J/(g- K)

$$D = \frac{\lambda}{\rho \cdot C_P}$$
$$= \frac{0.011}{(2.5)(0.86)} \frac{\text{cm}^2}{\text{s}}$$
$$= 0.0051 \frac{\text{cm}^2}{\text{s}}$$

For 1 cm thick BK7, heated from one side:

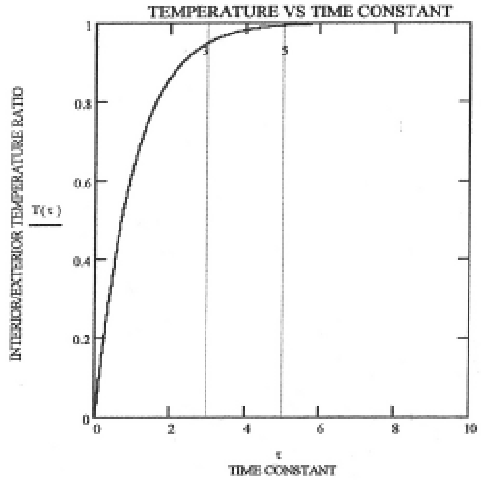
$$\tau = \frac{a^2}{D} = \frac{(1\,\text{cm})^2}{\left(0.0051 \frac{\text{cm}^2}{\text{s}}\right)} = 195\,\text{sec}$$ or ~ 3 min

Scaling: 25 mm glass takes 20 min

This the time it takes for the heat to travel through the glass. It does not include the coupling to the outside.

$$\Delta T(t) = \Delta T_0 e^{-t/\tau}$$

$\Delta T(\tau)/\Delta T_0$=0.37
$\Delta T(2\tau)/\Delta T_0$=0.14
$\Delta T(3\tau)/\Delta T_0$=0.05
$\Delta T(4\tau)/\Delta T_0$=0.02
$\Delta T(5\tau)/\Delta T_0$=0.007



Athermal System design

1. Control geometry

   Use low CTE materials

Kovar	~ 5 ppm/°C
Invar	~ 1 ppm/°C
Super Invar	~ 0.3 ppm/°C
Fused silica	~ 0.6 ppm/°C

   Practically zero CTE materials

   ULE

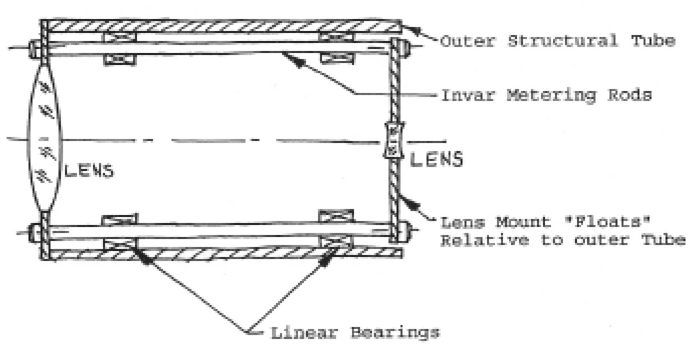
   Zerodur

   Athermalized Carbon Fiber Reinforced Plastic

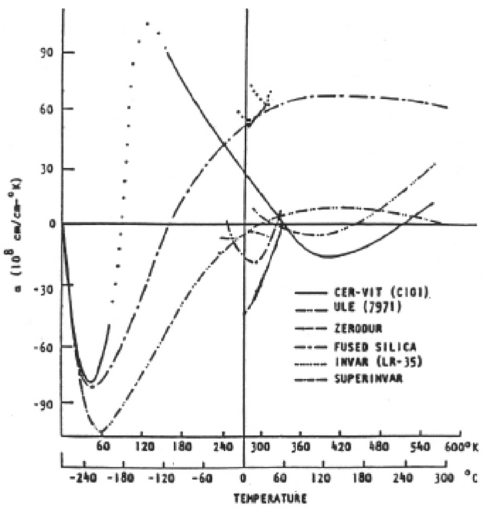
Composite truss for HST

Use of metering rods

- To avoid the structural inefficiency of low thermal expansion materials, use should be restricted to metering structures. The optical elements should be supported by a conventional structure and provided with mounts compliant in the direction along the optical axis. Use metering rods of low expansion material to tie the optical elements together. The metering rods maintain correct spacing as the main structure expands or contracts.



OPTICAL MATERIALS



Bibliography References: 2.2.2, 2.2.3.

(Vukabratovich) 17

OPTICAL MATERIALS

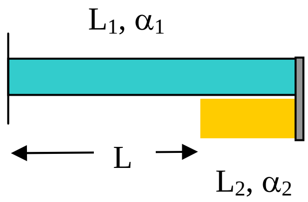
- INVAR

- Invar is an iron-nickel alloy, typically with about 36% nickel by weight.
- The thermal coefficient of expansion of invar may vary from -0.6 to 3.0 x 10^{-6} m/m-K between -70 to +100 °C. The thermal coefficient of expansion of invar can be limited to 0.8 to 1.6 x 10^{-6} m/m-K between 30 to +100 °C by careful control of the material during processing.
- A phase change occurs in invar at -20 °C, causing the thermal expansion coefficient to increase by a factor of 10. This phase change is reversible.
- Invar is unstable with respect to time (dimensional instability). The short term temporal instability may be as high as 11.0 x 10^{-6} m/m-day, with a time constant of about 100 days.
- For optimum thermal coefficient of expansion and long term stability, the so-called "MIT" or "Lement" heat treatment is suggested:
  - 1. 830 °C, 30 minutes, water quench
  - 2. 315 °C, 1 hour, air cool
  - 3. 95 °C, 48 hours, air cool
- Heavy machining or cold working may disturb the heat treatment of invar, and require another heat treatment cycle. Heavy machining is defined as any cut greater than 100 μm. Cold work, such as bead blasting, may also change the thermal coefficient of expansion of invar.

Bibliography References: 2.3.1, 2.3.2, 2.3.4, 2.3.8

(Vukabratovich) 18

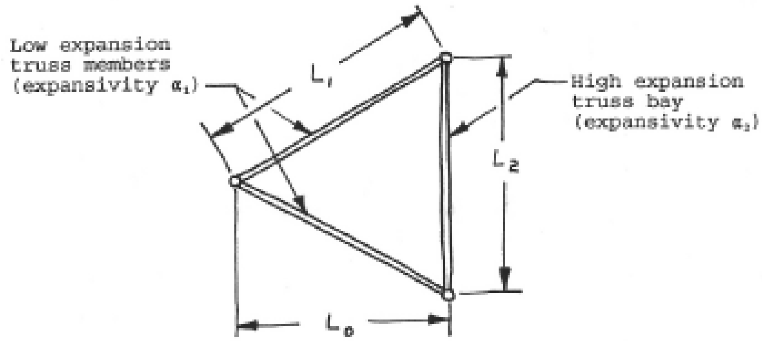
<u>Use different materials to compensate</u>



$$\alpha_1 L_1 - \alpha_2 L_2 = 0$$

$$L_1 - L_2 = L$$

- A type of athermal structure using the difference in the expansion coefficients of two (2) different materials is a bimetallic athermal truss. A high coefficient of expansion truss bay offsets a low coefficient of expansion truss member.



For L_o to remain constant for a temperature change (ΔT).

$$\frac{L_1}{L_2} = \frac{1}{2}\left[\frac{\alpha_2\ \Delta T\left(2+\alpha_2\Delta T\right)}{\alpha_1\ \Delta T\left(2+\alpha_1\Delta T\right)}\right]^{\frac{1}{2}} \approx \frac{1}{2}\sqrt{\frac{\alpha_2}{\alpha_1}}$$

For L_1 = Steel $\quad \alpha_1 = \dfrac{10 \times 10^{-6}}{K}$

L_2 = Aluminum $\quad \alpha_2 = \dfrac{23 \times 10^{-6}}{K}$

Then $\dfrac{L_1}{L_2} = 0.758$

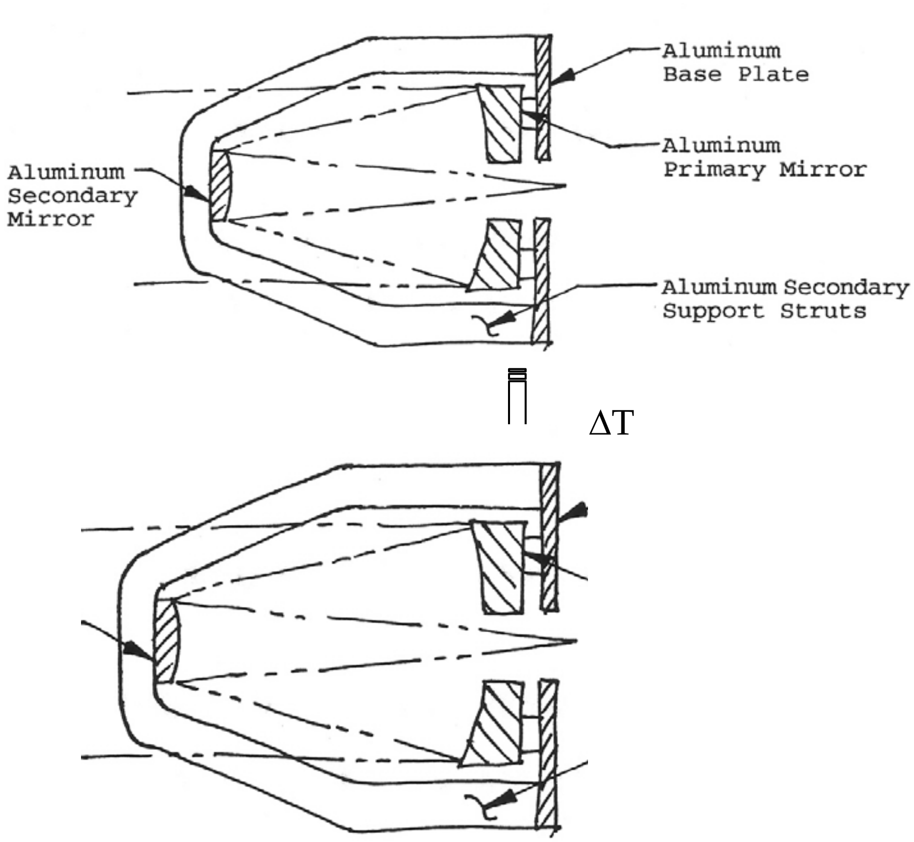
- This method is vulnerable to gradients

- Due to non-linear changes in thermal expansion coefficient with temperature, this method is useful only for a small temperature range.

Bibliography References: 3.4.21, 3.4.22

(Vukabratovich) 96

Make everything out of the same material (including the mirrors)



If all optical surfaces and spacing stay in proportion, then the system will still work!

Spitzer Telescope with mirrors and mechanics made of beryllium

Refractive materials:

OPTICAL MATERIALS

❖ THERM-OPTIC COEFFICIENT

❑ The index of refraction of optical materials changes with temperature. This change is given by:

$$n' = n + \left(\frac{dn}{dT}\right)\Delta T$$

Where:

n'	Is the refractive index after temperature change
n	Is the refractive index before temperature change
ΔT	Is the temperature change
dn/dT	Is the therm-optic coefficient

Bibliography References: 2.7.9, 2.7.10, 2.1.1, 2.8.1, 2.8.9

(Vukabratovich) 20

OPTICAL MATERIALS

THERM-OPTIC COEFFICIENTS

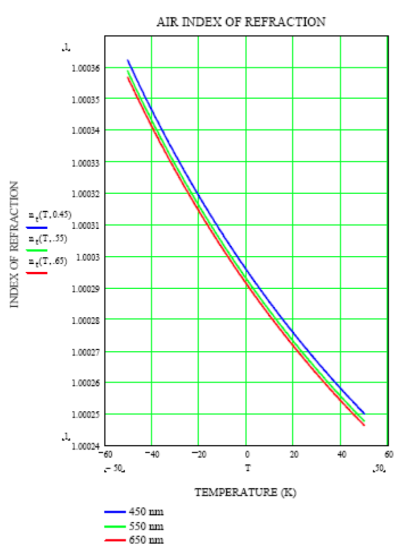
MATERIAL	n	λ (nm)	dn/dt (10^{-6} / K)
PK51	1.53019	546.1	-8.5
FK3	1.46619	546.1	-0.1
BK7	1.51872	546.1	3.0
LaK10	1.72340	546.1	5.0
SF5	1.67764	546.1	5.8
SF6	1.81265	546.1	11.6
FUSED SILICA	1.45850	587.6	8.1
CVD ZnSe	2.473	1150	59.7
CVD ZnS	2.279	1150	49.8
SILICON	2.38	10μM	162.0
KRS-5	2.37	10μM	-235.0
GERMANIUM	4.003	10μM	396.0

Bibliography References: 2.7.9, 2.7.10, 2.1.1, 2.8.1, 2.8.9

(Vukabratovich) 21

For lenses in air, we need to consider the effect of the changing refractive index of the air!

OPTOMECHANICAL DESIGN



Bibliography References: 3.4.11

(Vukabratovich) 85

OPTOMECHANICAL DESIGN

❑ Heat sources near the optical beam path can cause a distortion in the beam due to air turbulence. The amount of wave front distortion produced by a body of air of path length (L) which differs in temperature from the surrounding air by (Δ T) is given by:

$$\delta = (1.1)(L)(\Delta t \; x \; 10^{-6})$$

Where:

δ	Is in meters
L	Is in meters
Δt	Is in °C

❑ Heat sources should be kept away from the optical beam path. Failing this, heat sources should be insulated or located where hot air does not rise into the beam.

Bibliography References: 3.4.10, 3.4.8, 3.4.9, 3.4.7.

(Vukabratovich) 86

$n_d = 1.51680$	$\nu_d = 64.17$	$n_F - n_C = 0.008054$	**N-BK7 517642.251**
$n_e = 1.51872$	$\nu_e = 63.96$	$n_{F'} - n_{C'} = 0.008110$	

Refractive indices	λ [nm]	
$n_{2325.4}$	2325.4	1.48921
$n_{1970.1}$	1970.1	1.49485
$n_{1529.6}$	1529.6	1.50091
$n_{1060.0}$	1060.0	1.50669
n_t	1014.0	1.50731
n_s	852.1	1.50980
n_r	706.5	1.51289
n_C	656.3	1.51432
$n_{C'}$	643.8	1.51472
$n_{632.8}$	632.8	1.51509
n_D	589.3	1.51673
n_d	587.6	1.51680
n_e	546.1	1.51872
n_F	486.1	1.52238
$n_{F'}$	480.0	1.52283
n_g	435.8	1.52668
n_h	404.7	1.53024
n_i	365.0	1.53627
$n_{334.1}$	334.1	1.54272
$n_{312.6}$	312.6	1.54862
$n_{296.7}$	296.7	
$n_{280.4}$	280.4	
$n_{248.3}$	248.3	

Constants of dispersion formula	
B_1	$1.03961212 \cdot 10^{+00}$
B_2	$2.31792344 \cdot 10^{-01}$
B_3	$1.01046945 \cdot 10^{+00}$
C_1	$6.00069867 \cdot 10^{-03}$
C_2	$2.00179144 \cdot 10^{-02}$
C_3	$1.03580653 \cdot 10^{+02}$

Constants of formula dn/dT	
D_0	$1.86 \cdot 10^{-06}$
D_1	$1.31 \cdot 10^{-08}$
D_2	$-1.37 \cdot 10^{-11}$
E_0	$4.34 \cdot 10^{-07}$
E_1	$6.27 \cdot 10^{-10}$
λ_{TK}[μm]	0.170

Internal transmittance τ_i		
λ[μm]	τ_i [10mm]	τ_i [25mm]
2500	0.67	0.36
2325	0.79	0.56
1970	0.930	0.84
1530	0.992	0.980
1060	0.999	0.997
700	0.998	0.996
660	0.998	0.994
620	0.998	0.994
580	0.998	0.995
546	0.998	0.996
500	0.998	0.994
460	0.997	0.993
436	0.997	0.992
420	0.997	0.993
405	0.997	0.993
400	0.997	0.992
390	0.996	0.989
380	0.993	0.983
370	0.991	0.977
365	0.988	0.971
350	0.967	0.920
334	0.910	0.78
320	0.77	0.52
310	0.57	0.25
300	0.29	0.05
290	0.06	
280		
270		
260		
250		

Color code	
λ_{80}/λ_5	33/29

Remarks	

Relative partial dispersion	
$P_{s,t}$	0.3098
$P_{C,s}$	0.5612
$P_{d,C}$	0.3076
$P_{e,d}$	0.2386
$P_{g,F}$	0.5349
$P_{i,h}$	0.7483
$P'_{s,t}$	0.3076
$P'_{C',s}$	0.6062
$P'_{d,C'}$	0.2566
$P'_{e,d}$	0.2370
$P'_{g,F'}$	0.4754
$P'_{i,h}$	0.7432

Deviation of rel. partial dispersion ΔP from "Normal line"	
$\Delta P_{C,t}$	0.0216
$\Delta P_{C,s}$	0.0087
$\Delta P_{F,e}$	−0.0009
$\Delta P_{g,F}$	−0.0009
$\Delta P_{i,g}$	0.0035

Other properties	
$\alpha_{-30/+70°C}$ [10^{-6}/K]	7.1
$\alpha_{+20/+300°C}$ [10^{-6}/K]	8.3
Tg [°C]	557
$T10^{13.0}$ [°C]	557
$T10^{7.6}$ [°C]	719
c_p [J/(g·K)]	0.858
λ [W/(m·K)]	1.114
ρ [g/(cm^3)]	2.51
E [10^3(N/mm^2)]	82
μ	0.206
K [10^{-6}mm^2/N]	2.77
$HK_{0.1/20}$	610
HG	3
B	0
CR	2
FR	0
SR	1
AR	2
PR	2.3

Temperature coefficents of refractive index						
	$\Delta n_{rel}/\Delta T$ [10^{-6}/K]			$\Delta n_{abs}/\Delta T$ [10^{-6}/K]		
[°C]	1060.0	e	g	1060.0	e	g
−40/−20	2.4	2.9	3.3	0.3	0.8	1.2
+20/+40	2.4	3.0	3.5	1.1	1.6	2.1
+60/+80	2.5	3.1	3.7	1.5	2.1	2.7

Athermalizing systems with lenses

The refractive index of most materials, including air, varies with temperature. This causes the power in a lens to vary with temperature, which may cause a system focus to change.

The power ϕ in an optical surface is $\phi = \frac{\left(n_{glass} - n_{air}\right)}{R} = \frac{\left(n_r - 1\right)}{R}$

With n_r defined as n relative to air so $(n_{glass} – n_{air}) – (n_r – 1)$:

Taking derivative with temperature

$$\frac{d\phi}{dT} = \frac{dn_r}{dT}\frac{1}{R} - \frac{(n_r - 1)}{R^2}\frac{dR}{dT}$$

dividing through by ϕ,

$$\frac{1}{\phi}\frac{d\phi}{dT} = \frac{1}{(n_r - 1)}\frac{dn_r}{dT} - \frac{1}{R}\frac{dR}{dT}$$

Change in focal length goes as

$$f = \frac{1}{\phi} \quad \text{so} \quad \frac{1}{\phi}\frac{d\phi}{dT} = -\frac{1}{f}\frac{df}{dT}$$

Define β so $\Delta f = \beta f \Delta T$

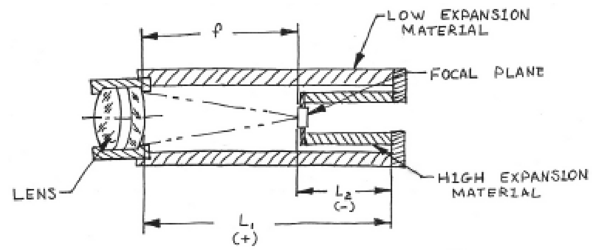
$$\beta = \frac{1}{R}\frac{dR}{dT} - \frac{1}{(n_r - 1)}\frac{dn_r}{dT}$$

$$= \alpha - \frac{1}{(n_r - 1)}\frac{dn_r}{dT}$$

GLASS TYPE	β (x 10^{-6})	MATERIAL TYPE	α (m/m – K x 10^{-6})
FK52	27.5	ALUMINUM	23
FK6	19.7	STAINLESS STEEL TYPE 310	16.6
FK5	10.77	STAINLESS STEEL TYPE 17-4 PH	10.8
PSK52	8.92	TITANIUM (6AL-4V)	8.0
BK7	0.98	INVAR (TYPE 36)	0.54
F2	0.68	INVAR (TYPE 36)	0.54

ibliography References: 3.4.4, 3.4.6

- Bi-metallic compensators can be used to produce very low or negative thermal coefficients of expansion:



- Some bi-metallic compensators for single elements:

Glass Type	β_g (m/m·kx10^{-6})	Material 1	α_1 (m/m·kx10^{-6})	L_1xf	Material 2	α_2 (m/m·kx10^{-6})	L_2xf
TiF6	20.94	Aluminum	23	0.678	Stainless Steel	16.6	0.322
BK1	3.28	Invar	0.54	0.829	Stainless Steel	16.6	0.171
LaKN9	0.32	Invar	0.54	1.01	Aluminum	23	-0.01
BAK4	-0.23	Invar	0.54	1.034	Aluminum	23	-.034
KzFS1	-2.89	Invar	0.54	1.153	Aluminum	23	-.153
ZnSe	-28.24	Stainless Steel	16.6	1.233	Plastic (ABS) (Polyurethane)	209	-.233
Silicon	-64.10	Stainless Steel	16.6	1.419	Plastic (ABS) (Polyurethane)	209	-.419
Germanium	-85.19	Stainless Steel	16.6	1.358	Plastic (Polyehtylene)	301	-.358

Bibliography References: 3.4.6, 3.4.17, 3.4.18, 3.4.19, 3.4.20, 3.4.21