Principles of Kinematic Constraint

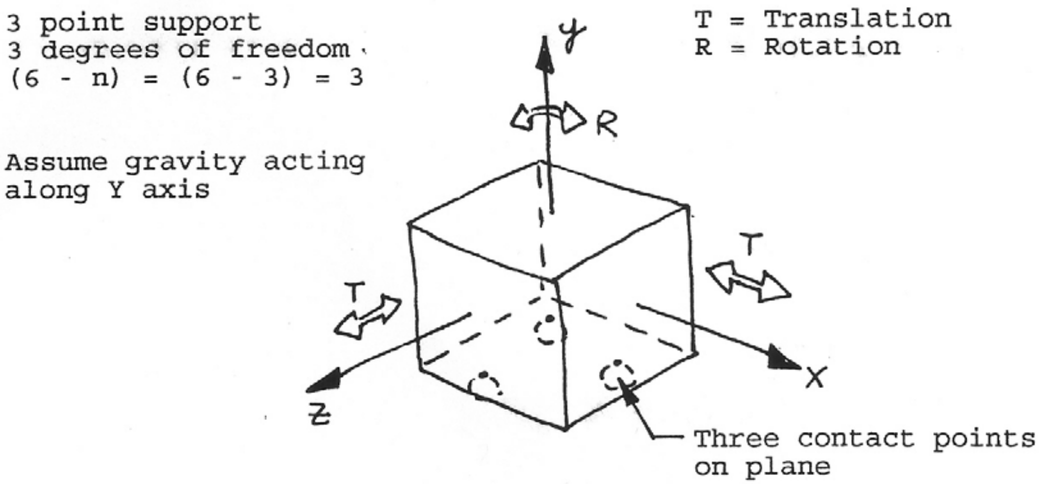
For holding a body (rigid thing) with the highest precision, we require:

- Full 6 DoF constraint
  - If 6 DoFs not fully constrained, then one is loose.

- No overconstraint
  - Any overconstraint can cause problems:
    - constraints can push against each other, resulting in stress and deformation.
    - constraints pushing against each other will “lurch” when forces exceed threshold

Kinematic constraint : All DoFs are constrained, and very strictly, none are overconstrained

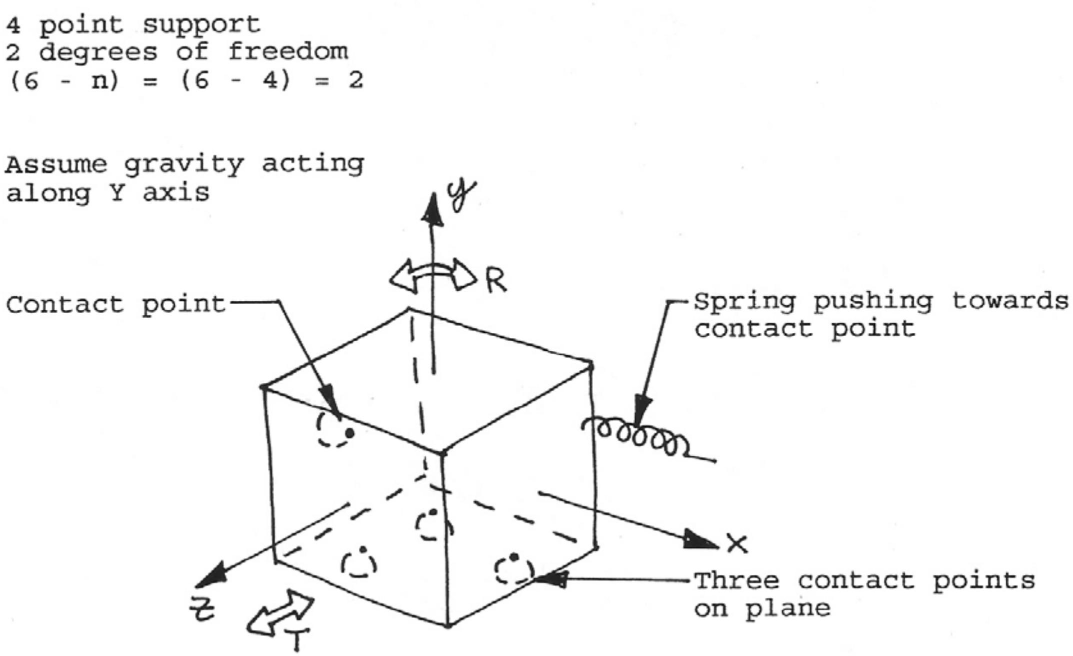
Semi-Kinematic : Slight overconstraint is allowed



What if you use 4 points in the plane?

What about 2 points?

What about 3 points in a line?



Balls provide position constraint.

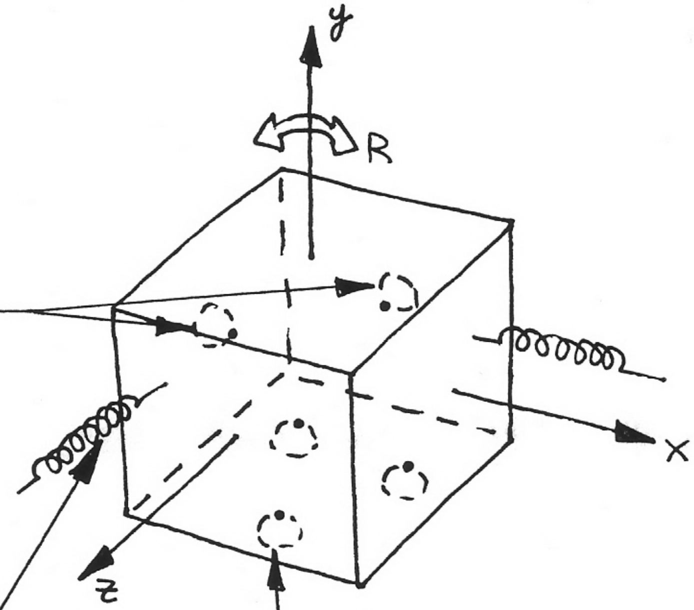
Springs, gravity provide preload. NO CONSTRAINT!

(Vukobratovich p. 68)

5 point support
1 degrees of freedom
(6 - n) = (6 - 5) = 1

Assume gravity acting along Y axis

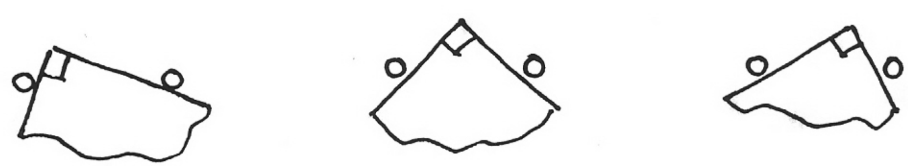
Two contact points



Springs pushing towards contact points

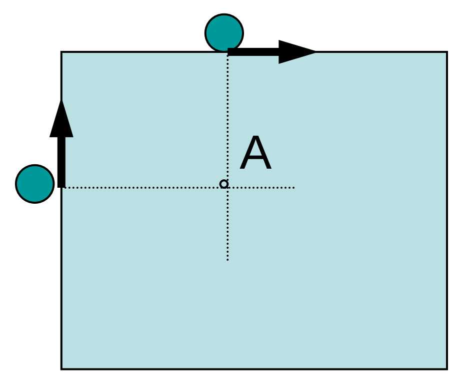
Three contact points on plane

Shows indeterminacy of upper two points.



One DoF left

Small motion : Rotation about point A



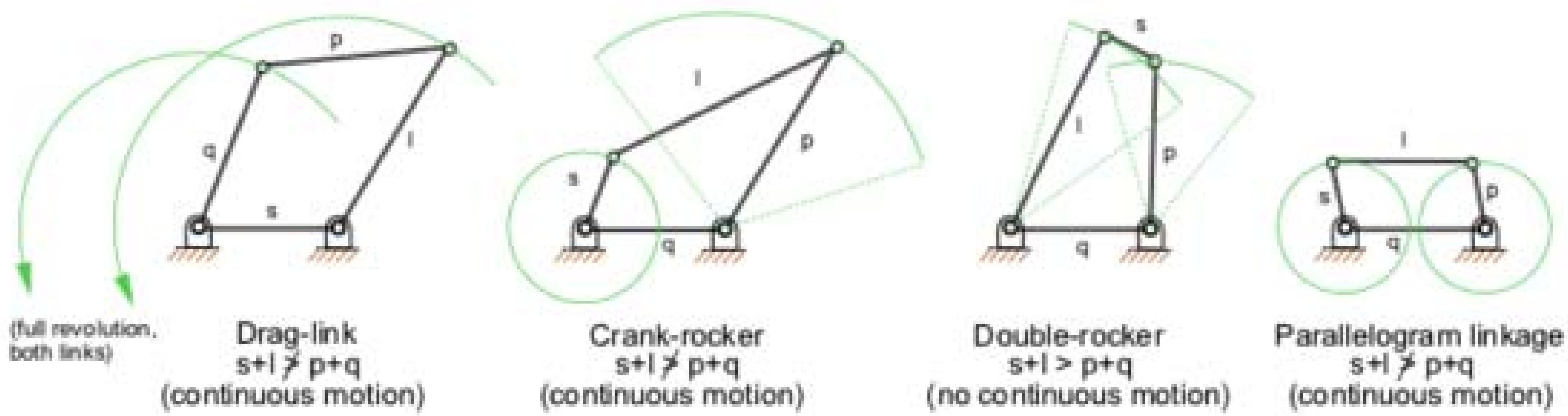
(Vukobratovich p. 69)

Instantaneous center of rotation

Same concept as four-bar linkage.

Instantaneous degree of freedom is rotation about a well defined point
– for small motions

For large motions, the geometry changes and the position of this instantaneous center of rotation moves.

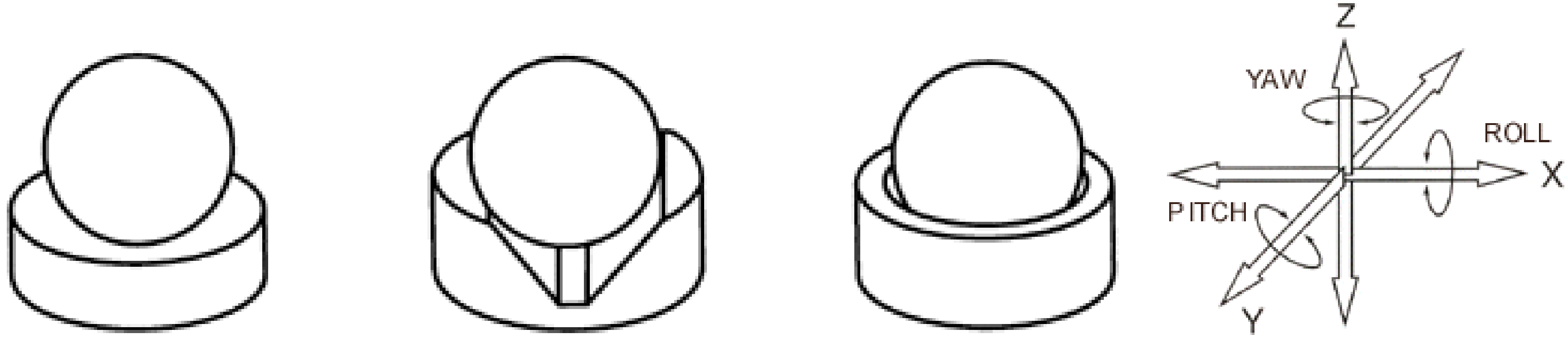


http://kmoddl.library.cornell.edu/resources.php?id=125

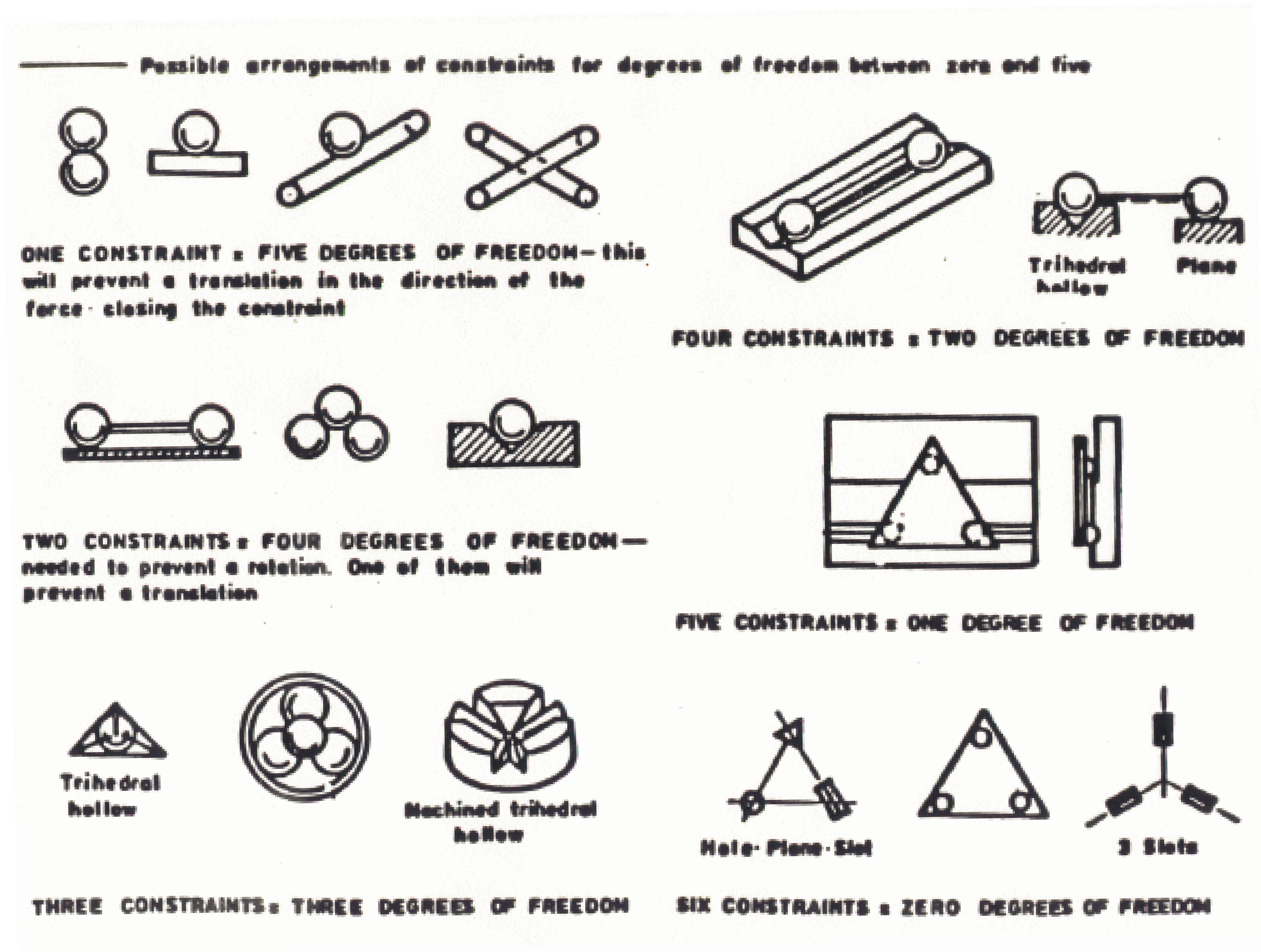
http://pergatory.mit.edu/2.007/lectures/2002/Lectures/Topic_04_Linkages.pdf

Use of balls

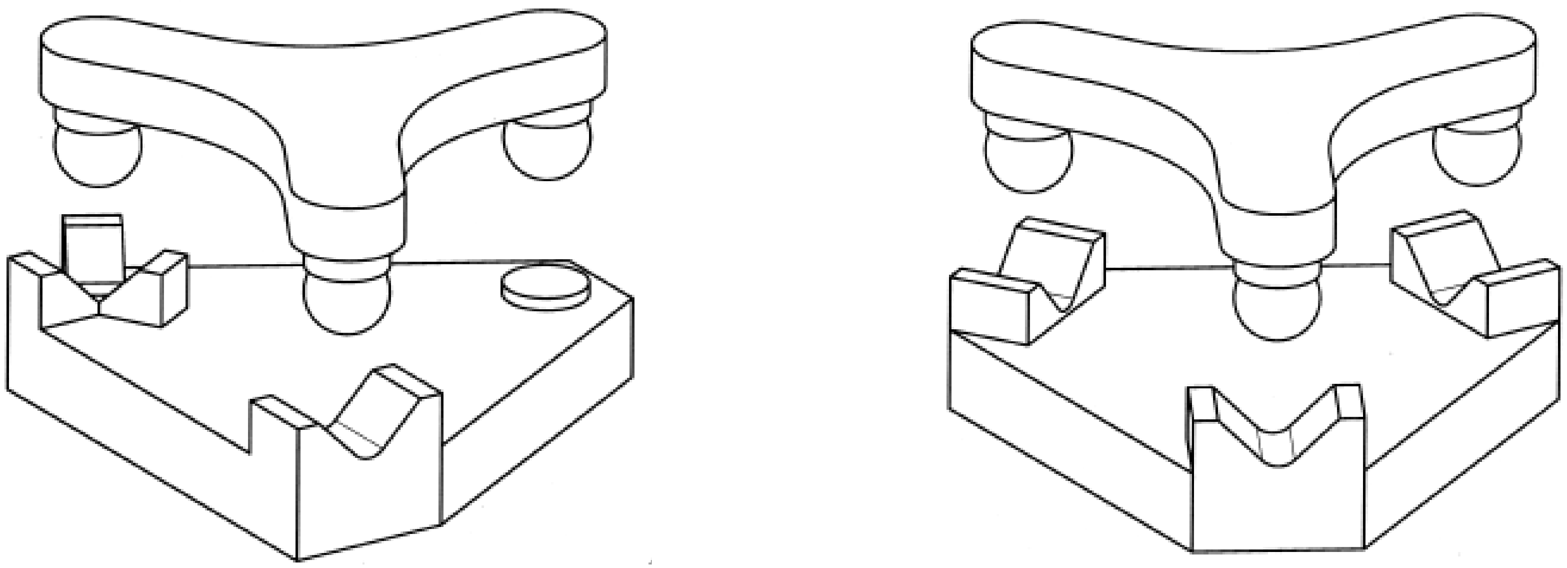
- Use symmetry of balls
- Material: Stainless steel, tungsten carbide, silicon nitride, diamond
- Constrain position in 1, 2, or 3 DoF



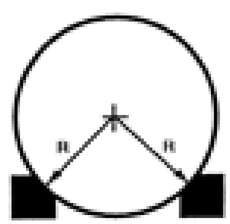
- Always leaves rotation about 3 axes about center of curvature, *If the ball is smooth*


Possible arrangements of constraints for degrees of freedom between zero and five
ONE CONSTRAINT = FIVE DEGREES OF FREEDOM—this will prevent a translation in the direction of the force closing the constraint
Trihedral hollow
Plane
FOUR CONSTRAINTS = TWO DEGREES OF FREEDOM
TWO CONSTRAINTS = FOUR DEGREES OF FREEDOM—needed to prevent a rotation. One of them will prevent a translation
FIVE CONSTRAINTS = ONE DEGREE OF FREEDOM
Trihedral hollow
Machined trihedral hollow
Hole-Plane-Slot
3 Slots
THREE CONSTRAINTS = THREE DEGREES OF FREEDOM
SIX CONSTRAINTS = ZERO DEGREES OF FREEDOM

Kinematic interface



(a)



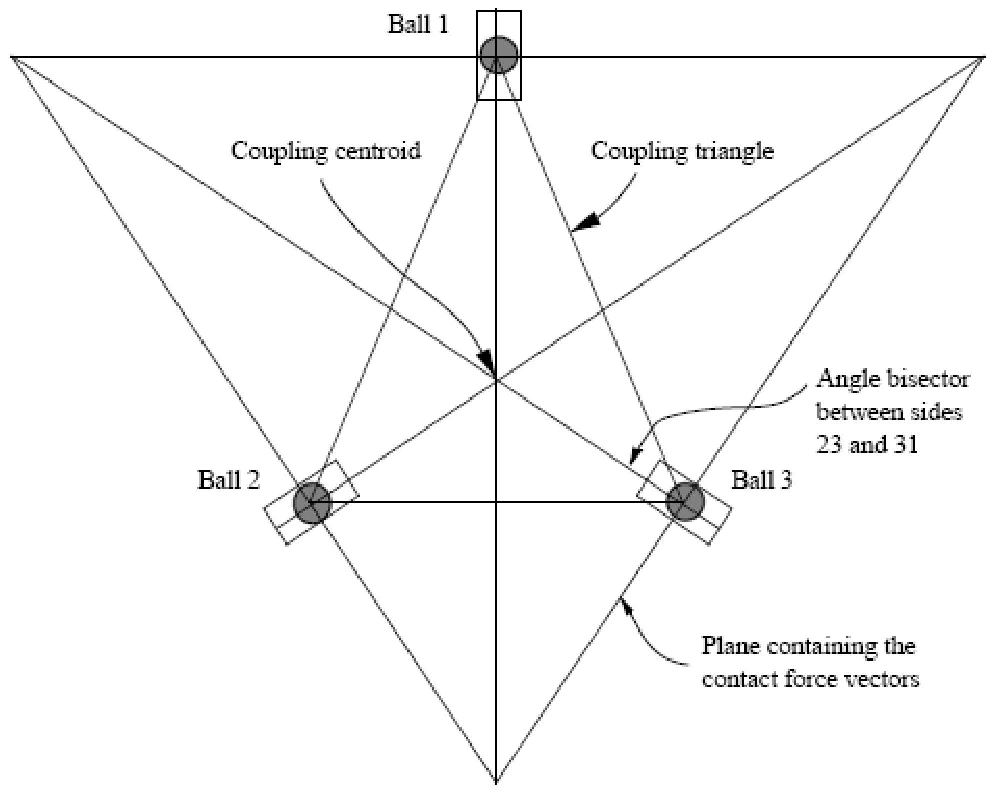
split Vee Block with
a large sphere

small spherical buttons
of a large spherical radius
and split Vee Block

(Hale and Slocum 2001)

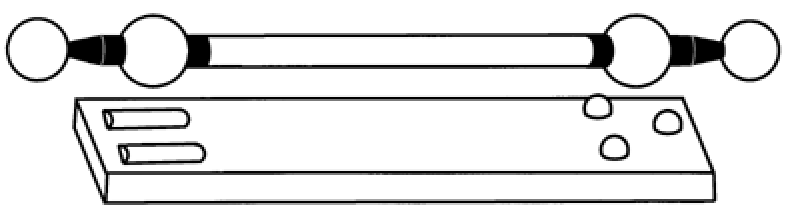
3 V geometry

- **Ideally, the normals to the contact planes should bisect the coupling triangle's angles:**



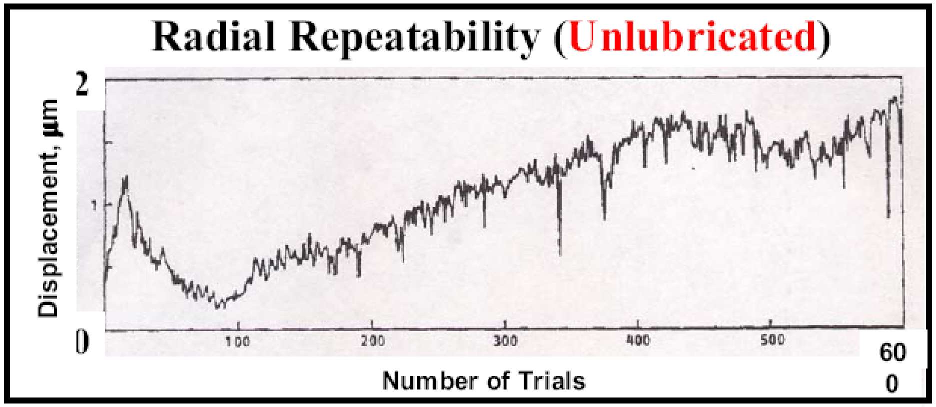
(Slocum 1994)

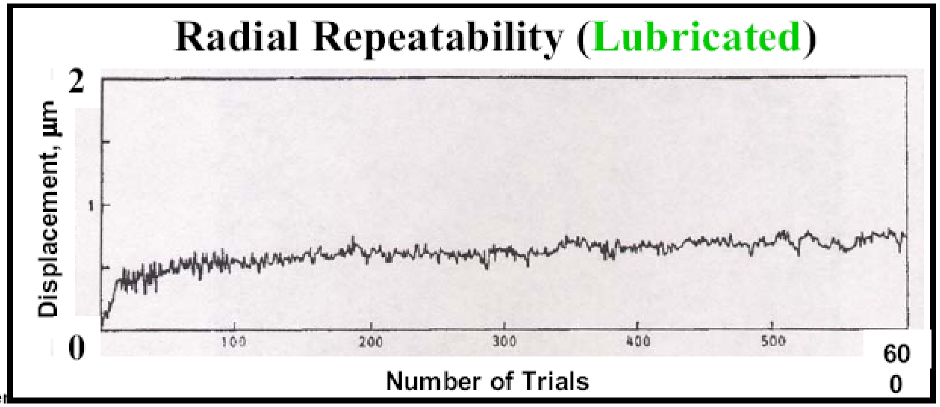
Kinematic hardware



Kinematic location

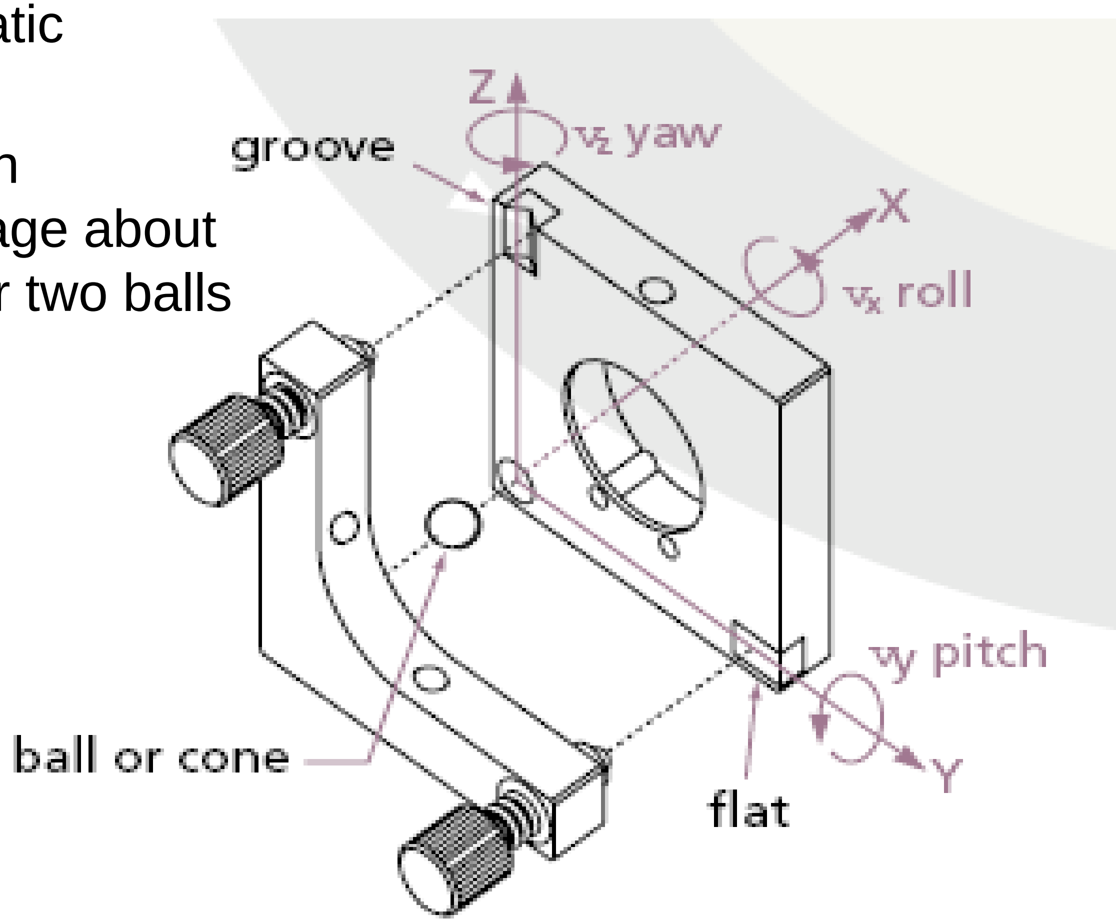
- Since the point contacts are well defined, the location is repeatable to sub-micron.
- Depends on friction, surface finish, loads.





Application of kinematic constraint for precision motion

- For three balls fixed, kinematic constraint
- Move one ball at a time (with micrometer) to rotate the stage about the axis defined by the other two balls
- Very stable
- Smooth motion



(Not shown, springs that hold this together)

Application of kinematic concepts for motion control

5 DoFs constrained using kinematic principles

Remaining DoF is used for the motion

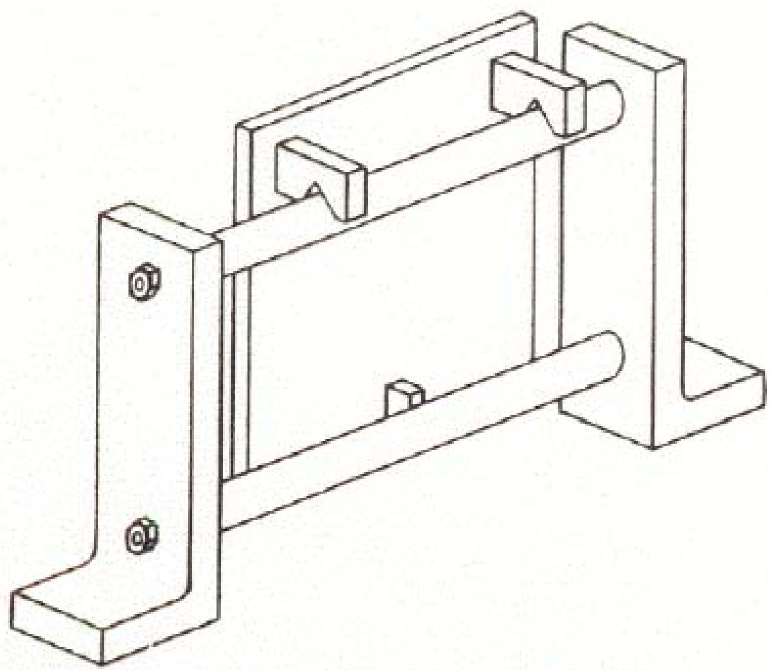
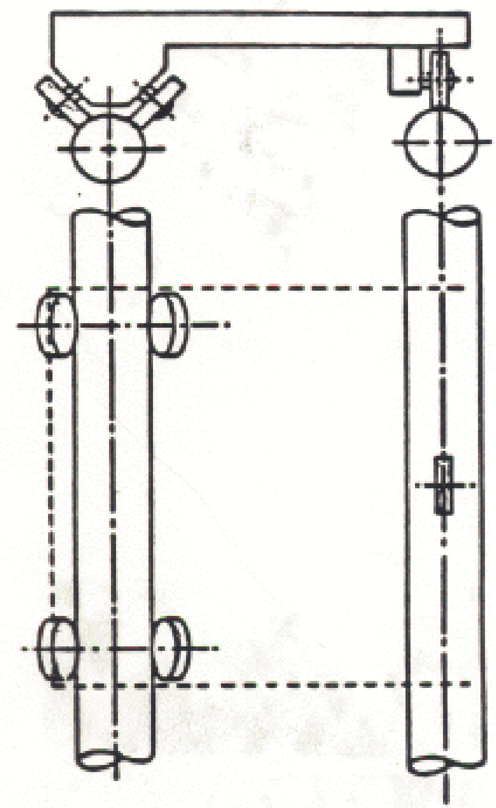


FIG. 6.2.—Kinematic design employing cylindrical surfaces as guides. Such surfaces can be accurately made.



Direction of V and preload

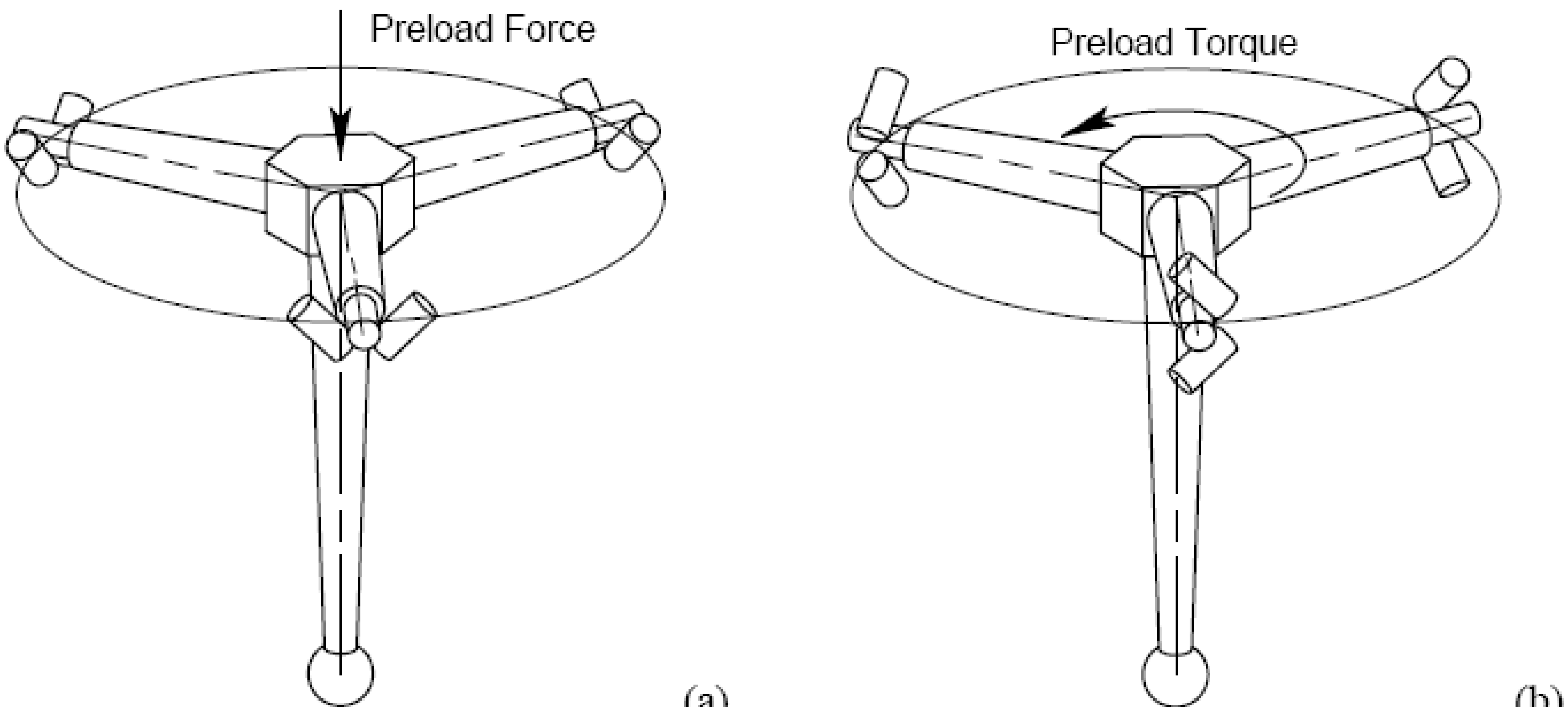


Figure 6-7 In (a), the moment required to unseat one vee while pivoting about the other two vees is a factor of two less than the moment required to unseat two vees while pivoting about the third vee. In (b), a moment applied about any axis in the plane of the vees produces equal reaction at all vees.

Problems with point and line contact

Nominally, the contact area is **zero** for a point or line

Really, the contact area comes from deformations and depends on the geometry and material properties.

More force causes more deformation which increases the contact area.

Non-point contact = not purely kinematic

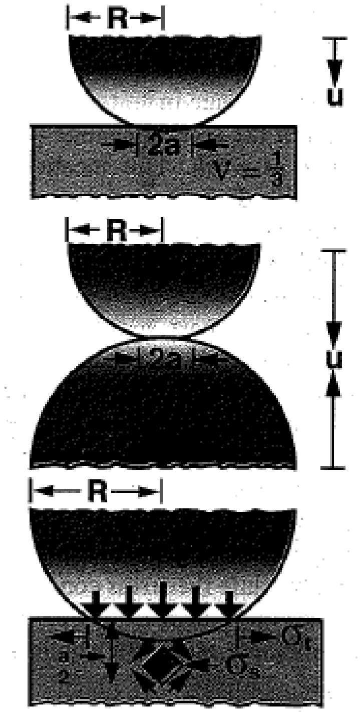
Stiffness = Force required for displacement is very low for the unloaded case. and very nonlinear. Preload is required.

Increased preloading makes stiffer, more stable interface in normal direction

But:

- Stress = Force/Area is very high and can damage the materials
- Tangential effects due to friction can be large

Contact stress



$$a = 0.7\left(\frac{F\,R}{E}\right)^{\frac{1}{3}}, \quad u = 1.0\left(\frac{F^2}{E^2 R}\right)^{\frac{1}{3}} \quad \nu = \tfrac{1}{3}$$

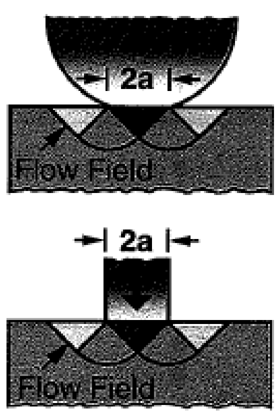
$$a = \left(\frac{3}{4}\,\frac{F}{E^*}\,\frac{R_1 R_2}{(R_1+R_2)}\right)^{\frac{1}{3}}$$

$$u = \left(\frac{9}{16}\,\frac{F^2}{(E^*)^2}\,\frac{(R_1+R_2)}{R_1 R_2}\right)^{\frac{1}{3}}$$

$$(\sigma_c)_{max} = \frac{3F}{2\pi a^2}$$

$$(\sigma_s)_{max} = \frac{F}{2\pi a^2}$$

$$(\sigma_t)_{max} = \frac{F}{6\pi a^2}$$



R_1, R_2 = Radii of spheres (m)
E_1, E_2 = Modulii of spheres (N/m^2)
ν_1, ν_2 = Poisson's ratios
F = Load (N)
a = Radius of contact (m)
u = Displacement (m)
σ = Stresses (N/m^2)
σ_y = Yield stress (N/m^2)

$$E^* = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)^{-1}$$

$$\frac{F}{\pi a^2} = 3\,\sigma_y$$

Stiffness

$$k = \frac{F}{u} \cong \left(E^2 RF\right)^{1/3}$$

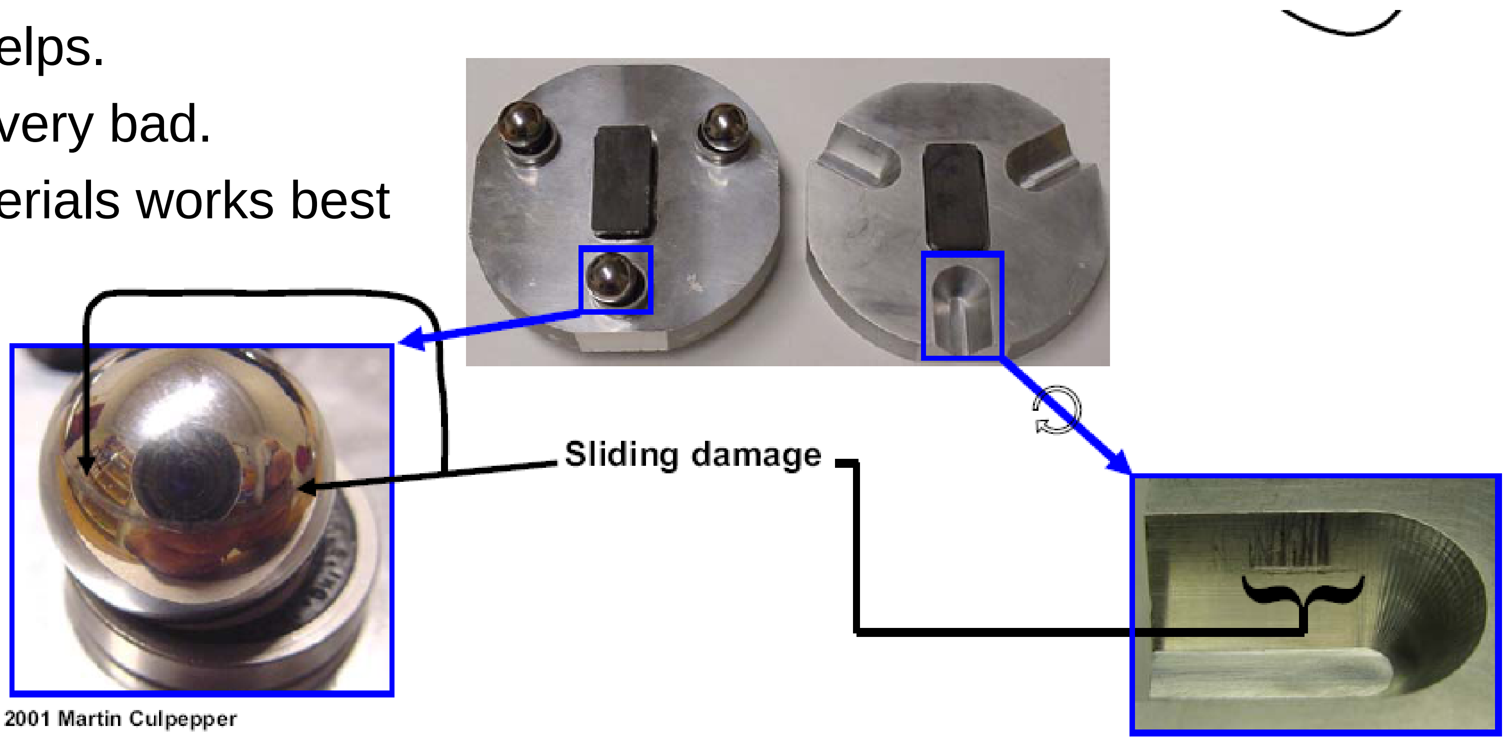
Stress

$$\left(\sigma_c\right)_{\max} \cong \left(\frac{E^2 F}{R^2}\right)^{1/3} \cong \frac{k}{R}$$

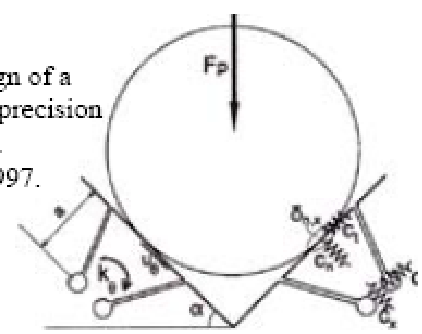
$$\tau_{\max} \cong \frac{\left(\sigma_c\right)_{\max}}{3}$$

Effect of contact stress

- Contact stress can cause fretting of the surface
- Lubrication helps.
- Aluminum is very bad.
- Different materials works best



Picture from:
Schouten, et. al., "Design of a kinematic coupling for precision applications", Precision Engineering, vol. 20, 1997.



Ball in V-Groove with Elastic Hinges

Repeatability as function of geometry

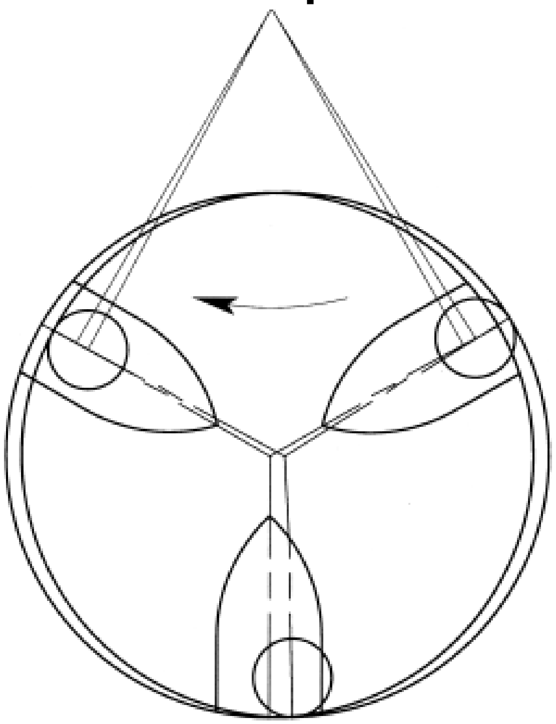
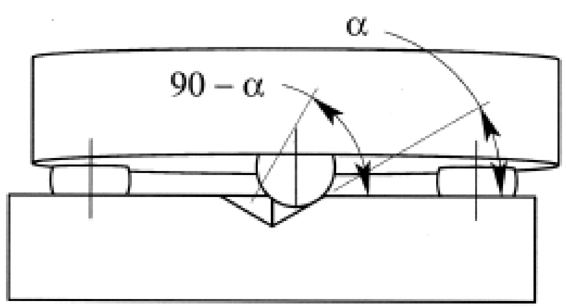


Fig. 5. The three-vee coupling slides on five constraints producing rotation about an instant center shown in the top view and also about an axis through the two seated balls.



Non-repeatability per ball/plane interface is

$$\rho \equiv \frac{f}{k} \approx \mu \left(\frac{2}{3R}\right)^{1/3} \left(\frac{P}{E}\right)^{2/3}$$

μ = friction coefficient
R = ball radius
P = load
E = Young's modulus

For the system (mostly horizontal):

$$\rho \cong \frac{\mu P}{18k\ sin^2\alpha\ \cos\alpha}\left(2\sqrt{3} + \cos\alpha + \sin 2\alpha\right)$$

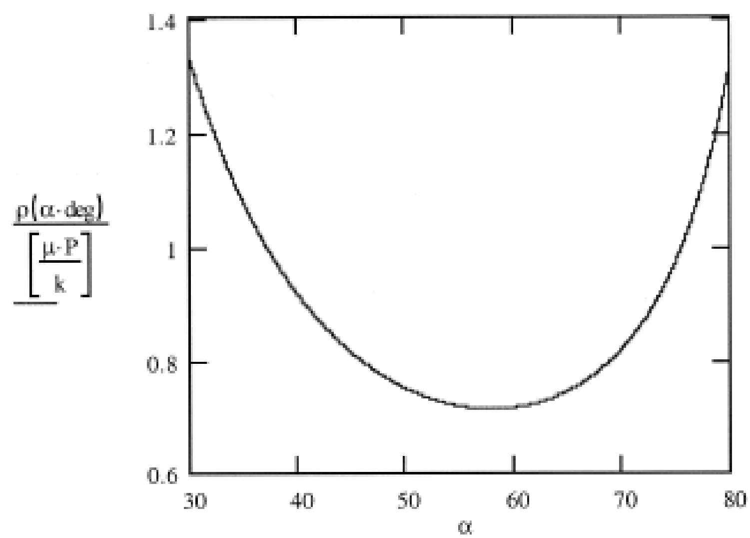
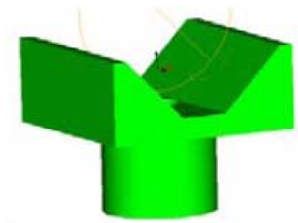


Fig. 6. The effect of the vee angle α on the repeatability of the symmetric three-vee coupling has a minimum of 0.71 at 58°.

Use geometry to reduce contact stress

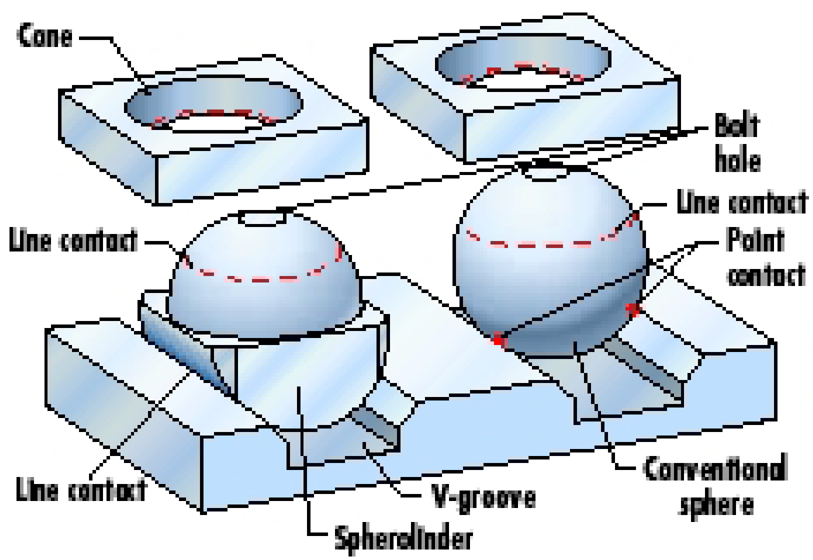
“Canoe ball”
1 meter ROC

(Baltek)

“Spherolinder”

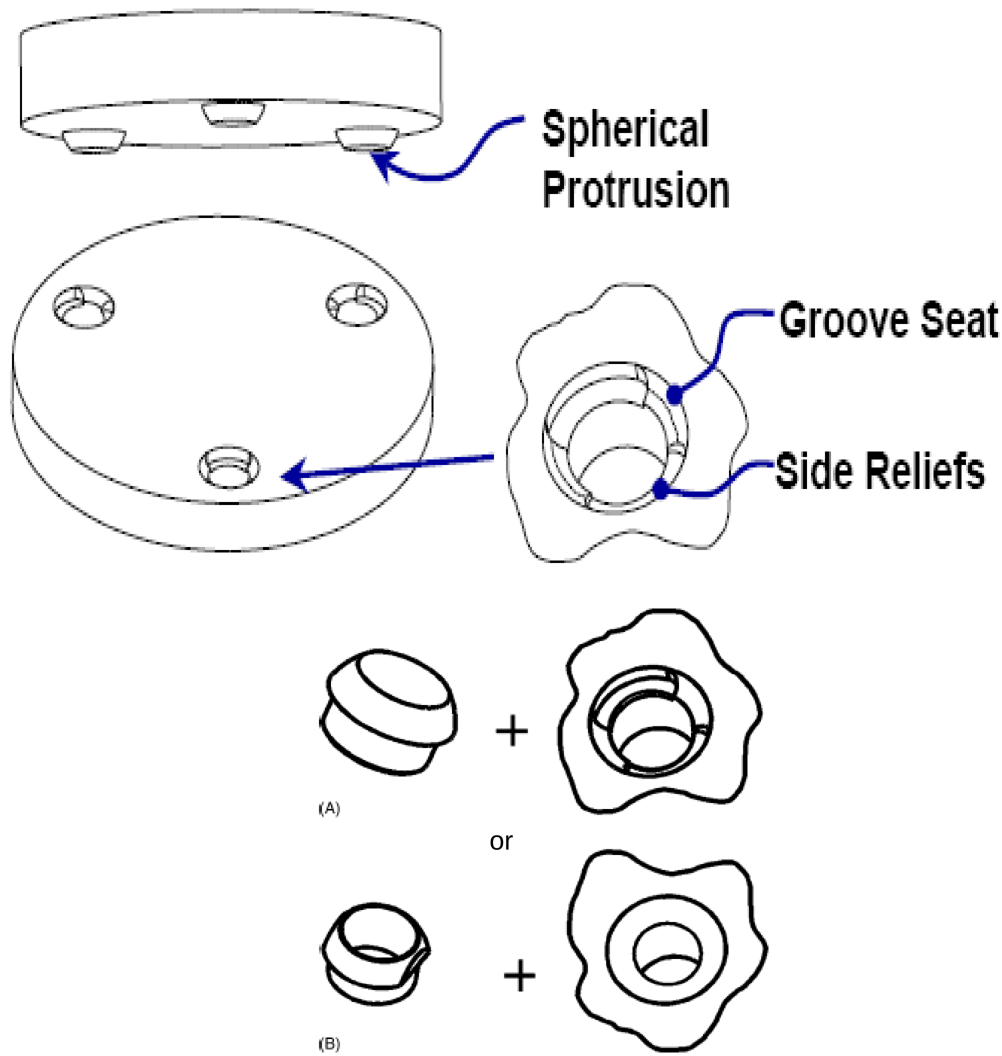
G2 Engineering



J. H. Burge
University of Arizona

Semi-kinematic design

L. Hale US Patent #6,065,898



Cylinders in V's

- Easy to make to high accuracy
- Leaves axial motion, clocking rotation unconstrained

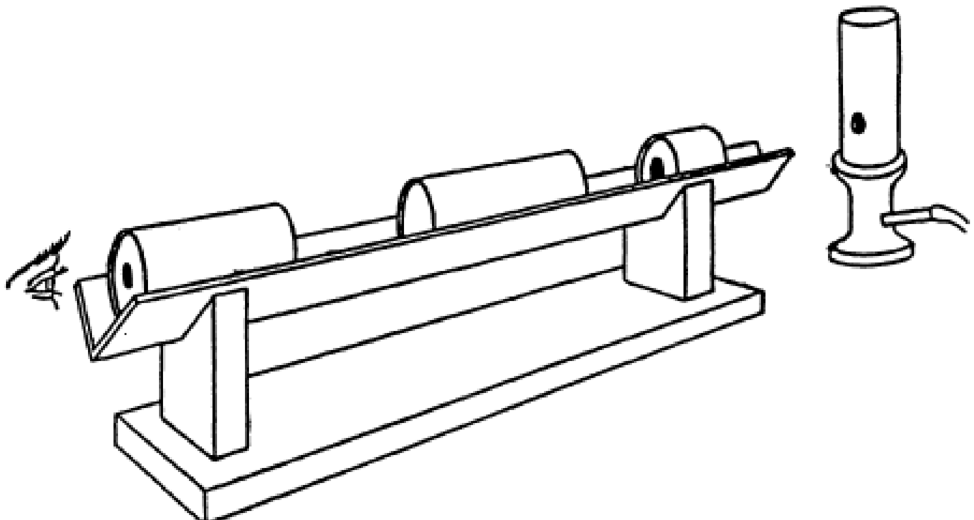
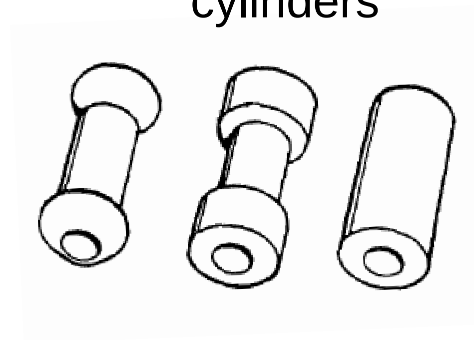


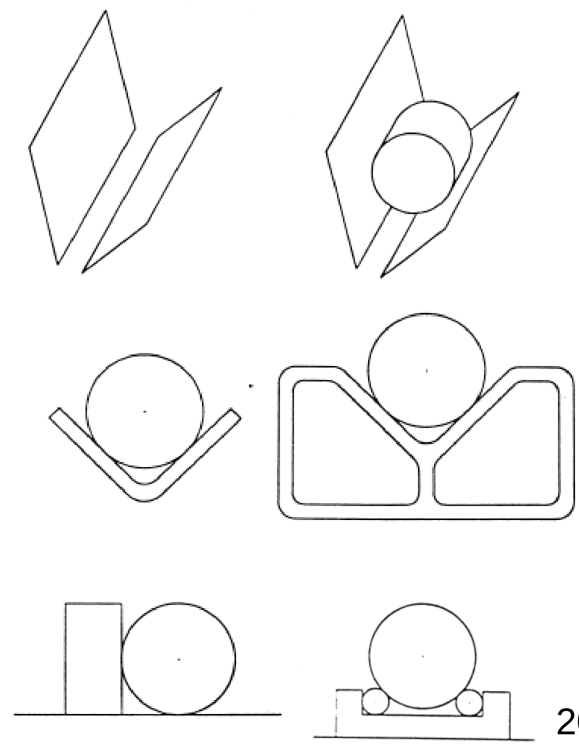
FIG. 59.—V Bench for Lenses of Uniform Diameter.

Fig 35 - From *Treatise on Practical Light*, 1911

cylinders



V's



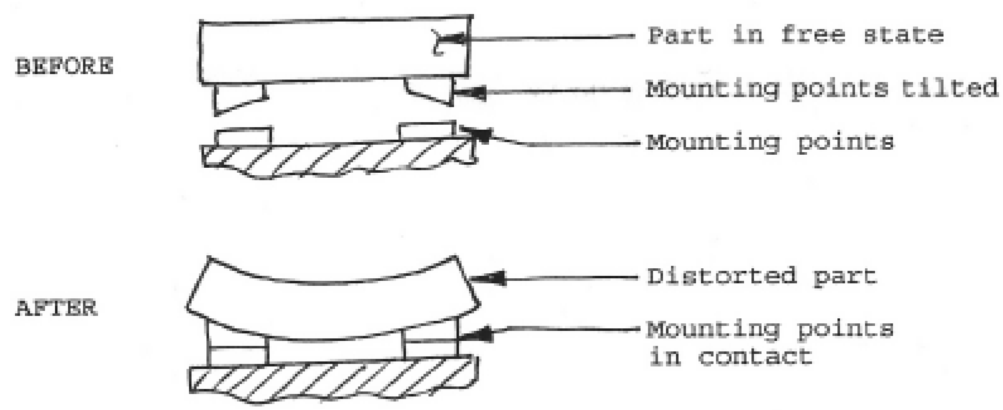
"Cylinders in Vs—An Optomechanical Methodology," Douglas S. Goodman, SPIE Proceedings 3132 *Optomechanical Design and Precision Instruments*, Santa Diego, CA, July, 1997

"More Cylinders in Vs," Douglas S. Goodman, SPIE Proceedings 4198, *Optomechanical Engineering*, Boston, MA, November, 2000

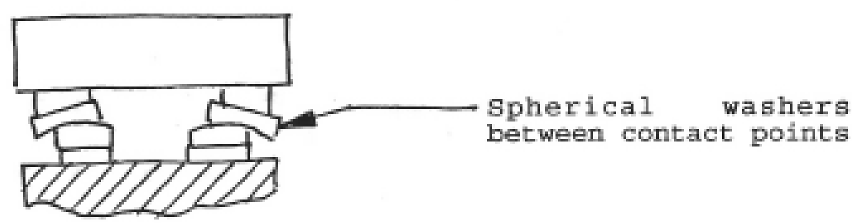
- A major problem with kinematic design is high stress in contact areas. Hertz contact stress theory is used to evaluate this problem.

- If stress is too high, use kinematic principles but replace point contacts with small area contacts. This is known as semi-kinematic design.

- A potential problem with semi-kinematic design is distortion of the part due to non-coplanarity of the mounting points.



- This problem is alleviated by making points very coplanar by introducing rotary compliance in the mounting points.



- Assembly procedures for semi-kinematic design are usually critical if part distortion is to be avoided.

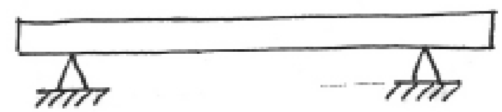
(Vukobratovich p. 76)

- Kinematic mounted parts may have excessive self-weight induced deflection between contact points. This requires additional support which nullifies kinematic design.

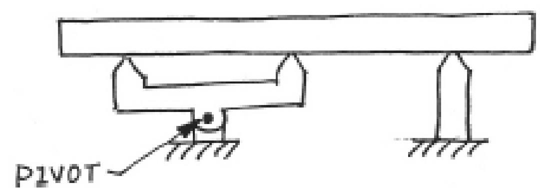
- One solution to the multi-point problem is a whiffle tree. This is a cascaded system of support where each level of support is kinematic.

- Consider a simple beam:

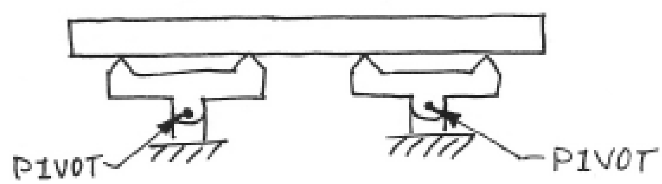
2 point support



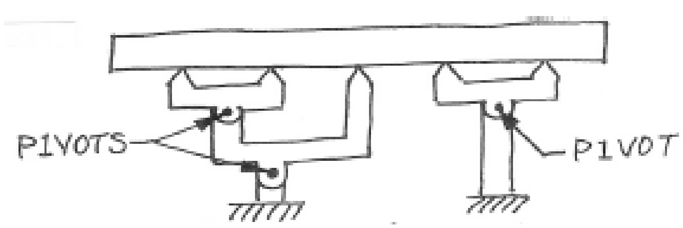
3 point support



4 point support



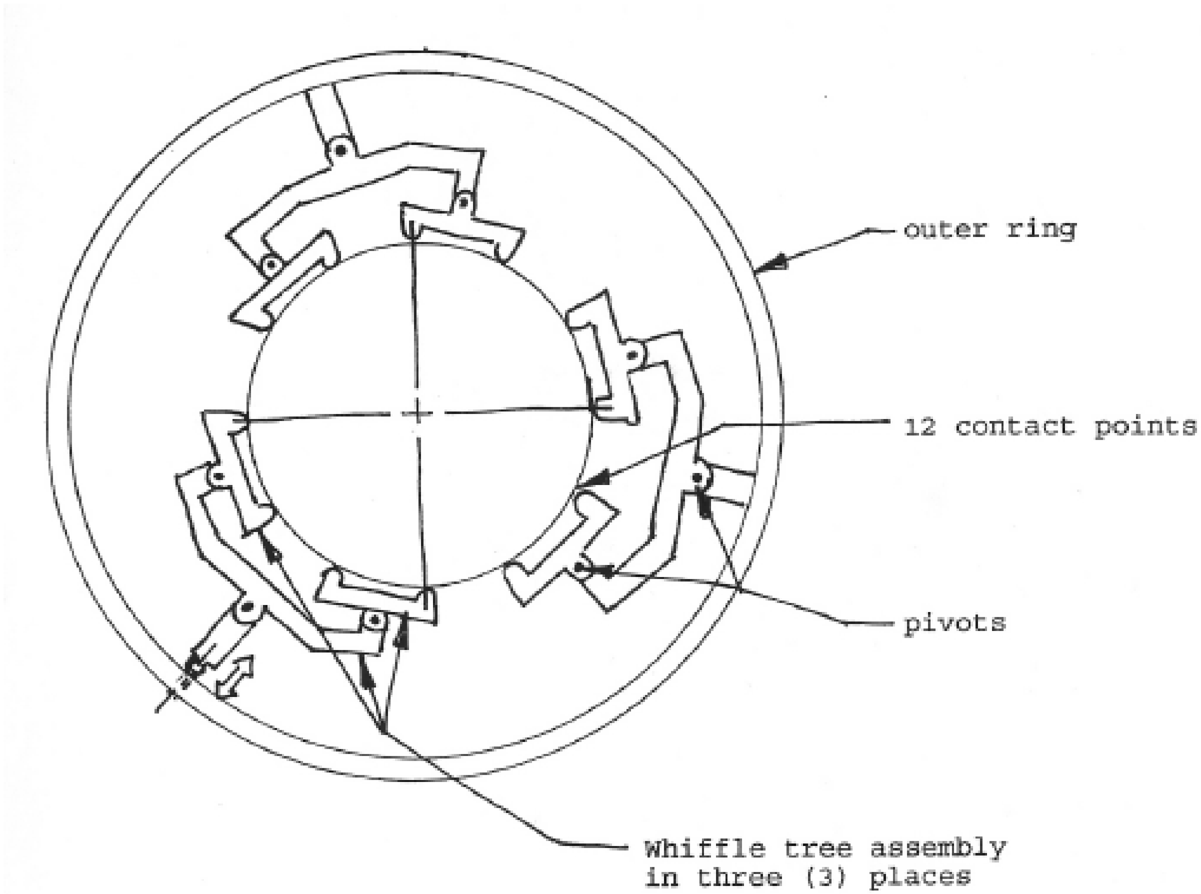
5 point support



NOTE: Pivots insure equal loads on each support.

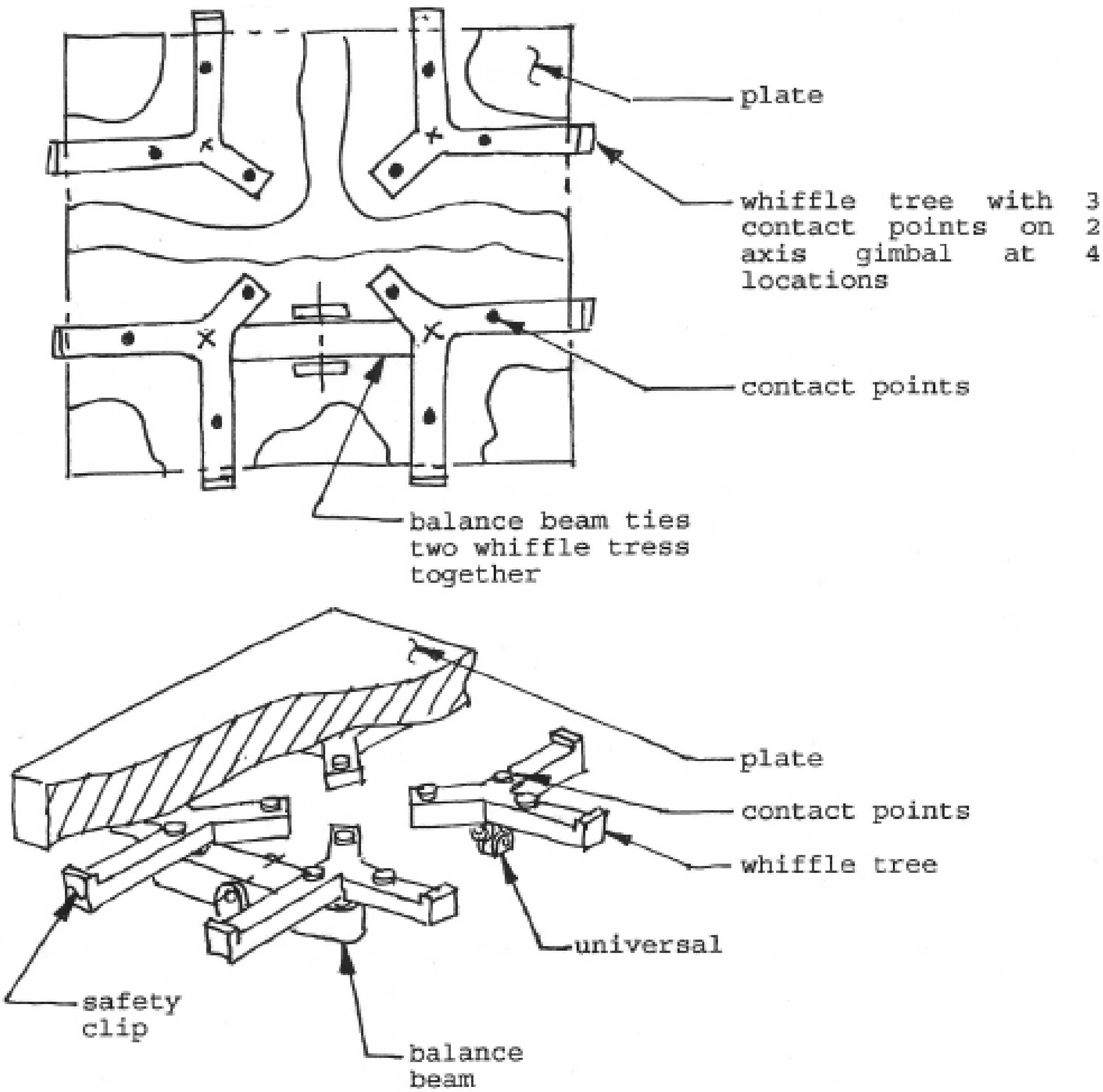
(Vukobratovich p. 78)

- The same approach can be used to support a disk around its edge.



- This approach gives an even distribution of forces yet preserves a three (3) point kinematic location system.

- The same approach also “floats” plates on whiffle trees.



(Vukobratovich p. 80)