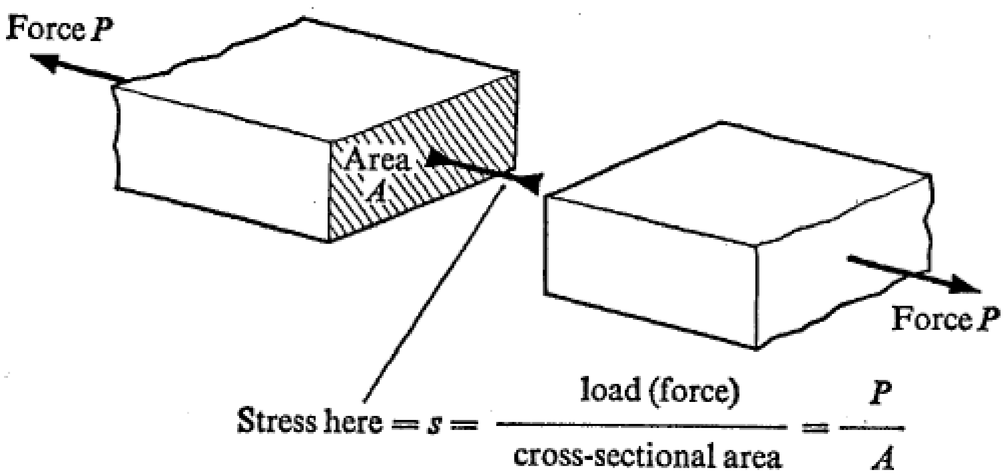
Stress/strain

Normal stress and strain

A normal stress σ, results when a member is subjected to an axial load applied through the centroid of the cross section. The average normal stress in the member is obtained by dividing the magnitude of the resultant internal force F by the cross sectional area A. Normal stress is

$$\sigma_{AVG} = \frac{Force}{Area} = \frac{P}{A}$$



Inasmuch as the stress σ acts in a direction perpendicular to the cut surface, it is referred to as a **NORMAL** stress. Thus, normal stressed may be either tensile or compressive. Our sign convention for normal stresses is:

Tensile stresses are positive (+)
Compressive stresses are negative (-)

Units of stress:

psi (or ksi or Msi)
Pa = N/m^2 MPa = N/mm^2
1 psi ~= 7000 Pa

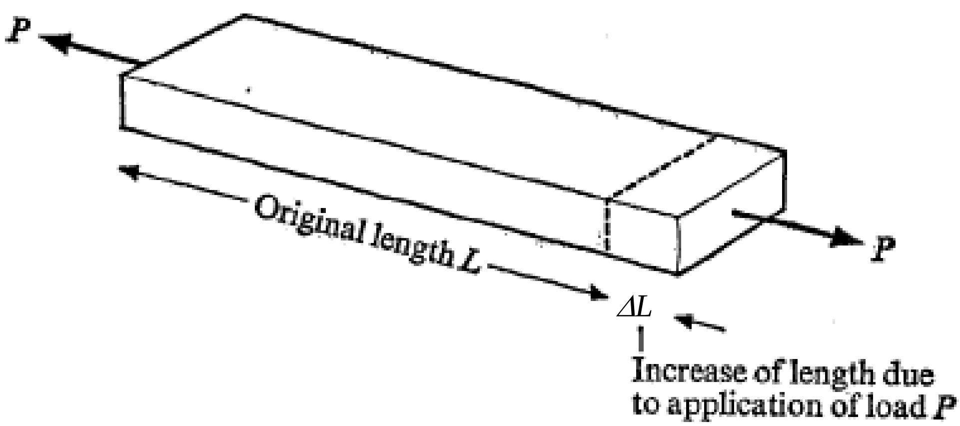
Deformation of Axial Members

For a prismatic bar of length L in tension by axial force F we define the stress:

$$\sigma = \frac{F}{A}$$

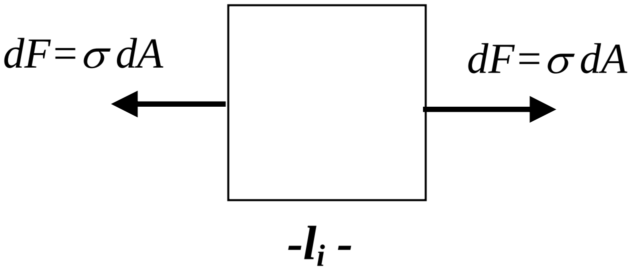
Now, define strain ε as normalized elongation:

$$\varepsilon = \frac{\Delta L}{L}$$



$$\text{Strain} = \frac{\text{increase of length}}{\text{original length}} = \frac{\Delta L}{L} = \varepsilon$$

Look at differential element



$$\varepsilon = \frac{\Delta l}{l}$$

$$L = \sum l_i$$

$$\Delta L = \sum \Delta l_i$$

For homogenous material and small deflections,

stress is proportional to strain

$$\sigma = \frac{dF}{dA} \cong \frac{F}{A}$$

$$\varepsilon E = \sigma$$

$$\varepsilon \triangleq \frac{\Delta L}{L} = \frac{\Delta l_i}{l_i}$$

E = Young's modulus or modulus of elasticity

E ~ 10,000,000 psi (10 Msi ~ 70 GPa) for aluminum

E ~ 10,000,000 psi (10 Msi ~ 70 GPa) for glass

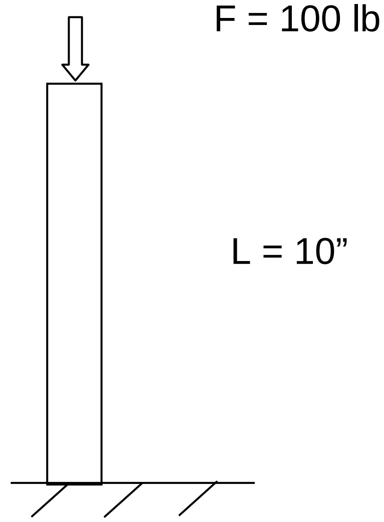
E ~ 30,000,000 psi (30 Msi ~200 GPa) for steel

Combining and solving for displacement, we obtain the following equation for the elongation (*deformation*) of the bar.

$$\Delta L = \frac{FL}{EA}$$

The above equation shows that deformation is proportional to the load and the length and inversely proportional to the cross sectional area and the elastic modulus of the material.

Consider the following example:



L = 10”   A = 1 in^2
W = 100 lbs
E = 10,000,000 psi (aluminum)

$$\Delta L = \frac{(100lb)(10'')}{(10Msi)(1in^2)} = 100\mu in = 0.0001''$$

or

$$\Delta L \cong \frac{(450N)(250mm)}{(70GPa)(625mm^2)} \cong 2.5\mu m$$

The product **EA** is known as the **axial rigidity** of the bar.

We can see that a bar is tension is analogous to an axially loaded spring. Recall for a spring F = K δ, where K is the spring stiffness. Likewise, the above equation can be expressed as follows:

$$F = \frac{AE}{L}\Delta L = K\Delta L$$

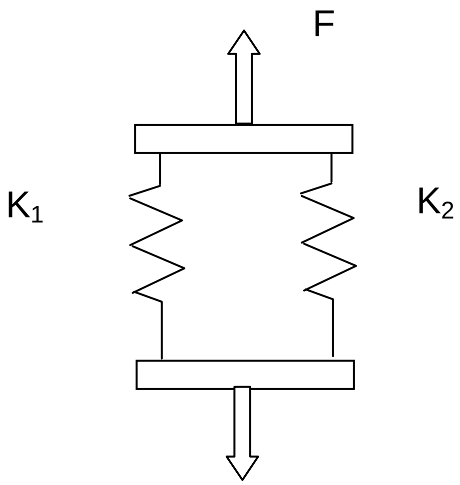
The quantity AE/L is the **stiffness K** of an axially loaded bar and is defined at the force required to produce a unit deflection.

In an analogous manner, the **compliance C** is defined as the deformation due to a unit load. Thus the compliance of a axially loaded bar is:

$$C = \frac{1}{K} = \frac{L}{EA}$$

Combining members

<u>Adding in parallel adds stiffness</u>



$$\Delta L = \frac{F_1}{K_1} = \frac{F_2}{K_2}$$

$$F = F_1 + F_2$$

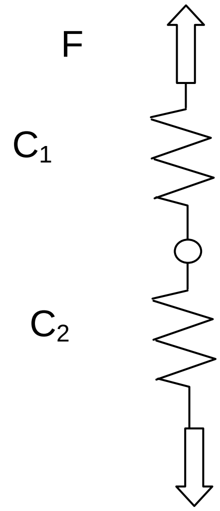
$$F = K_1 \Delta L + K_2 \Delta L$$

$$F = \left(K_1 + K_2\right) \Delta L$$

$$F = K_e \Delta L$$

$$K_e = K_1 + K_2$$

<u>Adding in serial adds compliance</u>



$$\Delta L = \Delta L_1 + \Delta L_2$$

$$F = F_1 = F_2$$

$$\Delta L_1 = C_1 F$$

$$\Delta L_2 = C_2 F$$

$$\Delta L = C_1 F + C_2 F$$

$$= \left(C_1 + C_2\right) F$$

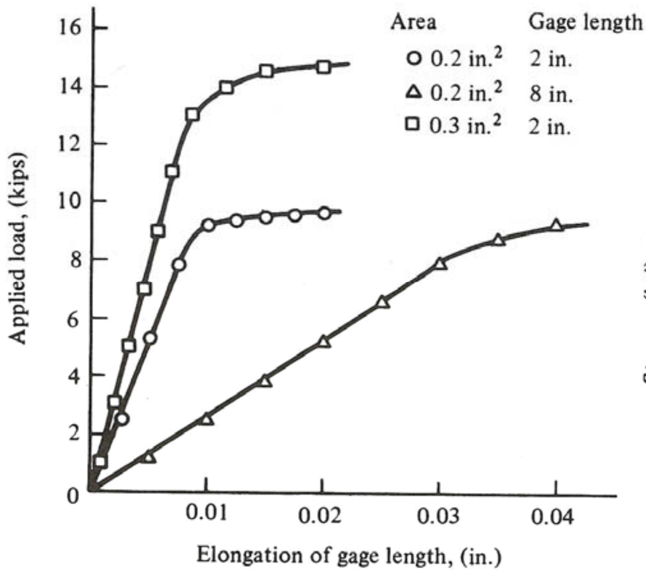
$$= C_e F$$

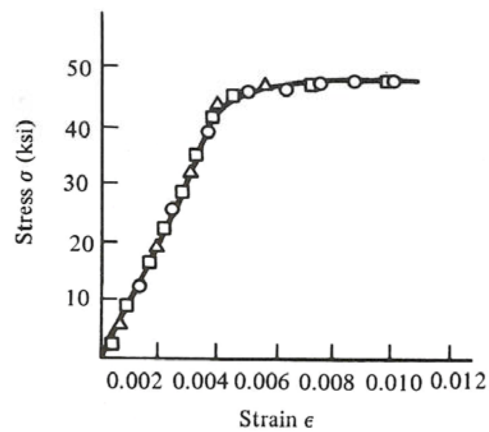
$$C_e = C_1 + C_2$$

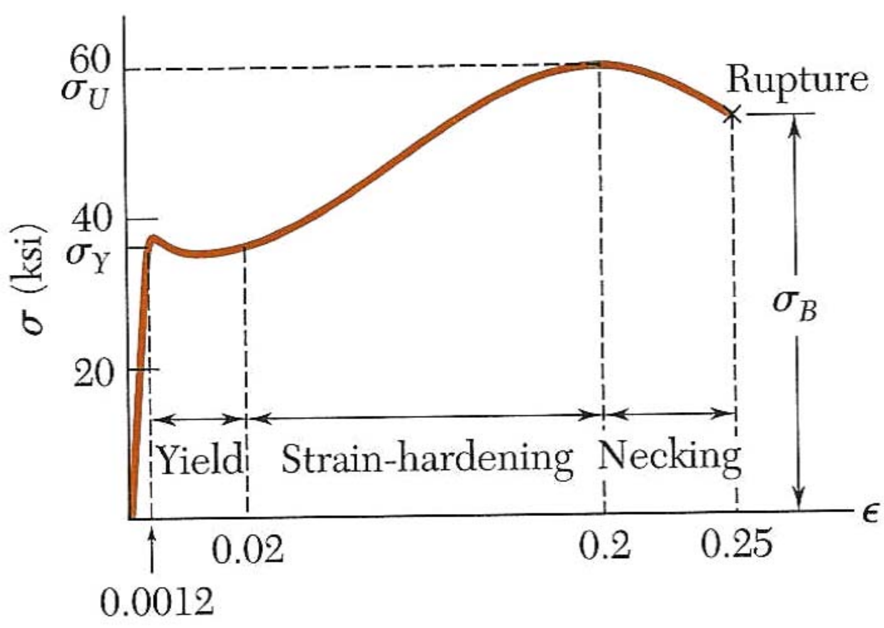
Materials

Load-deformation data

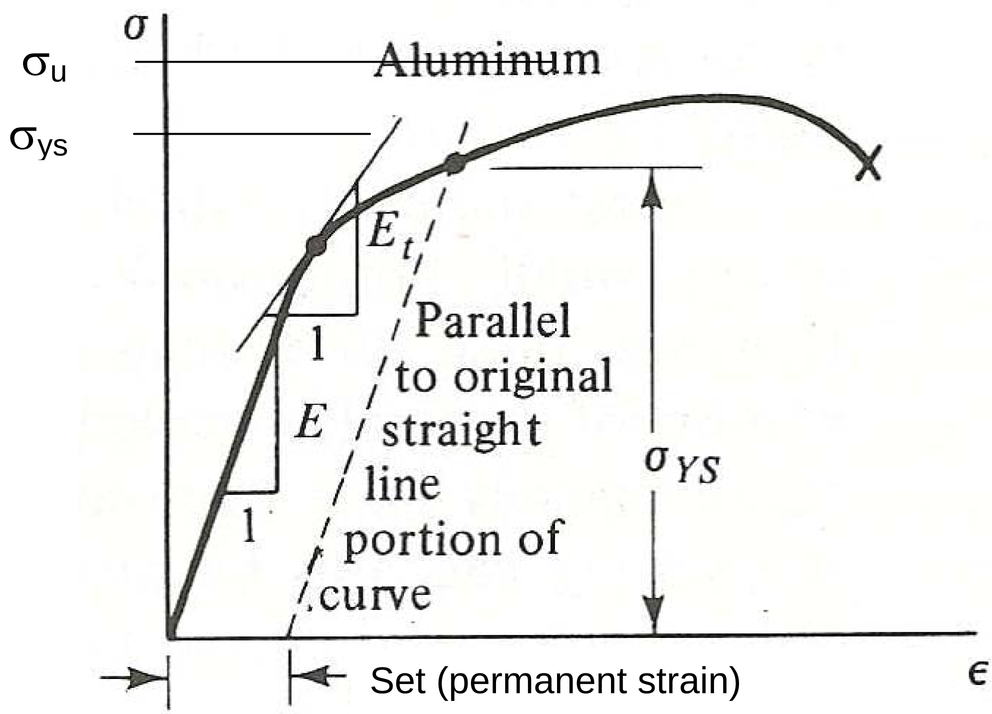
Stress-strain data







(a) Low-carbon steel



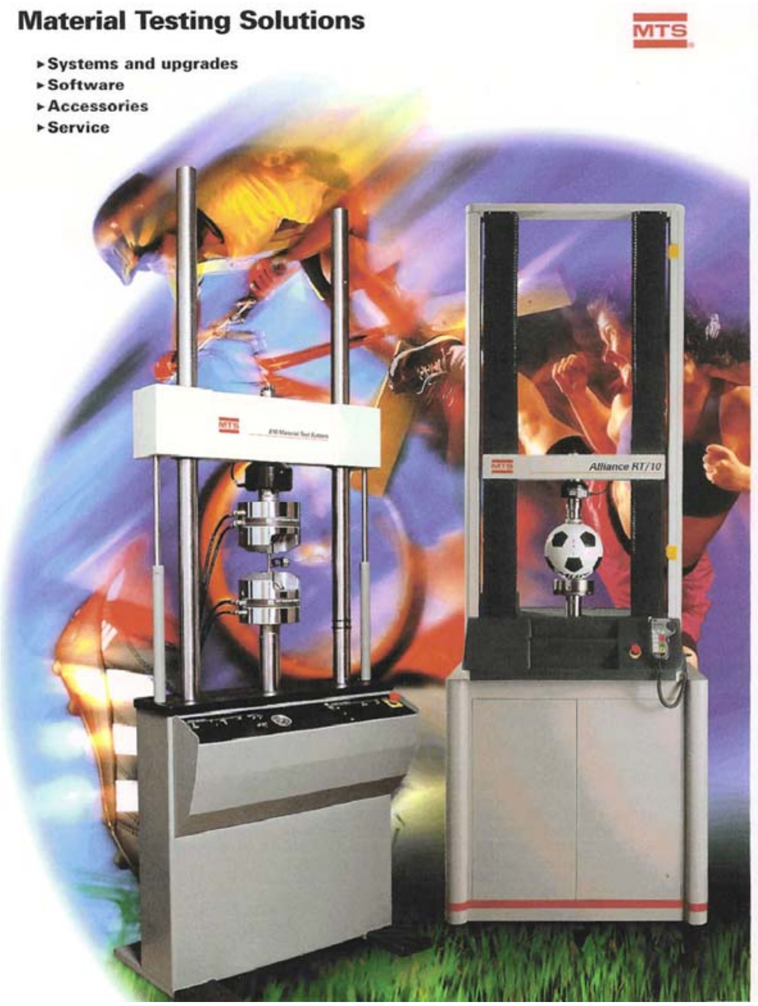
E=dσ/dε for small loads and small deflections

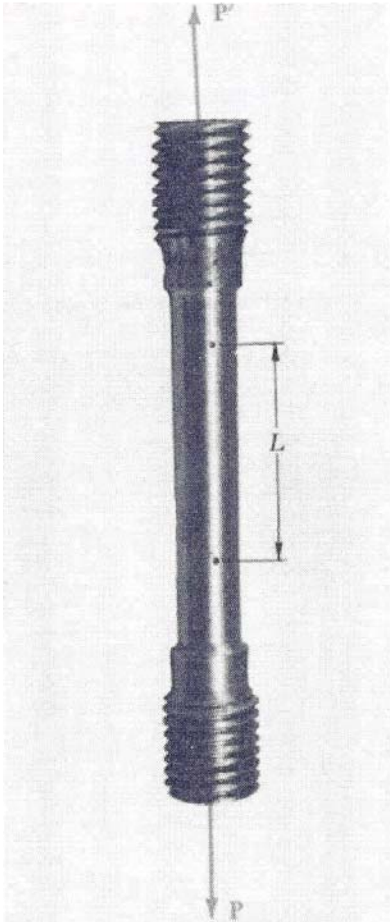
σ_{YS} ⇒ **Yield Strength** - The maximum stress that can be applied without exceeding a specified value of permanent strain (typically 0.2% = .002 in/in).

σ_{PEL} ⇒ **Precision elastic limit** or micro-yield strength - The maximum stress that can be applied without exceeding a permanent strain of 1 ppm or 0.0001%

σ_U ⇒ **Ultimate Strength** - The maximum stress the material can withstand (based on the original area).

Tensile Testing





One basic ingredient in the study of the mechanics of deformable bodies is the resistive properties of materials. These properties relate the stresses to the strains and can only be determined by experiment.

One of the simplest tests for determining mechanical properties of a material is the tensile test. In this test, a load is applied along the longitudinal axis of a circular test specimen. The applied load and the resulting elongation of the member are measured. In many cases, the process is repeated with increased load until the desired load levels are reached or the specimen breaks.

Load-deformation data obtained from tensile and/or compressive tests do not give a direct indication of the material behavior, because they depend on the specimen geometry.

However, using the relationships we previously discussed, loads and deformations may be converted to stresses and strains.

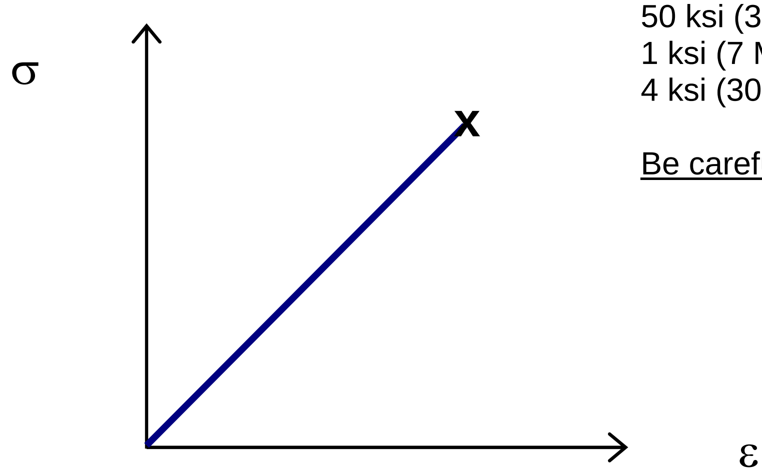
Strength of materials

(Defined as the stress which will cause the material to break.)

Material	Tensile strength p.s.i.	Tensile strength MN/m²
Metals		
STEELS		
Steel piano wire (very brittle)	450,000	3,100
High tensile engineering steel	225,000	1,550
Commercial mild steel	60,000	400
WROUGHT IRON		
Traditional	15,000–40,000	100–300
CAST IRON		
Traditional (very brittle)	10,000–20,000	70–140
Modern	20,000–40,000	140–300
OTHER METALS		
Aluminium: cast	10,000	70
wrought alloys	20,000–80,000	140–600
Copper	20,000	140
Brasses	18,000–60,000	120–400
Bronzes	15,000–80,000	100–600
Magnesium alloys	30,000–40,000	200–300
Titanium alloys	100,000–200,000	700–1,400

<u>Strength of glass:</u>

Depends on critical flaw size



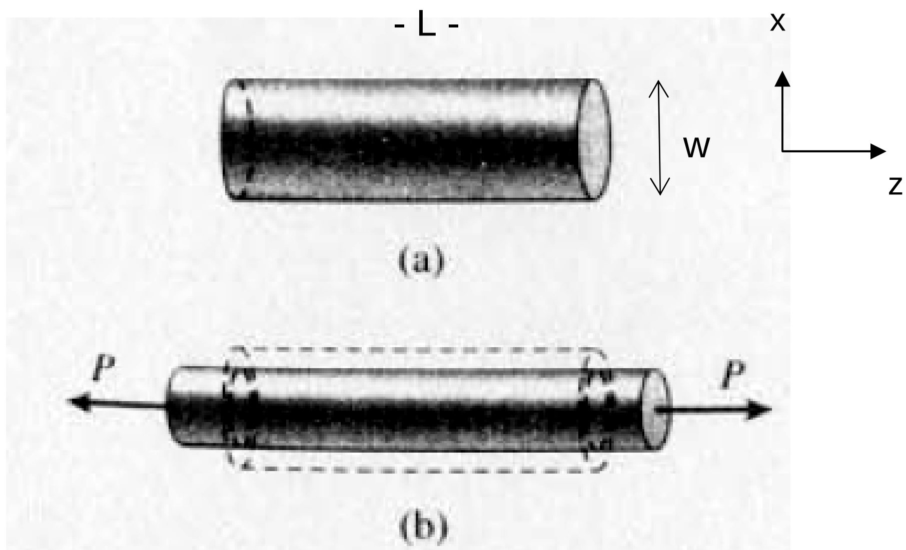
<u>Rule of thumb for glass strength</u>
50 ksi (350 MPa) compression
1 ksi (7 MPa) in tension
4 ksi (30 MPa) tensile (short duration)

<u>Be careful with this!</u>

Poisson's ratio – The ratio of lateral or transverse strain to the longitudinal strain.

Initial, unloaded state
Length L
Width w

Deformed state after loading
L' = L + ΔL
w' = w + Δw



Longitudinal strain = $\varepsilon_Z = \dfrac{\Delta L}{L}$

Transverse strain = $\varepsilon_x = \dfrac{\Delta w}{w}$

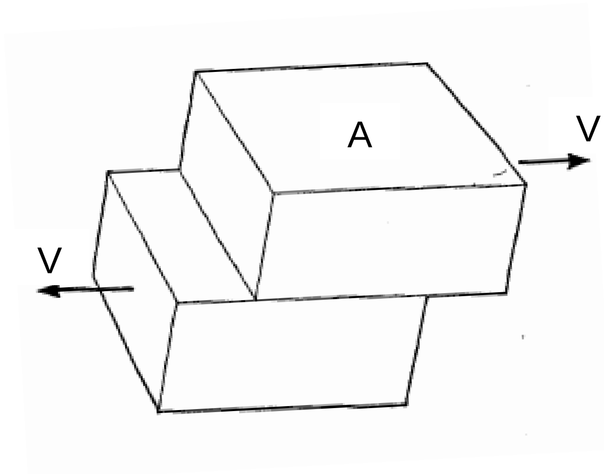
Poisson's ratio $$\nu = -\frac{\varepsilon_x}{\varepsilon_z}$$

Poisson's ratio for most materials ranges from 0.25 to 0.35.

Cork ⇒ $\nu \approx 0.0$
Steel ⇒ $\nu = 0.27 – 0.30$
Aluminum ⇒ $\nu = 0.33$
Rubber ⇒ $\nu \approx 0.5$ (limiting value for Poisson's ratio, volume is conserved)

Shear stress and strain

Shear force V
Force spread over area A



Shear stress = $\tau = \frac{V}{A}$

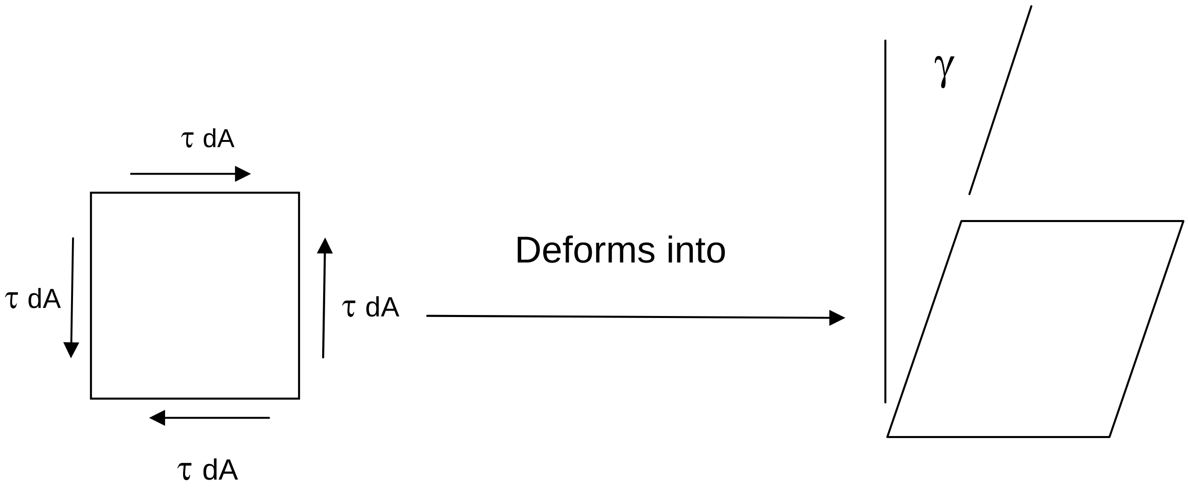
For a small element

$$\sigma = \frac{dF}{dA}$$

$$\tau = \frac{dV}{dA}$$

Units of psi or Pa (1 ksi =~ 7MPa=7 N/mm^2)

Shear strain



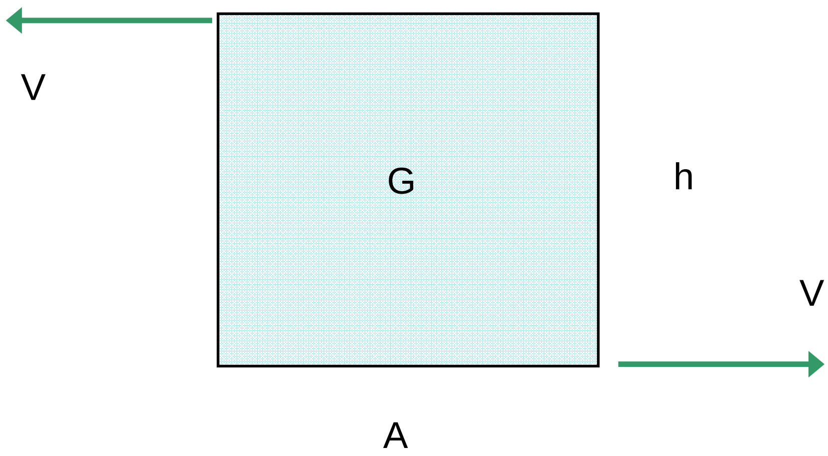
Shear strain $\gamma = \frac{\tau}{G}$

G = shear modulus or modulus of rigidity

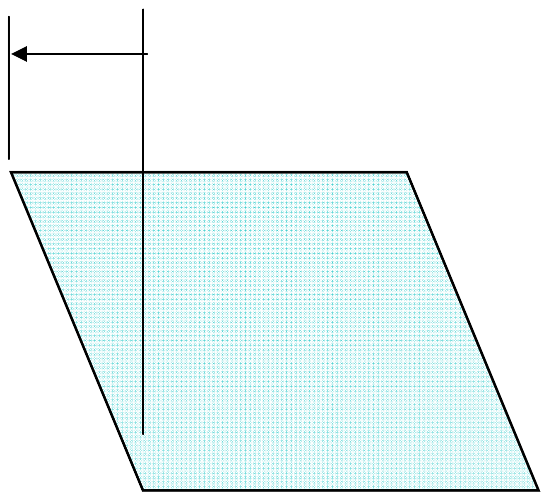
For linear, isotropic materials

$$G = \frac{E}{2(1+\nu)}$$

Shear stiffness



$$\delta x = \gamma h = \frac{\tau}{G} h = \frac{V/A}{G} h = \frac{Vh}{GA}$$



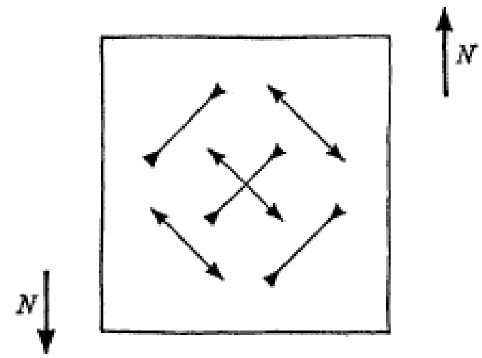


Figure 4. Shear will produce tension and compression stresses in directions at 45° to the plane of shearing.

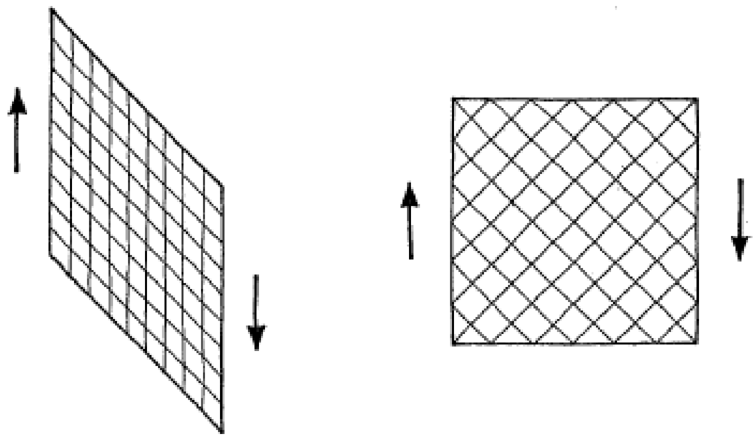


Figure 5. Thus a system like the one on the right is 'rigid' in shear, and systems like the one on the left are floppy.

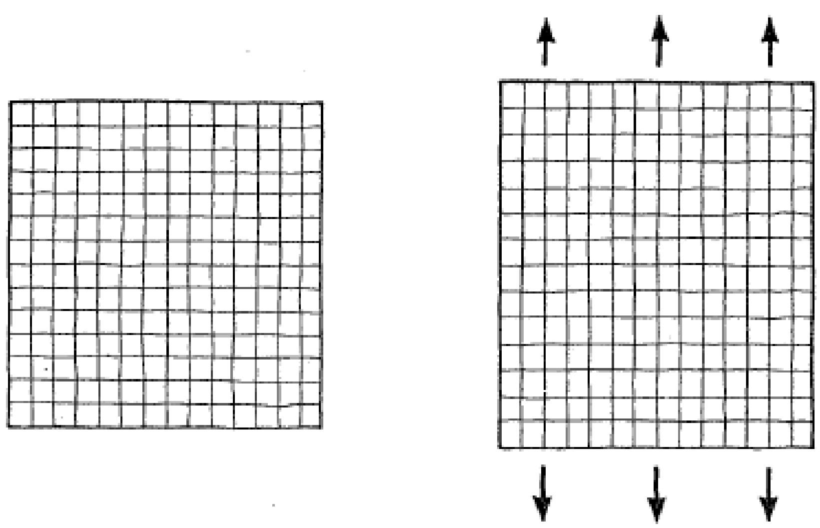
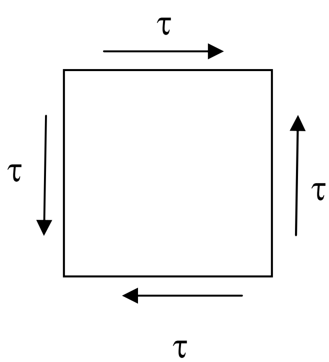


Figure 6. When cloth is pulled parallel to the warp or the weft threads, the 'material' is 'stiff' and the lateral contraction is quite small.

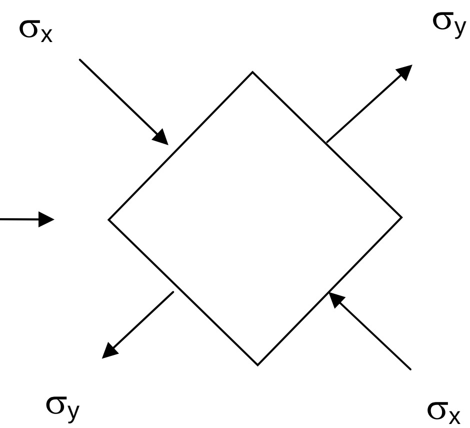
Shear and normal stress

Pure shear



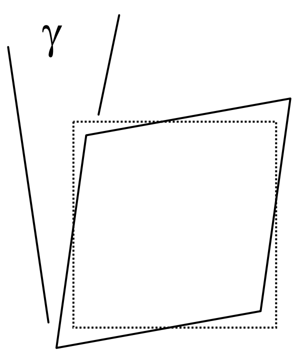
45° Coordinate rotation

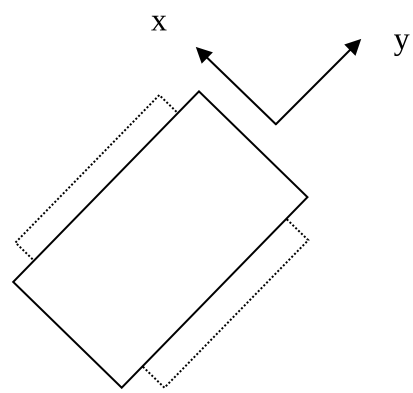
Principal stresses



Static equilibrium requires

$$\tau = (-)\sigma_x = \sigma_y$$





$$\gamma = \frac{\tau}{G}$$

$$\varepsilon_y = \frac{dy}{dl} = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E}$$

$$\varepsilon_x = \frac{dx}{dl} = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$$

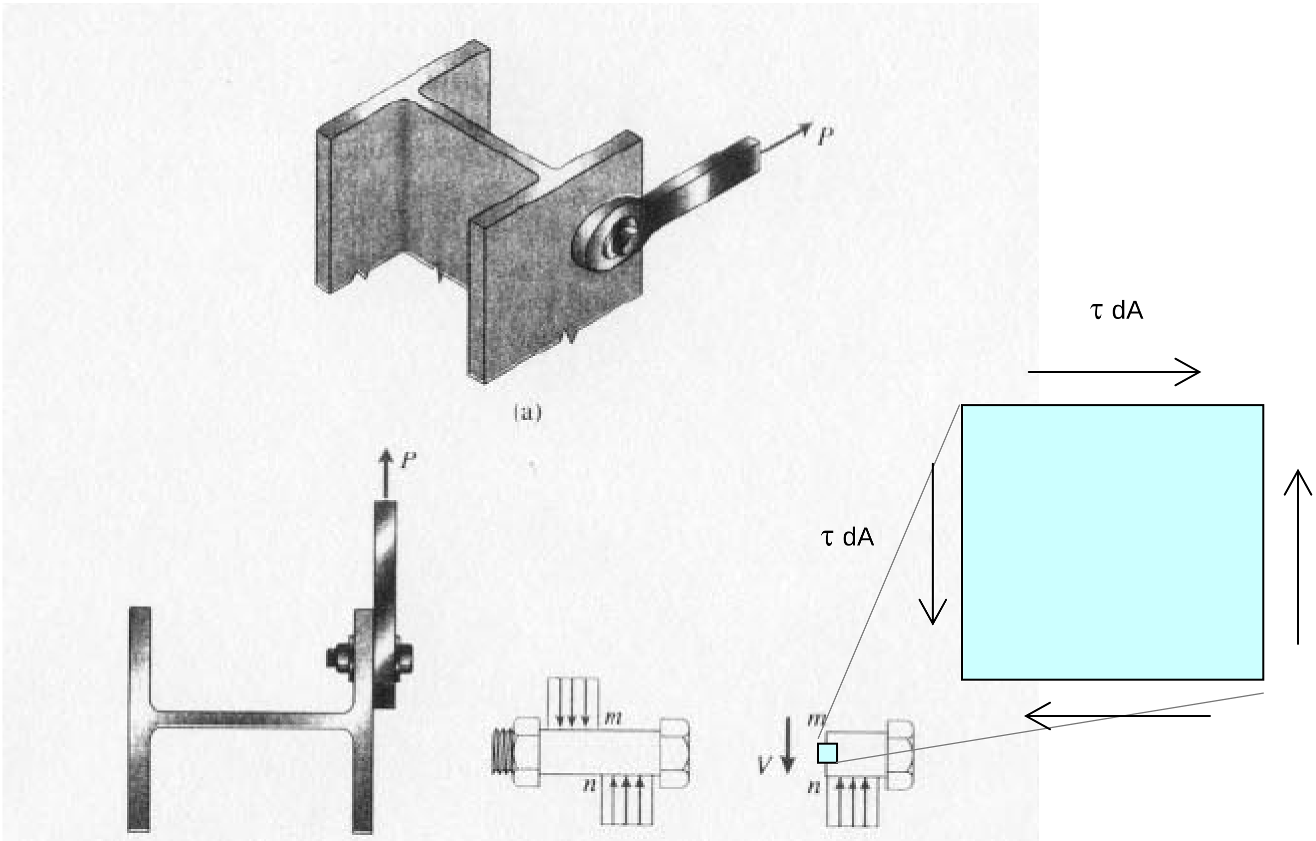
$$(\sigma_x, \varepsilon_x < 0)$$

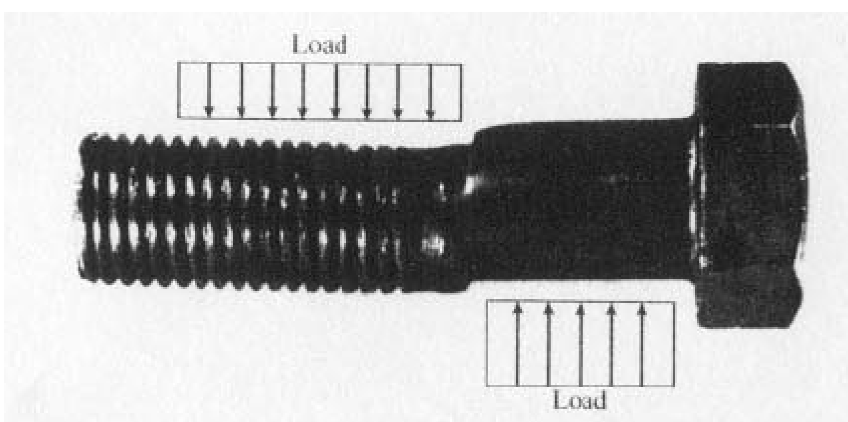
Equivalent, using $G = \dfrac{E}{2(1+\nu)}$

For more general cases, use **Mohr's circle** to transform coordinates and see transformation of normal and shear stress

<u>Principal Axes</u> are oriented so that the shear stress is zero

Shear strength





Bulk Modulus

Defines compressibility of material

For an element in uniform hydrostatic pressure P (in all directions)

Change in volume per unit volume =*ΔV/V*

Can you show the following:

$$\frac{\Delta V}{V} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$= \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Bulk modulus E_B is a material property, defined as

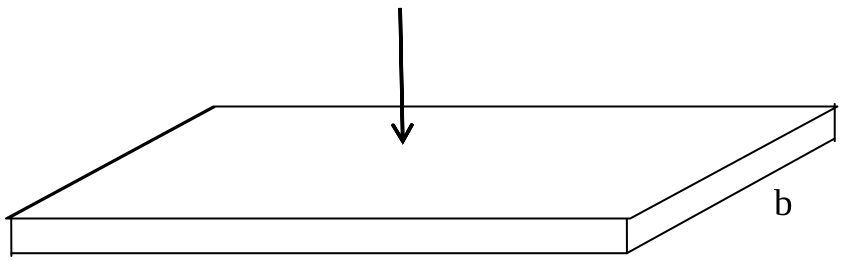
$$E_B \triangleq \frac{-P}{\left(\frac{\Delta V}{V}\right)}$$

For isotropic materials,

$$E_B = \frac{E}{3(1-2\nu)}$$

For case of soft rubber between two stiff plates

Force F, Area A (= a b)



$$\Delta x \cong 0,\ \Delta y \cong 0,\ \Delta A \cong 0$$ Constrained layer

$$\Delta V \cong A \cdot \Delta t, \frac{\Delta V}{V} = \frac{\Delta t}{t}$$

$$P = -E_B \cdot \frac{\Delta V}{V} = \frac{F}{A}$$ by definition of E_B

$$-E_B \frac{\Delta t}{t} = \frac{F}{A}$$

$$\Delta t \cong -\frac{F\,t}{E_B\,A}$$

$$E_B = \frac{E}{3(1-2\nu)}$$ for soft rubber, ν ~0.5, E_B blows up

For RTV rubber, E ~ 300 psi and E_B~100,000 psi
The constraint makes the rubber seem 300 times stiffer!