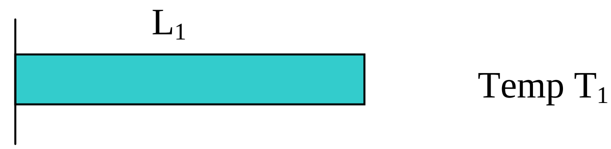
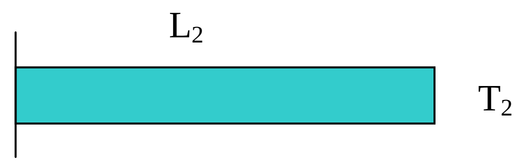
Thermal Expansion

Materials expand or contract with changing temperature.



Change the temperature



$$L_2 - L_1 = \alpha L \left(T_2 - T_1 \right)$$

$$\Delta L = \alpha L \Delta T$$

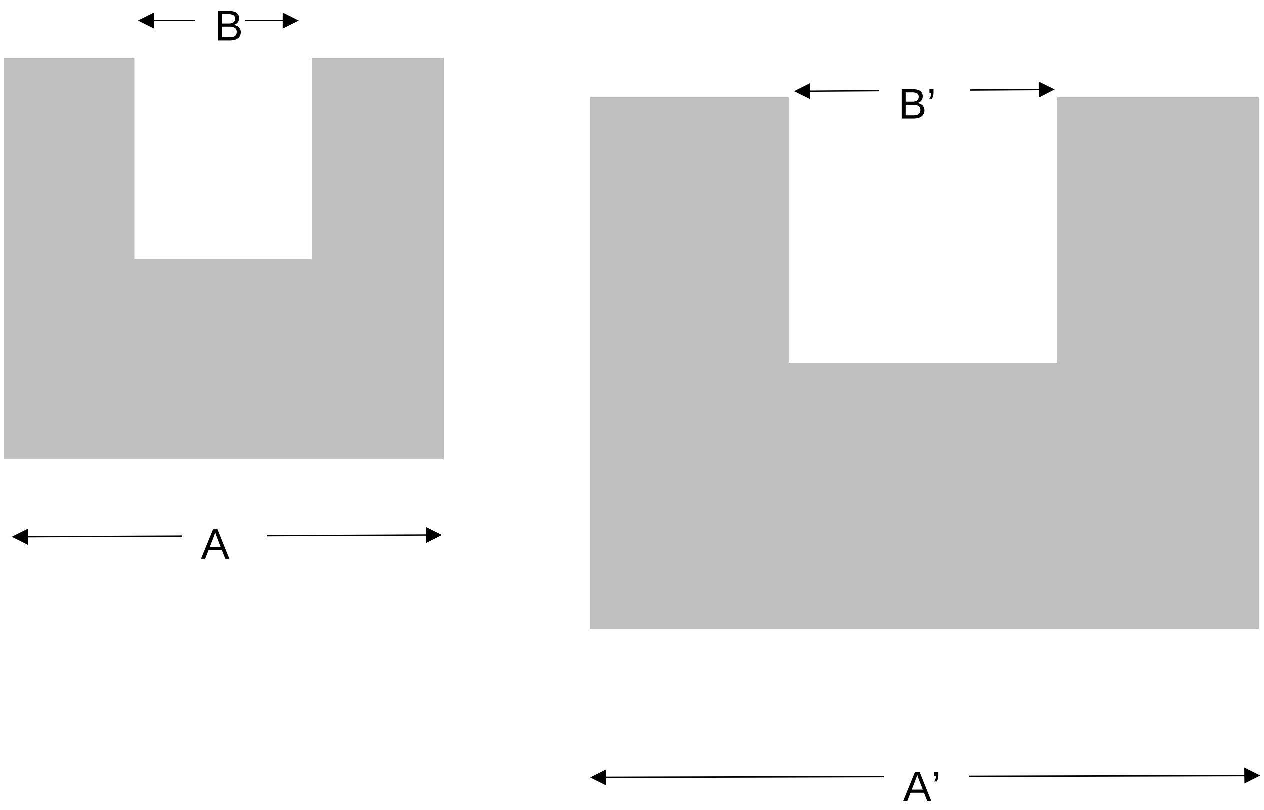
$$\varepsilon \triangleq \frac{\Delta L}{L} = \alpha \Delta T$$

thermal strain

α is the Coefficient of Thermal Expansion CTE

Aluminum	~23 ppm/°C
Optical Glass	~3 – 10 ppm/°C

Isotropic materials, temperature changes cause ALL dimensions to scale proportionally:



$$\Delta A = A\alpha\Delta T$$

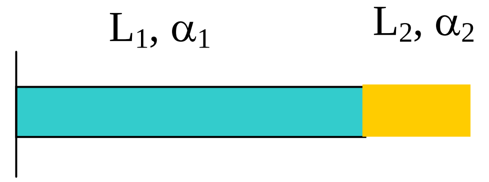
$$\Delta B = B\alpha\Delta T$$

Inside dimensions scale the same as outside dimensions

Low CTE materials:

Borosilicate glass (Pyrex)	~3 ppm/°C
Fused silica	0.6 ppm/°C
Invar	~1 ppm/°C
Super Invar	~0.3 ppm/°C
Zerodur (Schott)	0
ULE (Corning)	0
CFRP	(can be tuned to 0)

Athermalizing -- Combining two materials:

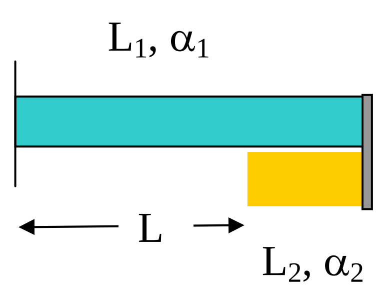


$$L = L_1 + L_2$$
$$\Delta L = \Delta L_1 + \Delta L_2$$
$$= \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T$$
$$= (\alpha_1 L_1 + \alpha_2 L_2)\Delta T$$

To athermalize over a distance L, use two materials so

$$\alpha_1 L_1 + \alpha_2 L_2 = 0$$
$$L_1 + L_2 = L$$

Using materials with $\alpha > 0$, requires $L < 0$



$$\alpha_1 L_1 - \alpha_2 L_2 = 0$$
$$L_1 - L_2 = L$$

Thermal stress

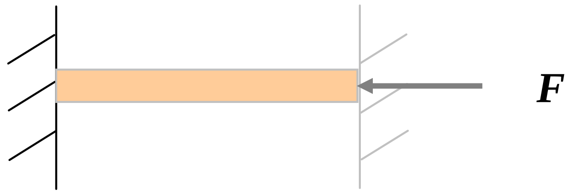
Use superposition

Calculate stress due to temperature change

1. Determine expansion, as if unconstrained

$$\Delta L = L\alpha\Delta T$$

2. Add reaction force that provides constraint by pushing back ΔL

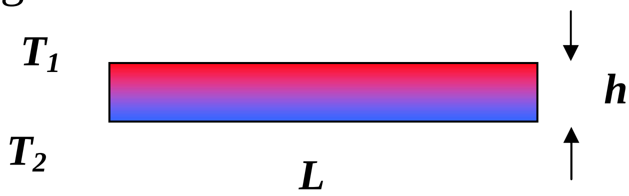


$$\Delta L = \frac{FL}{EA}$$

$$\sigma = \frac{F}{A} = E\alpha\Delta T$$

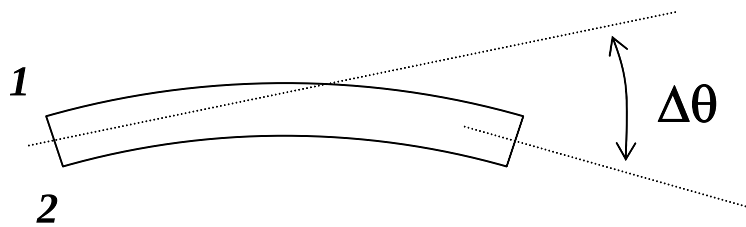
Solve more general problems the same way.

Temperature gradients cause distortions



Gradient = $\frac{\partial T}{\partial y} = \frac{T_1 - T_2}{h}$

This will cause the beam to bend in an arc, in the same way that the applied moment did.



The arc length along surface 1 is longer than the arc length along surface 2 by the amount

$$L_1 - L_2 = L\alpha\left(T_1 - T_2\right)$$

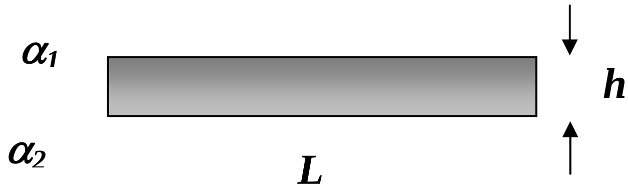
By geometry: $$\Delta\theta = \frac{L_1 - L_2}{h}$$

or $$\Delta\theta = \frac{\alpha L \Delta T}{h}$$

More generally, $$\Delta\theta = \alpha L \frac{\partial T}{\partial y}$$

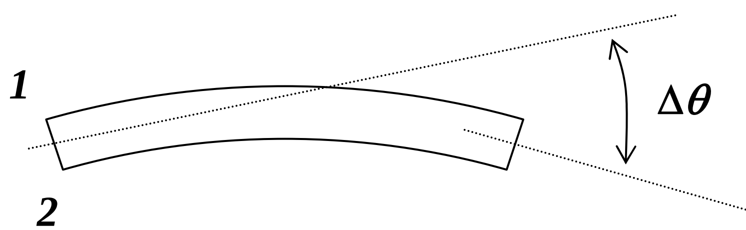
Integrate for deflection

Materials with inhomogeneous CTE, coupled with a bulk temperate change behave the same way as above:



CTE Gradient = $\dfrac{\partial \alpha}{\partial y} = \dfrac{\alpha_1 - \alpha_2}{h}$

This will cause the beam to bend in an arc, in the same way that the thermal gradient did.



If the temperature is changed uniformly, the arc length along surface 1 is longer than the arc length along surface 2 by the amount

$$L_1 - L_2 = L\Delta T\left(\alpha_1 - \alpha_2\right)$$

By analogy : $$\Delta\theta = L\frac{\Delta\alpha}{h}\Delta T$$

and $$\Delta\theta = L\frac{\partial\alpha}{\partial y}\Delta T$$

Generally: $$\Delta\theta = L\frac{\partial\left(\alpha T\right)}{\partial y}$$

Integrate to get deflections $\delta_y(z)$

Use boundary conditions:

$$\frac{d}{dz}\delta_y(z) = \theta(z) = \theta_0 + \Delta\theta(z)$$

$$\Delta\theta(z) = z\,\frac{\partial(\alpha T)}{\partial y}$$

$$\delta_y(z) = \int_{z=0}^{z} \Delta\theta(z)dz = \theta_0 z + \frac{1}{2}\frac{\partial(\alpha T)}{\partial y} z^2 + C$$

For <u>cantilever</u>: at z=0, θ=0 and δ_y=0

So θ_0=0 and C=0

$$\delta_y(L) = \frac{1}{2}\frac{\partial(\alpha T)}{\partial y} L^2$$

For simple support at the ends:

δ_y=0 at z=0 and at z=L

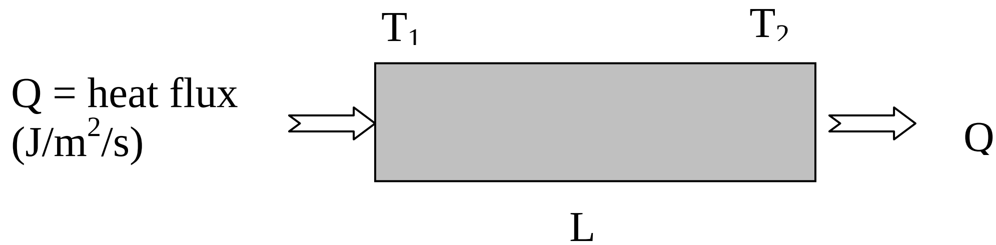
So $\theta_0 = -\frac{L}{2}\frac{\partial(\alpha T)}{\partial y}$ and C=0

max deflection at L/2 is

$$\delta_y\left(\frac{L}{2}\right) = \frac{1}{2}\frac{\partial(\alpha T)}{\partial y}\left(\frac{L}{2}\right)^2 = \frac{1}{8}\frac{\partial(\alpha T)}{\partial y} L^2$$

Heat flow causes thermal gradients

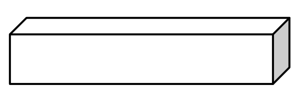
For steady state: $Q_{in} = Q_{out}$



Define thermal conductivity λ :
$$Q = \lambda \frac{(T_1 - T_2)}{L}$$

	λ
Glass	1.1 W/(m K)
Aluminum	170 W/(m K)
Copper	390 W/(m K)
Stainless steel	16 W/(m K)

Heat flow
$$H = Q \bullet A = \frac{A\lambda\Delta T}{L}$$



Apply 1 W through 10 cm long bar of Al, A = 1 cm^2

$$\Delta T = \frac{HL}{A\lambda} = \frac{(1W)(0.1m)}{(0.0001m^2)\left(170\, {W}/{m \cdot K}\right)} = 6°K = 10°F$$

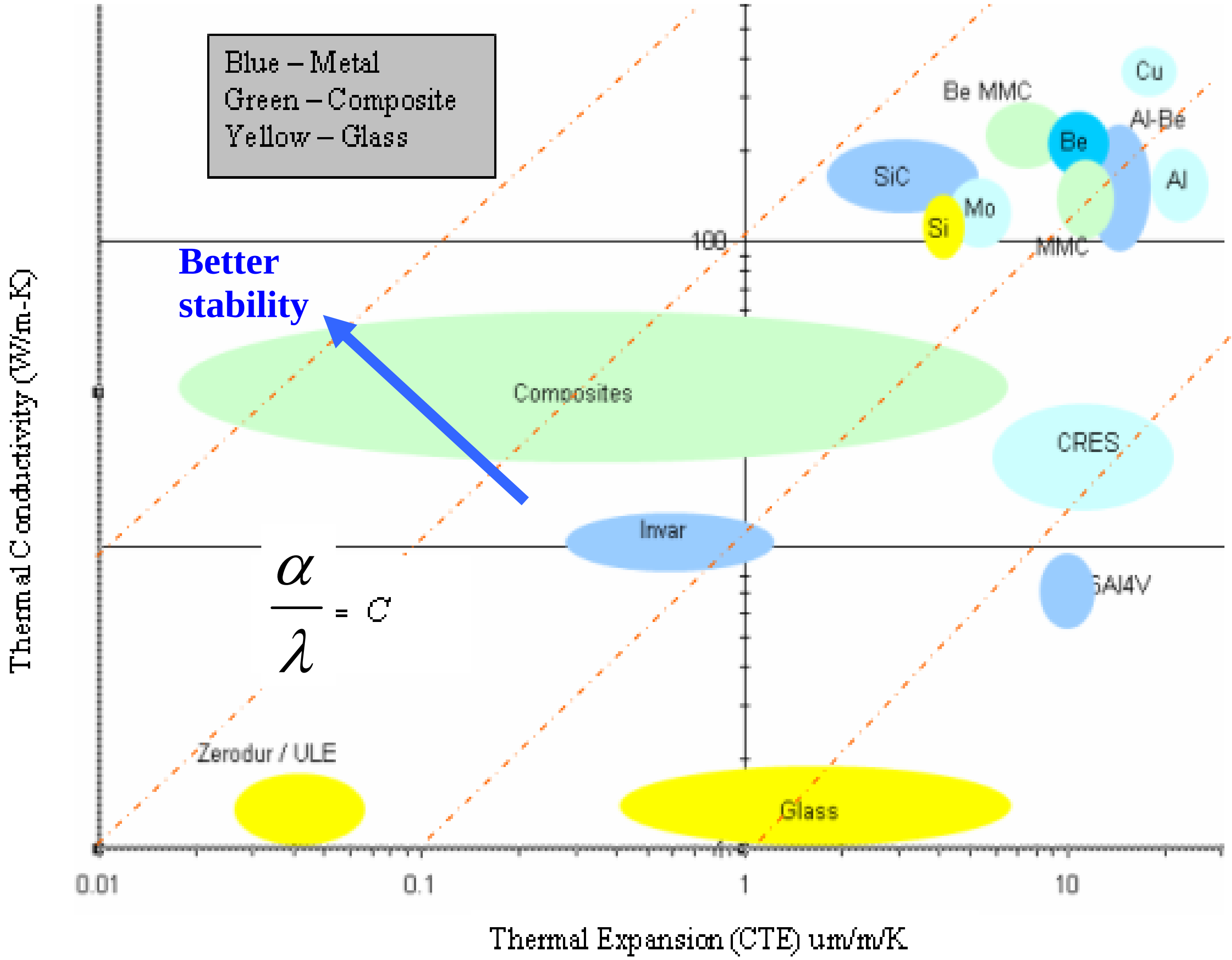
Steady state thermal distortion

Distortion due to temperature gradient ∇T is always proportional to $\alpha \nabla T$

For a constant heat source, with power *H*, the thermal gradient is proportional to $\nabla T \propto \dfrac{H}{\lambda}$

So the distortion will be proportional to $\dfrac{\alpha}{\lambda}$

This provides a **figure of merit** to compare sensitivity to steady state heat loading



- Lines of constant thermal stability are shown
- Performance improves toward upper left corner
- SiC has the largest thermal stability due to high conductivity, even though large CTE
- Zerodur and ULE have very high thermal stability due to their extremely low CTE, even though poor conductor
- Titanium and CRES are very poor for thermal stability

Transient heating

- Transient heat flux is when the temperature distribution changes with time
- Thermal diffusivity (D) is the ratio of thermal conductivity to heat capacity

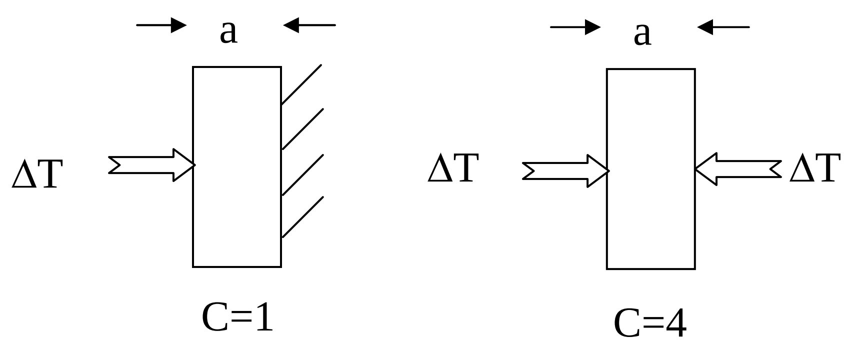
$$D = \frac{\lambda}{\rho \cdot C_P}$$

$$\frac{\partial T}{\partial t} = D \cdot \nabla^2 T$$

Larger thermal diffusivity means quicker response to temperature changes.

The thermal time constant τ governs the rate at which the transient response decays exponentially ($1-e^{-t/\tau}$)

$$\tau \cong \frac{a^2}{CD}$$



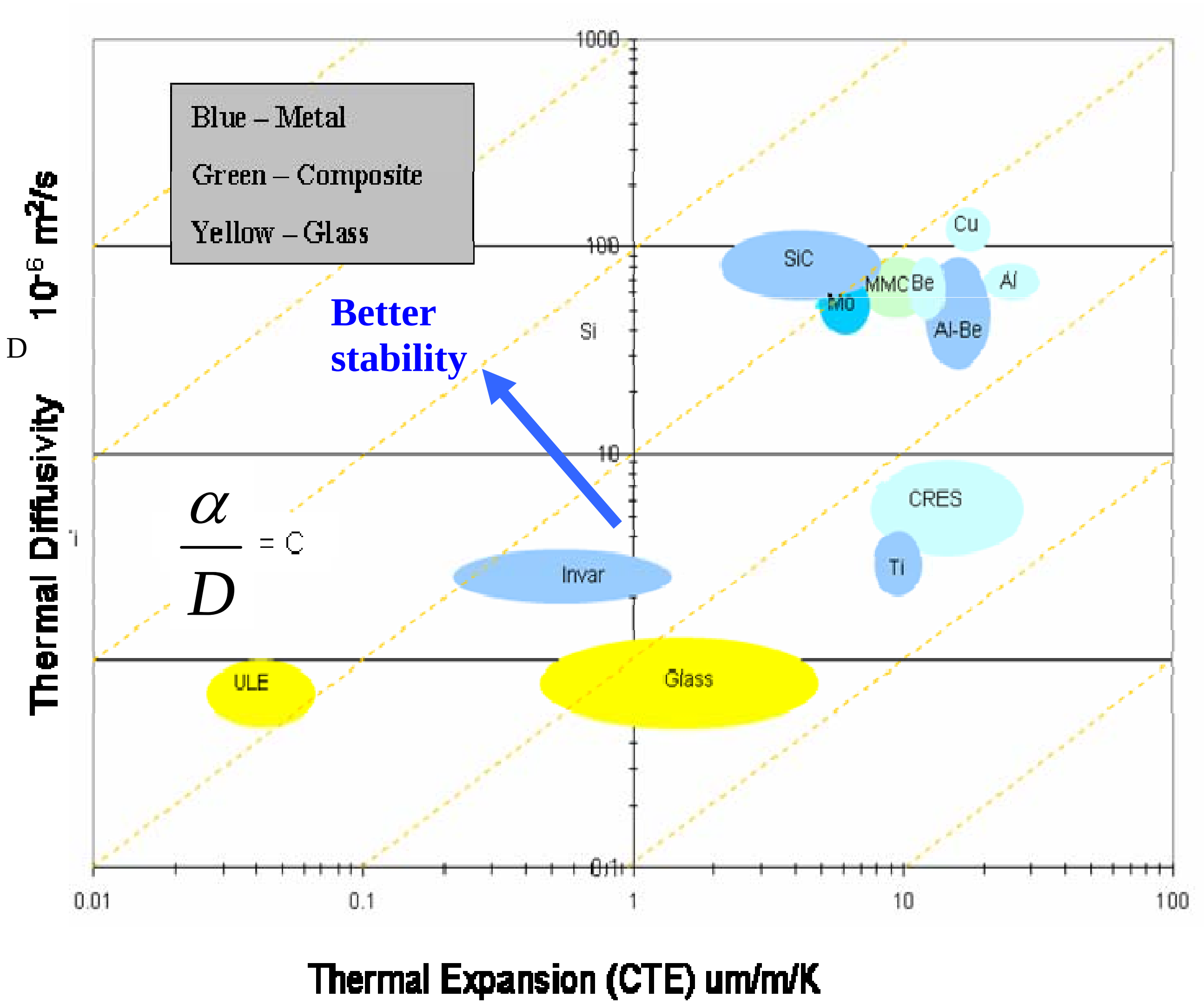
After a given time from a transient heat impulse, the temperature gradient will be proportional to 1/D.

Again, the thermal distortion is proportional to $\alpha\,\Delta T$, so the transient distortion will be proportional to

$$\frac{\alpha}{D}$$

This provides a merit function for transient thermal stability

Thermal stability under transient heating



- Lines of constant thermal stability are shown
- Performance improves toward upper left corner
- SiC has the largest transient thermal stability
- ULE has high stability due to its extremely low CTE
- Glass, Titanium and CRES are very poor

Thermal time constant:

For BK7 glass:
λ = 1.1 W/(m – K) = 0.011 J/s /(cm – K)
ρ = 2.5 g/cm^3
c_p = 0.86 J/(g- K)

$$D = \frac{\lambda}{\rho \cdot C_P} = \frac{0.011}{(2.5)(0.86)} \frac{\text{cm}^2}{\text{s}} = 0.0051 \frac{\text{cm}^2}{\text{s}}$$

For 1 cm thick BK7, heated from one side:

$$\tau = \frac{a^2}{D} = \frac{(1\,\text{cm})^2}{\left(0.0051 \frac{\text{cm}^2}{\text{s}}\right)} = 195\,\text{sec}$$

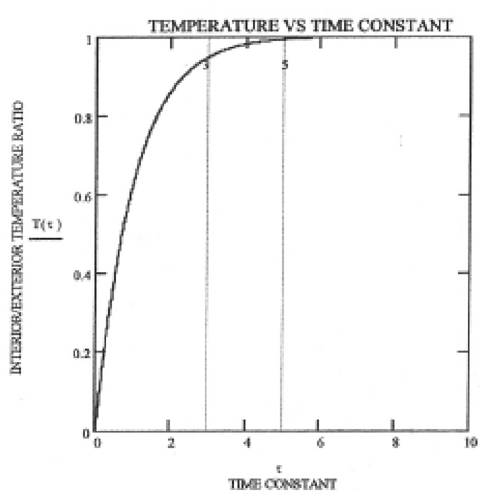
or ~ 3 min

Scaling: 25 mm glass takes 20 min

This the time it takes for the heat to travel through the glass. It does not include the coupling to the outside.

$$\Delta T(t) = \Delta T_0 e^{-t/\tau}$$

$\Delta T(\tau)/\Delta T_0$=0.37
$\Delta T(2\tau)/\Delta T_0$=0.14
$\Delta T(3\tau)/\Delta T_0$=0.05
$\Delta T(4\tau)/\Delta T_0$=0.02
$\Delta T(5\tau)/\Delta T_0$=0.007



Athermal System design

1. <u>Control geometry</u>

    Use low CTE materials

Kovar	~ 5 ppm/°C
Invar	~ 1 ppm/°C
Super Invar	~ 0.3 ppm/°C
Fused silica	~ 0.6 ppm/°C

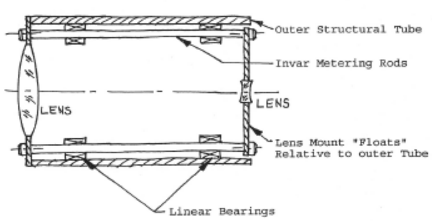
    Practically zero CTE materials

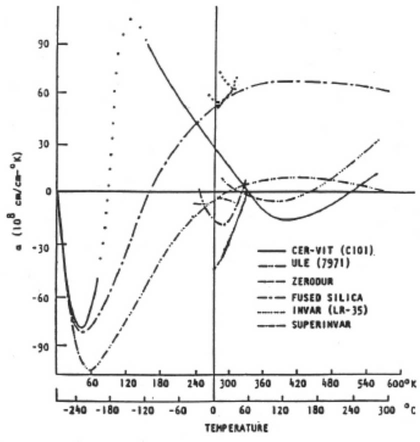
    - ULE
    - Zerodur
    - Athermalized Carbon Fiber Reinforced Plastic

Composite truss for HST

Use of metering rods

- To avoid the structural inefficiency of low thermal expansion materials, use should be restricted to metering structures. The optical elements should be supported by a conventional structure and provided with mounts compliant in the direction along the optical axis. Use metering rods of low expansion material to tie the optical elements together. The metering rods maintain correct spacing as the main structure expands or contracts.





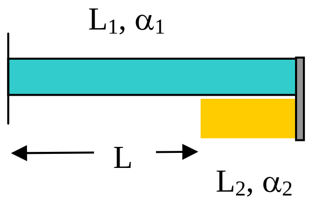
Bibliography References: 2.2.2, 2.2.3.

- INVAR

- Invar is an iron-nickel alloy, typically with about 36% nickel by weight.
- The thermal coefficient of expansion of invar may vary from -0.6 to 3.0 x 10^{-6} m/m-K between -70 to +100 °C. The thermal coefficient of expansion of invar can be limited to 0.8 to 1.6 x 10^{-6} m/m-K between 30 to +100 °C by careful control of the material during processing.
- A phase change occurs in invar at -20 °C, causing the thermal expansion coefficient to increase by a factor of 10. This phase change is reversible.
- Invar is unstable with respect to time (dimensional instability). The short term temporal instability may be as high as 11.0 x 10^{-6} m/m-day, with a time constant of about 100 days.
- For optimum thermal coefficient of expansion and long term stability, the so-called "MIT" or "Lement" heat treatment is suggested:
  - 1. 830 °C, 30 minutes, water quench
  - 2. 315 °C, 1 hour, air cool
  - 3. 95 °C, 48 hours, air cool
- Heavy machining or cold working may disturb the heat treatment of invar, and require another heat treatment cycle. Heavy machining is defined as any cut greater than 100 µm. Cold work, such as bead blasting, may also change the thermal coefficient of expansion of invar.

Bibliography References: 2.3.1, 2.3.2, 2.3.4, 2.3.8

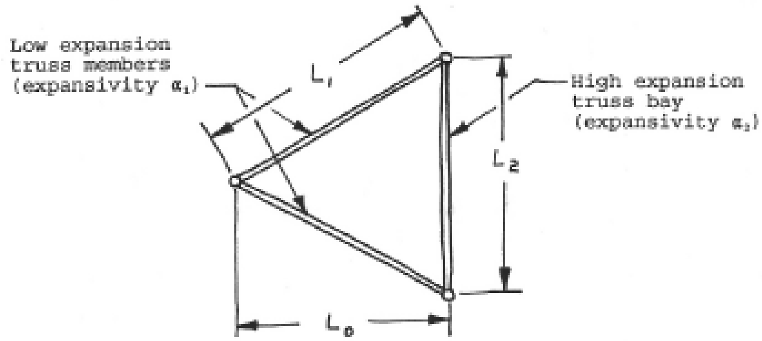
Use different materials to compensate



$$\alpha_1 L_1 - \alpha_2 L_2 = 0$$

$$L_1 - L_2 = L$$

- A type of athermal structure using the difference in the expansion coefficients of two (2) different materials is a bimetallic athermal truss. A high coefficient of expansion truss bay offsets a low coefficient of expansion truss member.



For L_o to remain constant for a temperature change (ΔT).

$$\frac{L_1}{L_2} = \frac{1}{2}\left[\frac{\alpha_2\ \Delta T\left(2+\alpha_2 \Delta T\right)}{\alpha_1\ \Delta T\left(2+\alpha_1 \Delta T\right)}\right]^{\frac{1}{2}} \approx \frac{1}{2}\sqrt{\frac{\alpha_2}{\alpha_1}}$$

For L_1 = Steel $\alpha_1 = \frac{10 \times 10^{-6}}{K}$

L_2 = Aluminum $\alpha_2 = \frac{23 \times 10^{-6}}{K}$

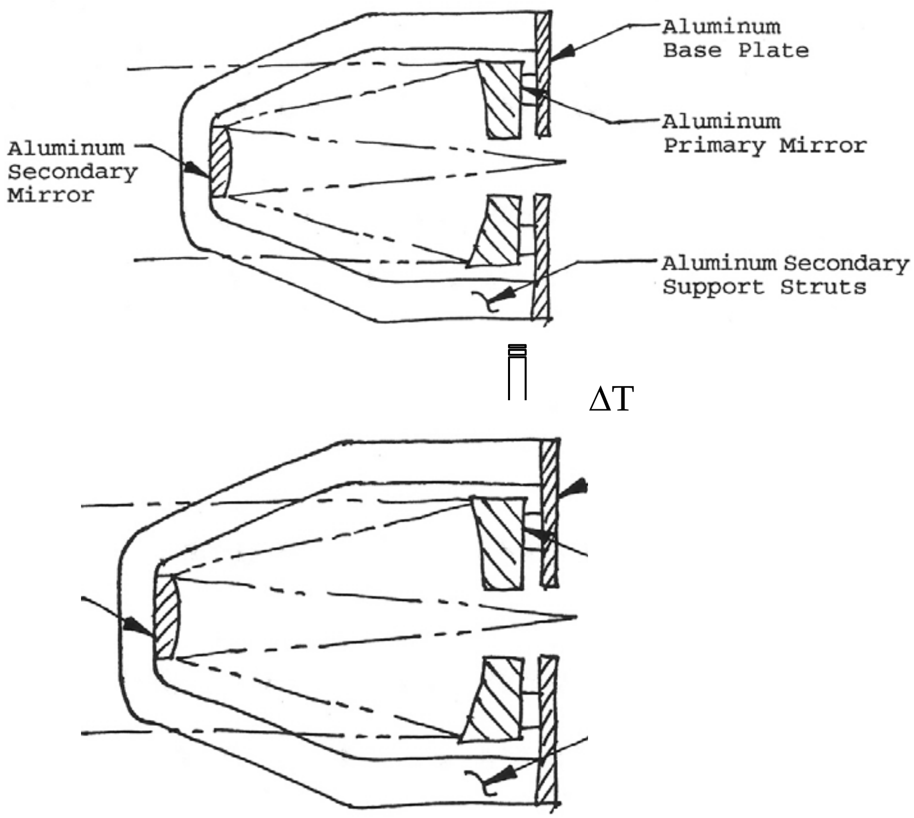
Then $\frac{L_1}{L_2} = 0.758$

- This method is vulnerable to gradients
- Due to non-linear changes in thermal expansion coefficient with temperature, this method is useful only for a small temperature range.

Bibliography References: 3.4.21, 3.4.22

(Vukabratovich) 96

Make everything out of the same material (including the mirrors)



If all optical surfaces and spacing stay in proportion, then the system will still work!

Spitzer Telescope with mirrors and mechanics made of beryllium