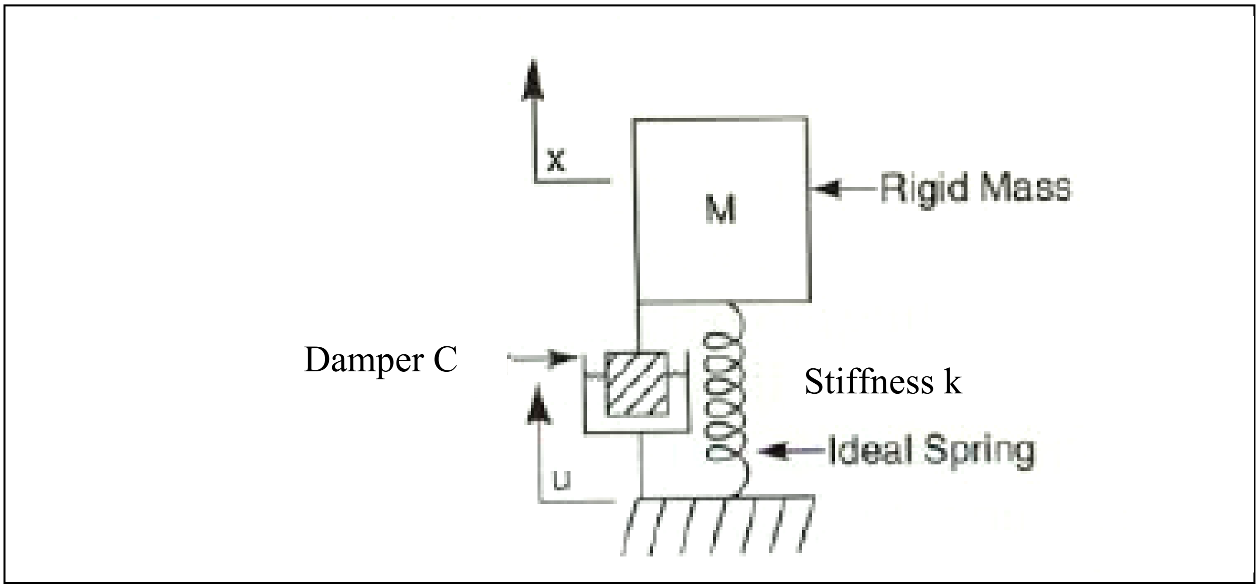
17. Vibration Isolation



Each degree of freedom can be represented as a simple mode that has a mass, stiffness, and damping.

This can then be modeled using the simple second order differential equation.

$$F = ma$$

$$-k(x-u) - C(\dot{x}-\dot{u}) = m\ddot{x}$$

$$\omega_n = \sqrt{k/m} \quad \text{(in radians/sec)}$$

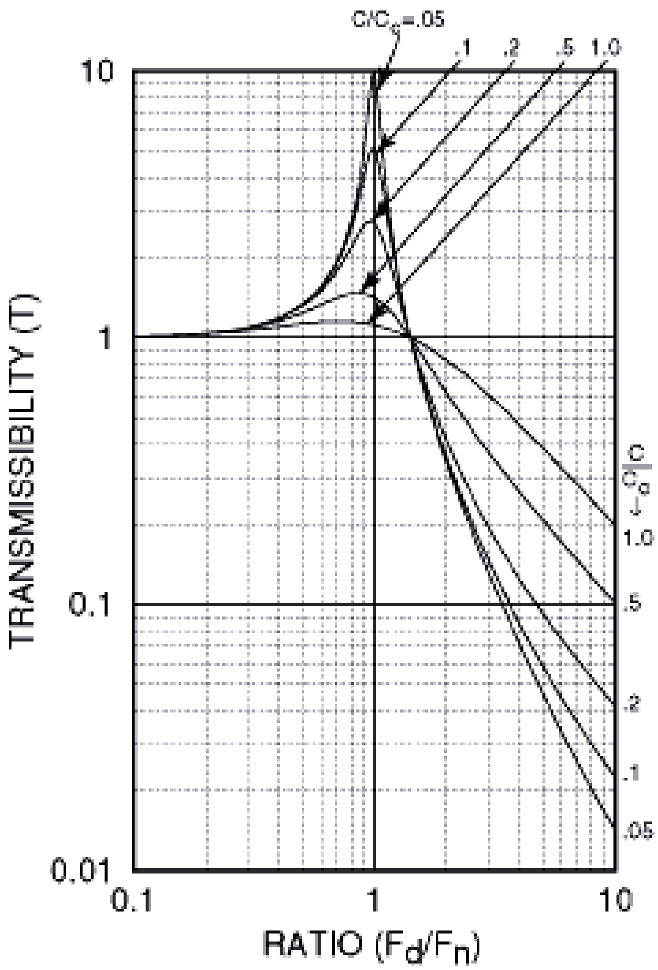


Figure 8 Family of transmissibility curves for a single degree of freedom system.

$$T = \frac{isolated\ motion}{base\ motion} = \frac{x}{u} = \sqrt{\frac{1+\left(2\frac{\omega}{\omega_n}C_R\right)^2}{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\frac{\omega}{\omega_n}C_R\right)^2}}$$

the critical damping coefficient (C_c) is given by: $C_c = 2m\,\omega_n$

The critical damping ratio (C_R) is given by: $C_R = C/C_c$

Where: C Is the system damping

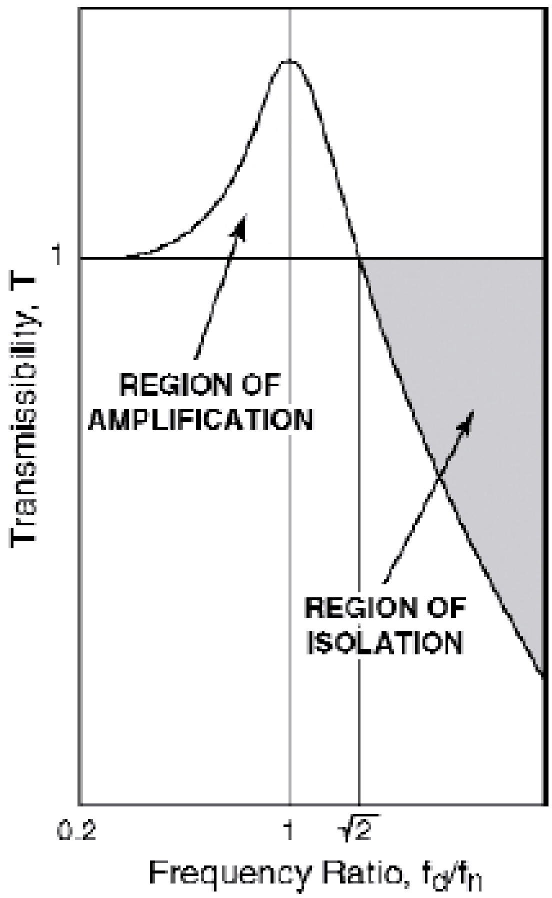


Figure 6 Typical transmissibility curve for an isolated system where fd = disturbance frequency and fn = isolation system natural frequency.

$$T = \frac{isolated\ motion}{base\, motion} = \sqrt{\frac{1 + \left(2\dfrac{\omega}{\omega_n} C_R\right)^2}{\left(1 - \dfrac{\omega^2}{\omega^2{}_n}\right)^2 + \left(2\dfrac{\omega}{\omega_n} C_R\right)^2}}$$

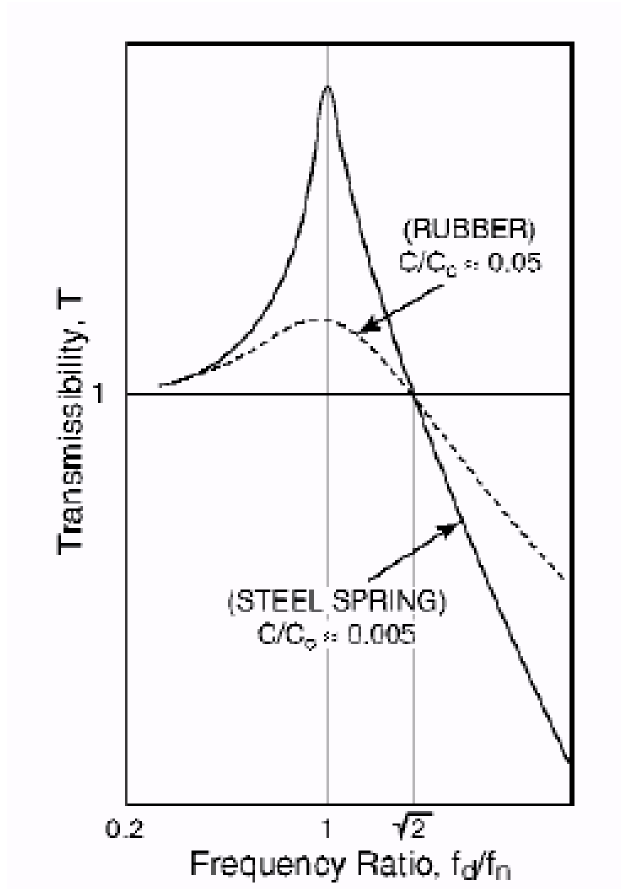
Damping

$$Q = \frac{1}{2C_R}$$

Q is the maximum amplification at resonance

Also, Width of resonance

$$\frac{\Delta\omega_{FWHM}}{\omega_n} = 2C_R = \frac{1}{Q}$$



Material	Approx Damping Factor C/Cc
Steel Spring	0.005
Elastomers:	-
Natural Rubber	0.05
Neoprene	0.05
Butyl	0.12
Barry Hi Damp	0.15
Barry LT	0.11
Barry Universal	0.08
Friction Damped Springs	0.33
Metal Mesh	0.12
Air Damping	0.17
Felt and Cork	0.06

Table 1 Damping factors for materi commonly used for isolators

<u>Rule of thumb:</u>
Assembled structures, optimized for rigidity
~2% damping, Q = 25

Steel honeycomb core $Q = \frac{1.4\times10^{-5}}{3.9\times10^{-6}} \cong 4$

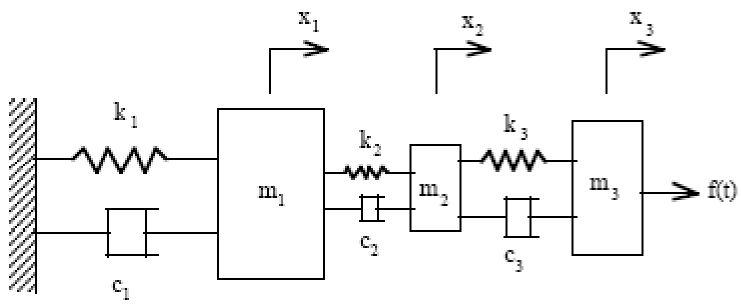
Aluminum honeycomb core $Q = \frac{2.2\times10^{-4}}{1.9\times10^{-5}} \cong 12$

Granite block $Q = \frac{2.3\times10^{-4}}{5\times10^{-7}} \cong 460$

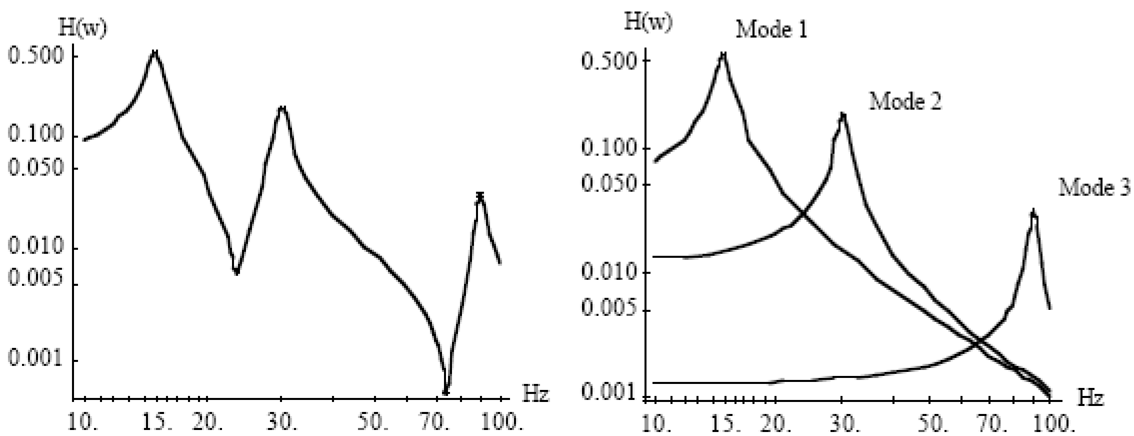
J. H. Burge
University of Arizona

Dynamics of a Multiple Degree of Freedom System

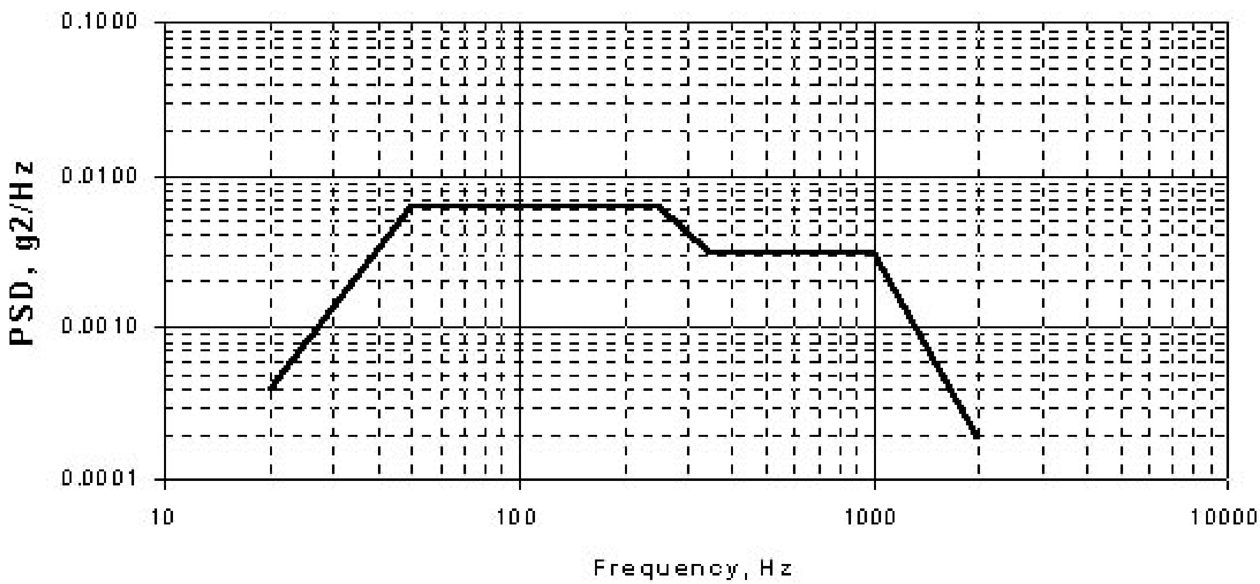
- **A sample MDOF system:**



- **A typical frequency response plot (individual contributions shown to illustrate mode superposition):**



Using power spectral density



Start with base motion with power spectral density PSD_{BASE}.

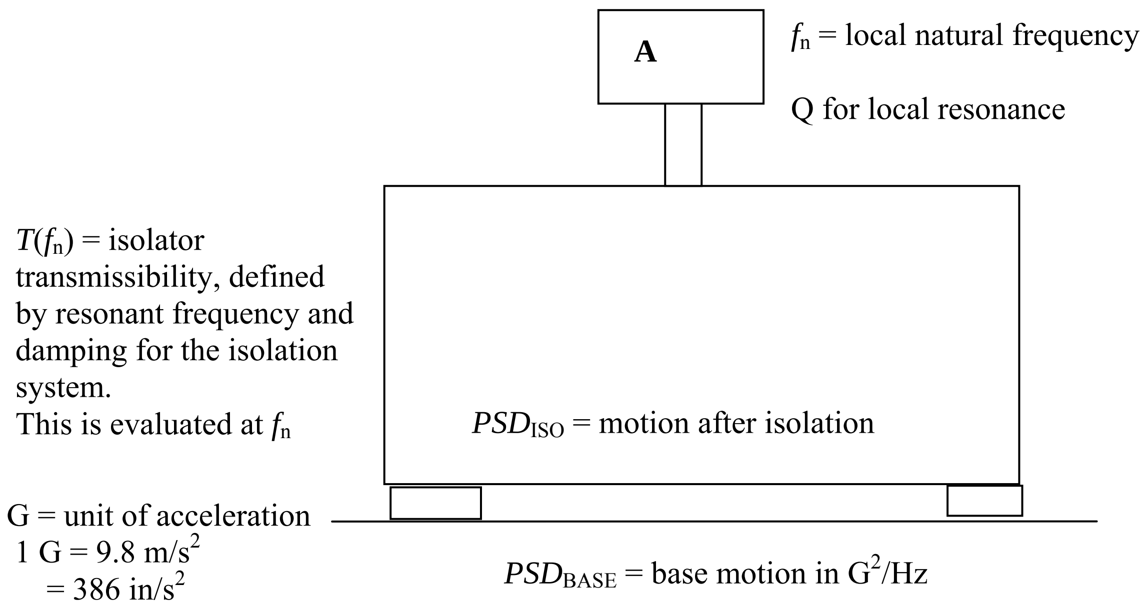
10^{-10} G^2/Hz: relatively quiet lab
10^{-9} G^2/Hz: light traffic
10^{-8} G^2/Hz: light manufacturing

The isolators remove most of this. The *PSD* after isolation is

$$PSD_{ISO} = T^2 \; PSD_{BASE}$$

Where *T* is the transmissibility of the isolators

Consider object A being isolated. It will ring at its lowest resonance $f_{n.}$, driven by the vibration that leaks through the isolators



The root mean square acceleration for object **A** is given by the Miles equation:

$$a_{rms} = \sqrt{\frac{\pi}{2} f_n \cdot Q \cdot PSD_{ISO}}$$

where f_n is the natural frequency for A
Q is the maximum amplification at resonance for A
PSD_{ISO} is the PSD that drives A

(PSD in G^2/Hz -- a_{rms} will be in G's)

A G is a unit of acceleration which equals 9.8 m/s^2

Converting from a_{rms} acceleration in G's to displacement δ_{rms}

$$\delta_{rms} = \frac{a_{rms}}{\left(2\pi f\right)^2} g$$

g = 9.8 m/s^2 or 386 in/s^2

Combining

$$\delta_{rms} = g \cdot T(f_n)\sqrt{\left(\frac{1}{32\pi^3}\right)\left(\frac{Q}{f_n^3}\right) \cdot PSD_{BASE}}$$

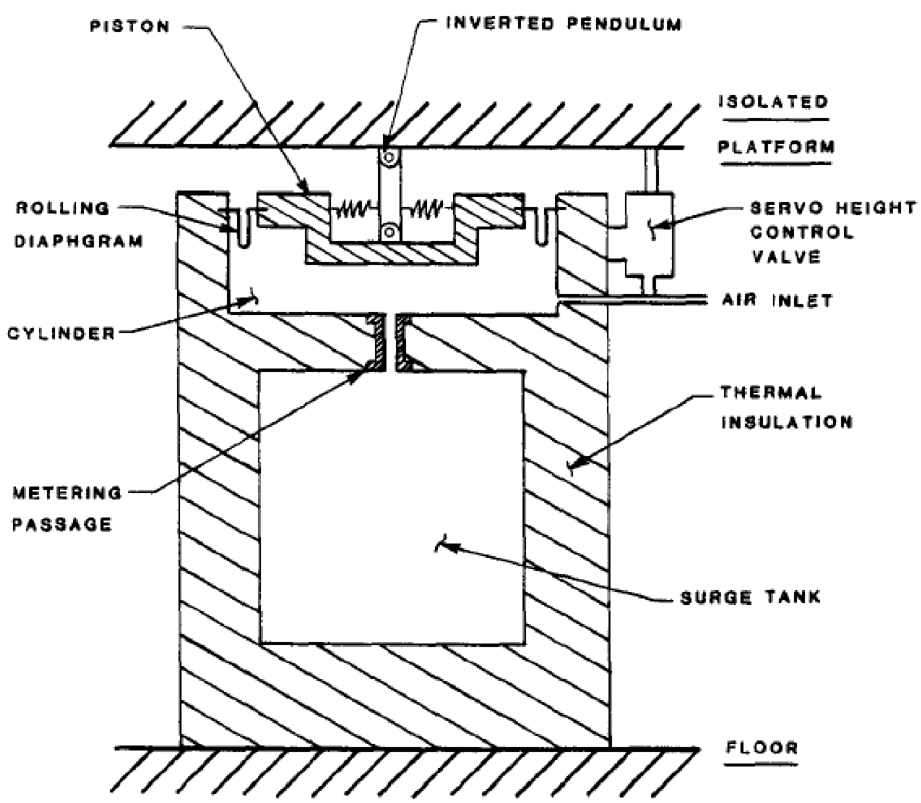
This gives the approximate motion of A with respect to the rest of the platform. Most of this motion will occur at A's resonant frequency f_{n}.

High performance vibration isolation.

Use heavy table top and very soft springs. With soft springs, a small change in loading causes a large motion of the system. This must be accommodated. Also damping must be added.

Air springs, actively change the pressure to maintain geometry

Using negative stiffness element, balance with regular spring



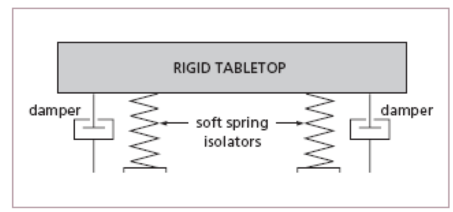
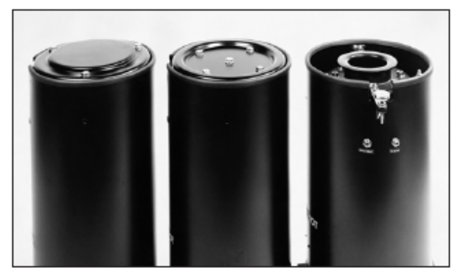


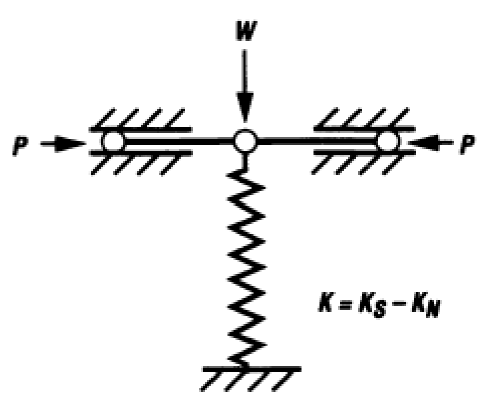
Figure 31.27 **Ideal seismic mounting of an optical table, consisting of weak spring supports with added damping**



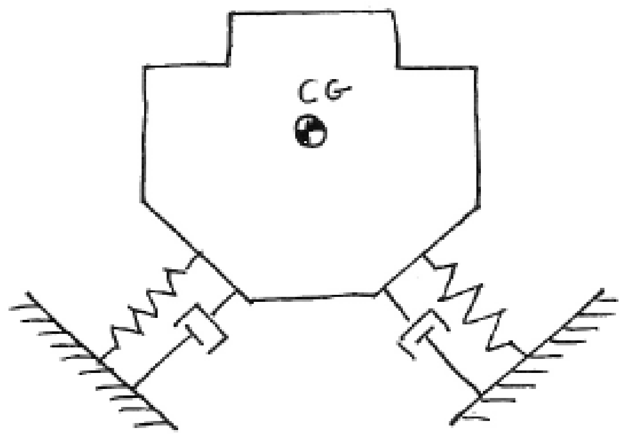
Tabletop supports are available in rigid, passive and self-leveling versions.

Zero stiffness spring

Balances positive and negative



- To avoid exciting rotational modes of vibration, the Center of Gravity of equipment mounted on the isolation system should be kept as low as possible. Alternately, line of action of isolators can pass through the Center of Gravity.



- Earthquake stops should be provided. Isolator frequency is often close to earthquake frequencies, so destructive resonance is possible.

- "Rattle Space" between the isolated mass and the structure should be provided. Rattle space can be estimated by:

$$X_{Rattle} = Q\ (X\ /\omega^2)$$

Where:

X Is the magnitude of the acceleration of the exciting force

- Whenever possible, buy an off-the-shelf isolation system rather than designing one from scratch. System design can be complex and subtle.

Bibliography References: 3.84, 3.94.

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Shock Loading

Just handling, expect 3 G's of shock loading

If dropped to the floor, an item can see very high accelerations for a short period of time.

Simple approximations, for item dropped from height *h*:

Velocity $v = \sqrt{2gh}$

Stopped in time $$\tau \cong \frac{1}{\omega_n} = \frac{1}{2\pi f_n}$$

Peak acceleration $$a_p \cong \frac{1}{\sqrt{2}} \frac{v}{\tau} = \omega_n \sqrt{gh}$$

Convert acceleration to A_g, which is in units of G's

$$a_p = A_g \cdot g$$

$$A_g = \omega_n \sqrt{\frac{h}{g}}$$

$$= \sqrt{\frac{g}{\delta_{selfweight}}} \times \sqrt{\frac{h}{g}} = \sqrt{\frac{h}{\delta_{selfweight}}}$$