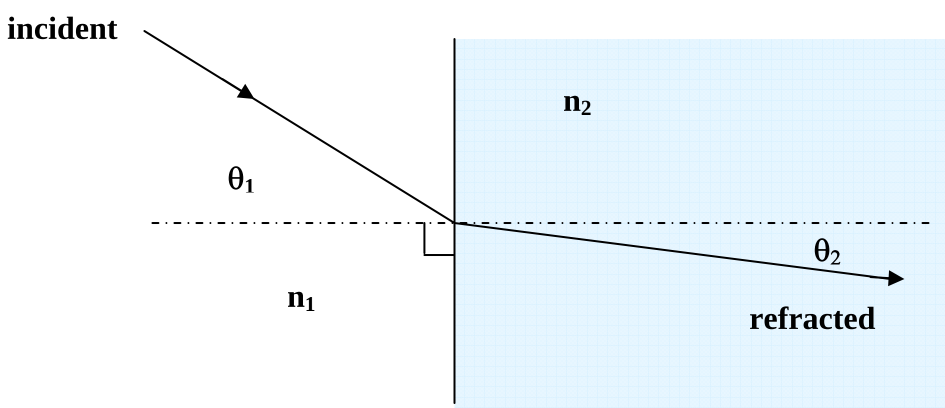
2) First order optics

Motion of optical elements affects the optical performance?

1. **by moving the image**
2. **higher order things (aberrations)**

The first order effects are most important

Snell's law for refraction



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

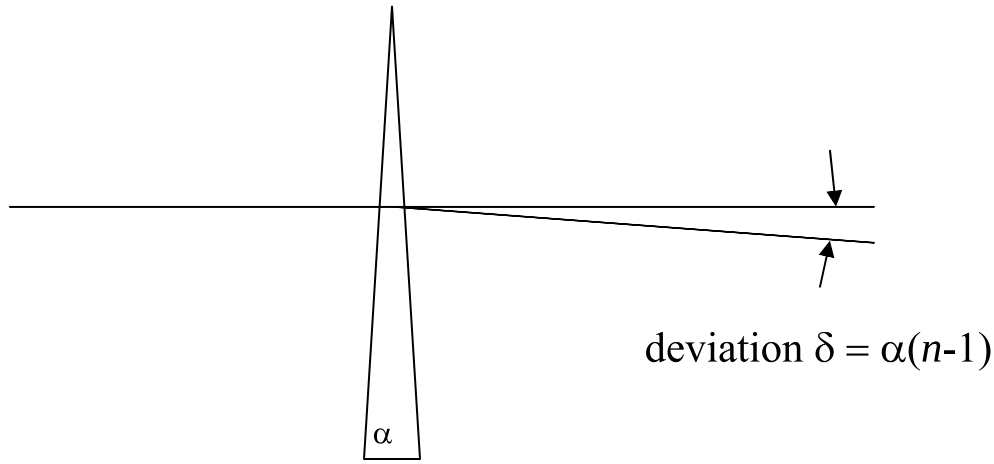
In air n = 1.000 so

$$\frac{\sin\theta_1}{n_2} = \sin\theta_2$$

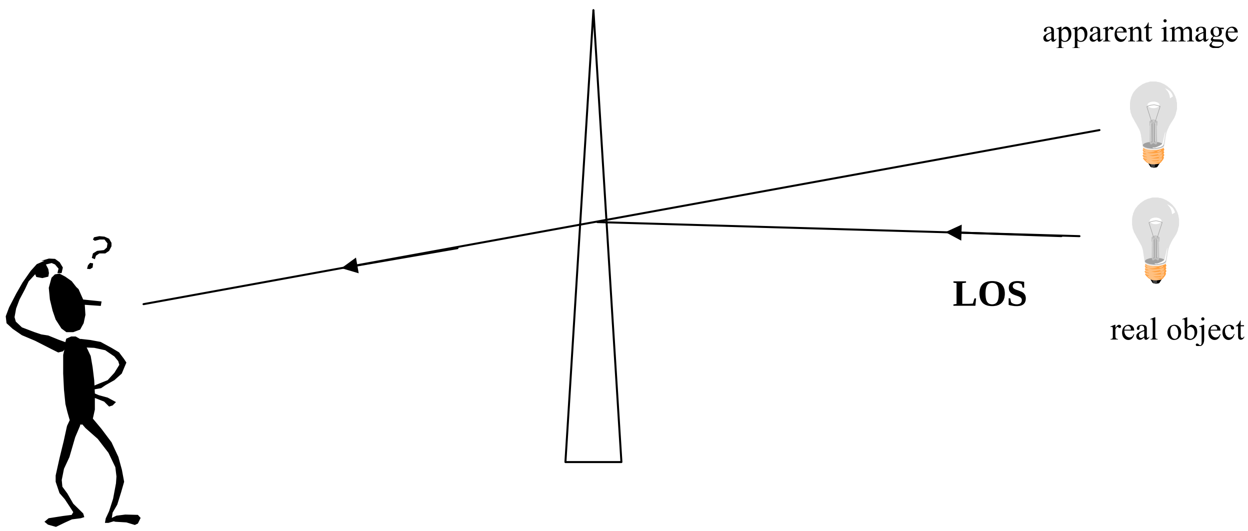
Reciprocity: Works the same from left to right as right to left, same coming and going.

Small angle approximation: $n_1\theta_1 = n_2\theta_2$

Small angle prism in air



Define Line of Sight (LOS): Where the optical system is looking

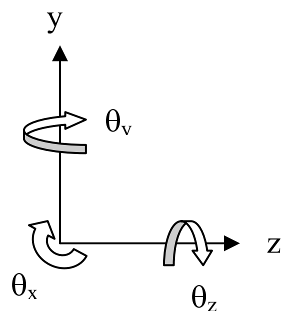


One easy way to determine the line of sight is to imagine that your optical system is "projecting" like a laser projector. Light travels the same path in either direction. Your Line of Sight will be defined by this imaginary projected beam

A rigid body always has 6 degrees of freedom:

Translation in x, y, and z

Rotation about x-axis, y-axis, z-axis



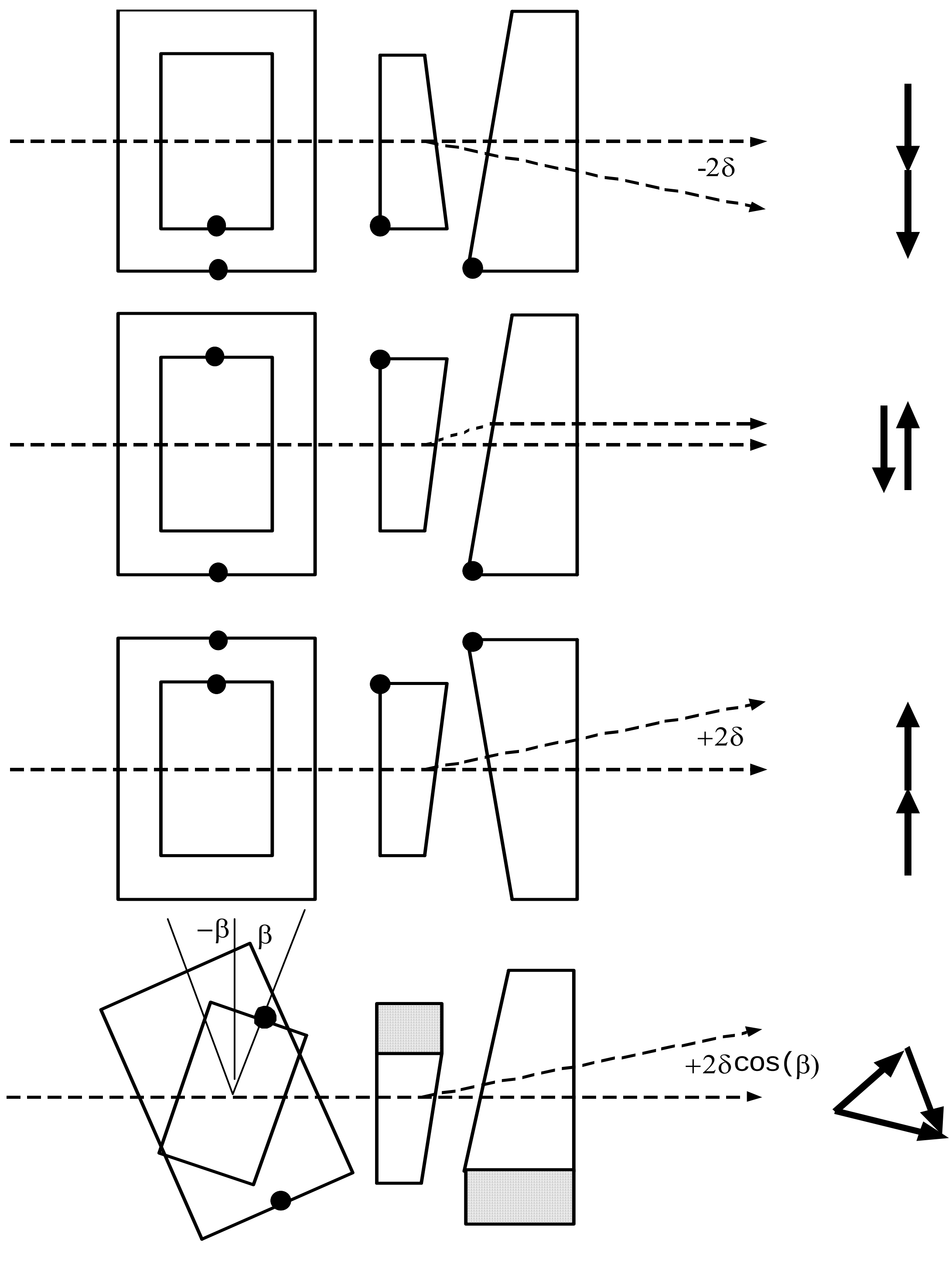
(x axis out of page)

Motion of thin prism:

The only motion that affects the line of sight light is θ_z, rotation about the optical axis.

Risley prisms

Steer the line of sight by using rotation of prisms
Rotation of one prism moves LOS in a circle
Separate rotation of a second prism allows two-axis control of LOS

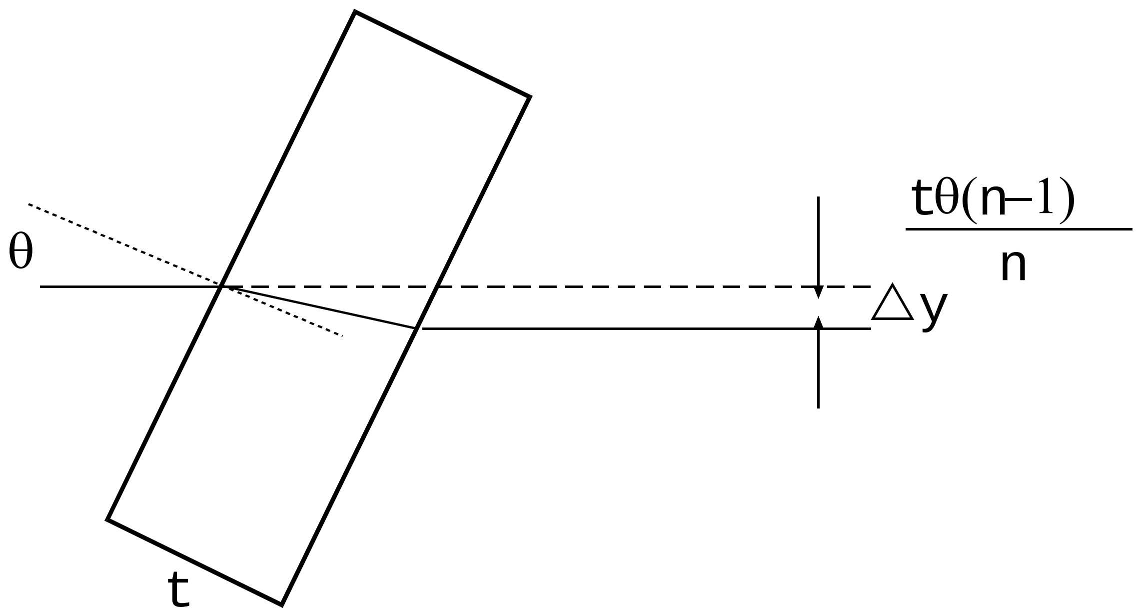


Plane Parallel Plate

No angular change of line of sight

However, tilted plate causes a linear deviation

Approximately:

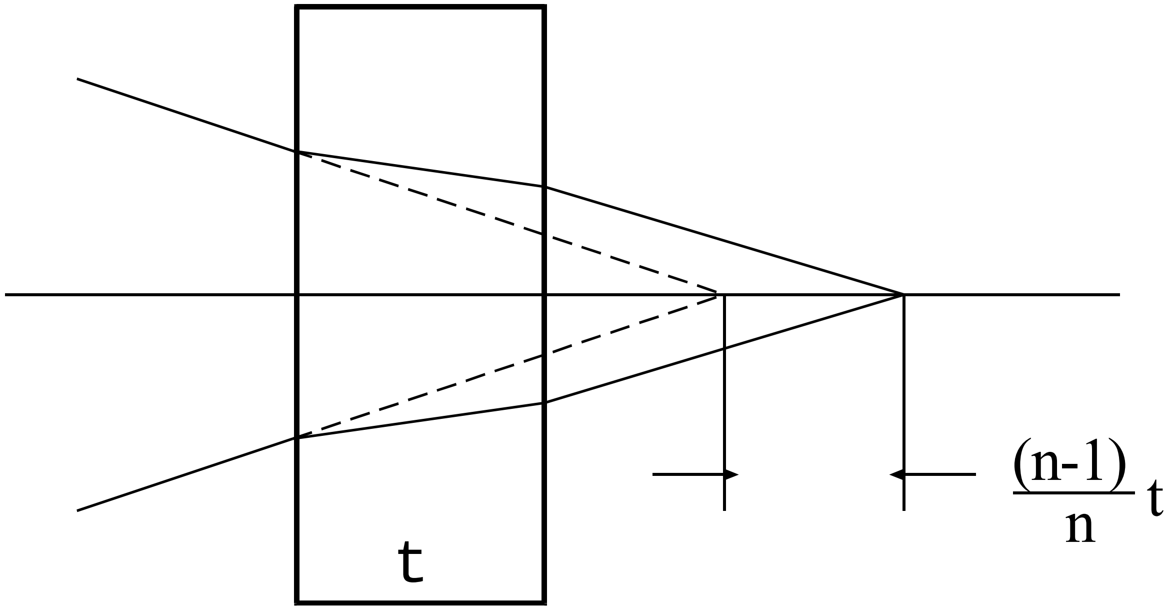


For glass, 45° tilt, $\Delta y \cong \frac{t}{3}$

Plane Parallel Plate

Focus shift in a converging or diverging beam

Approximately:

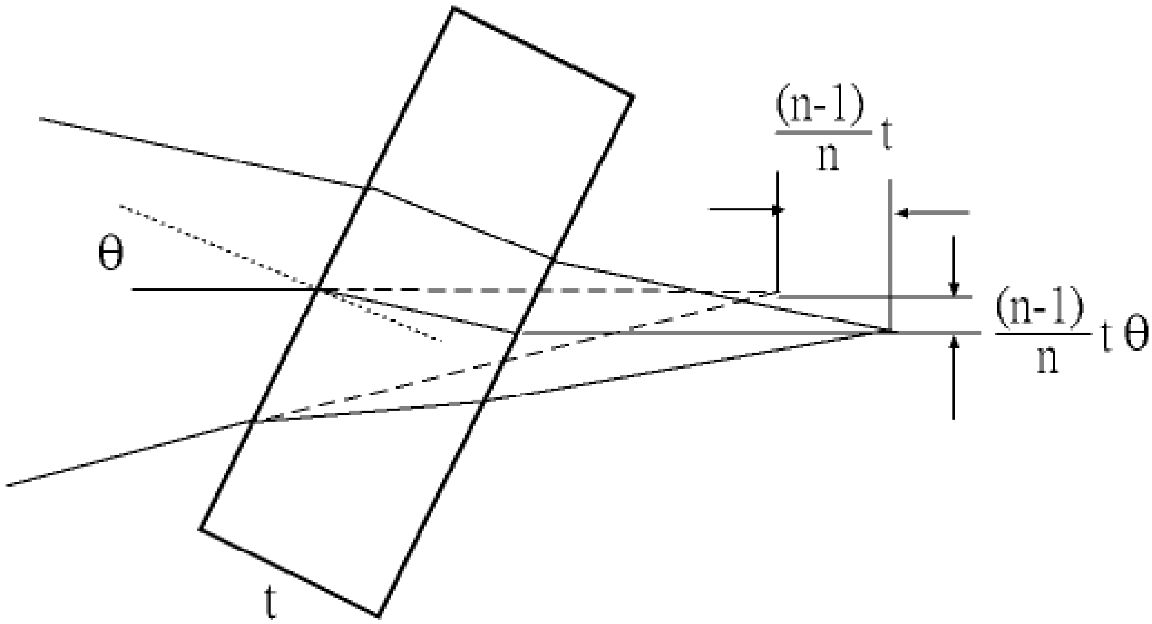


For glass, $\Delta z \cong \frac{t}{3}$

Does not depend on position or orientation

Plane Parallel Plate

in a converging or diverging beam : **causes aberrations**



$$W_{SA} = -\frac{t(n^2 - 1)}{(f/\#)^4 128 n^3}$$

$$W_{COMA} = -\frac{t\theta(n^2 - 1)}{(f/\#)^3 16 n^3}$$

$$W_{ASTIG} = -\frac{t\theta^2(n^2 - 1)}{(f/\#)^2 8 n^3}$$

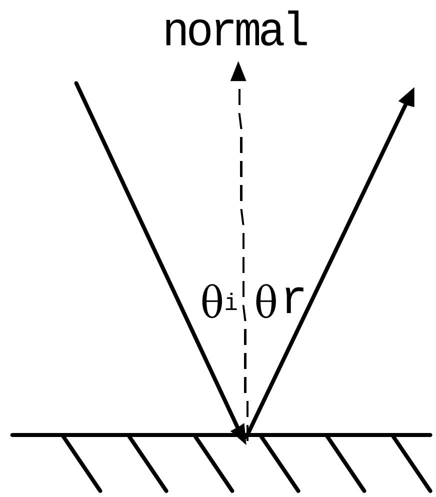
transverse color

$$W_{x\lambda} = \frac{t\theta(n - 1)}{n^2 v}$$

longitudinal color

$$W_{z\lambda} = -\frac{t(n - 1)}{n^2 v}$$

Reflection from a Plane mirror



Law of reflection

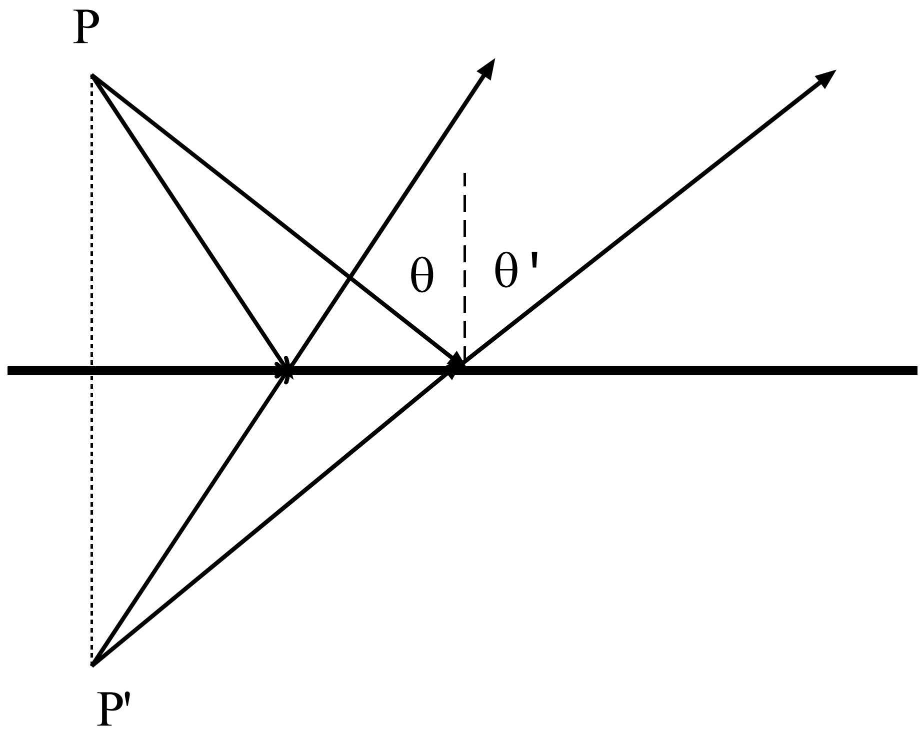
$$\theta_i = \theta_r$$

Reflected ray in plane with incident ray and surface normal

In vector form:

$$\hat{k}_r = \hat{k}_i - 2(\hat{k}_i \bullet \hat{n})\hat{n}$$

Plane mirror creates “mirror image”



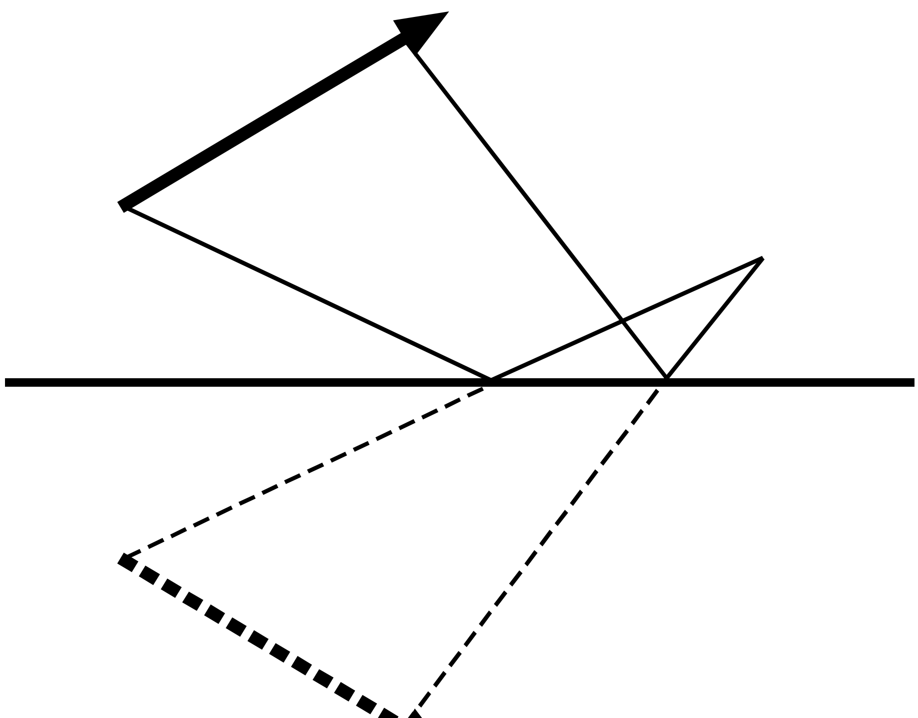
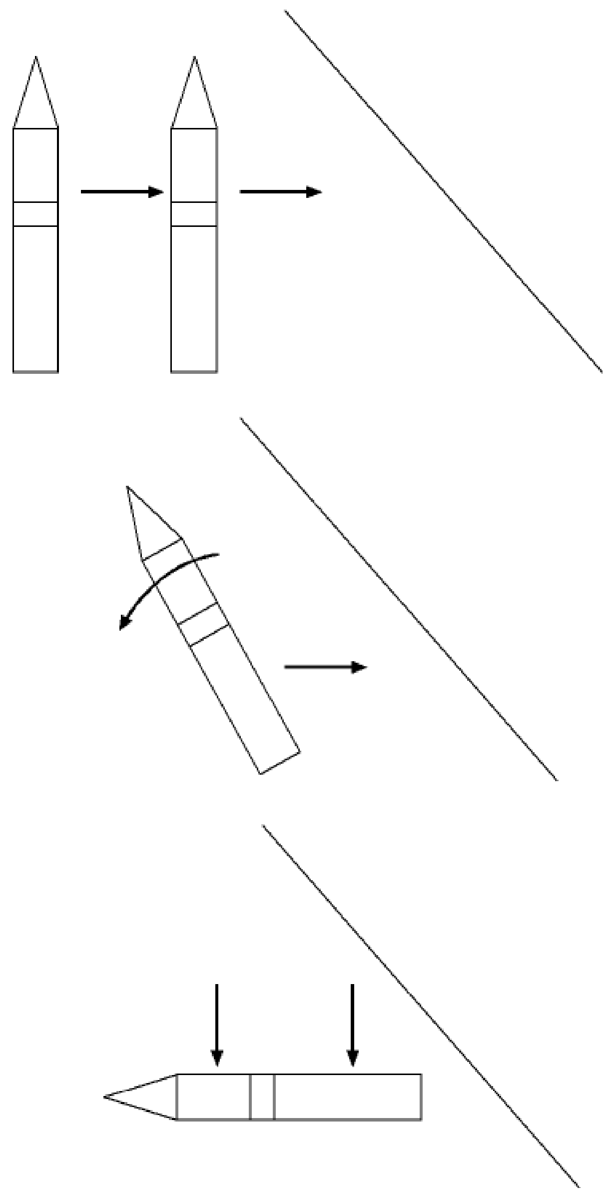
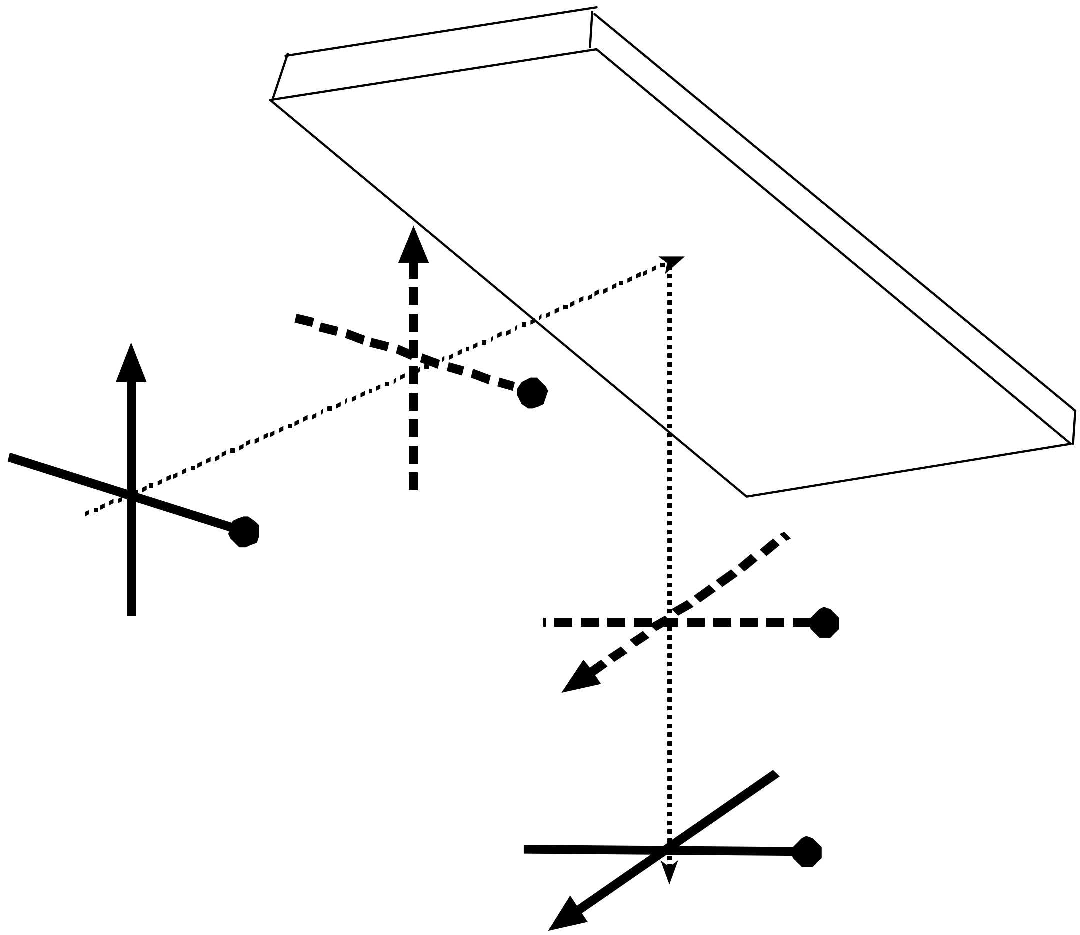
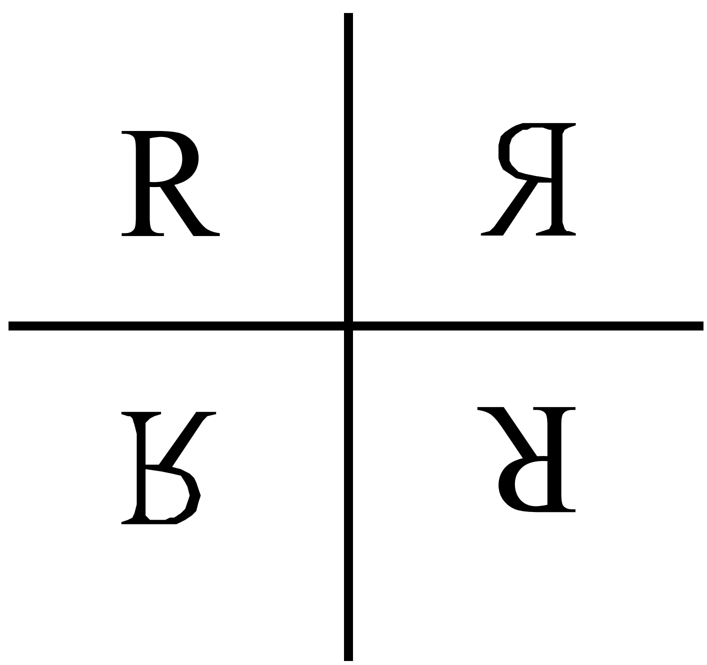


Image orientation

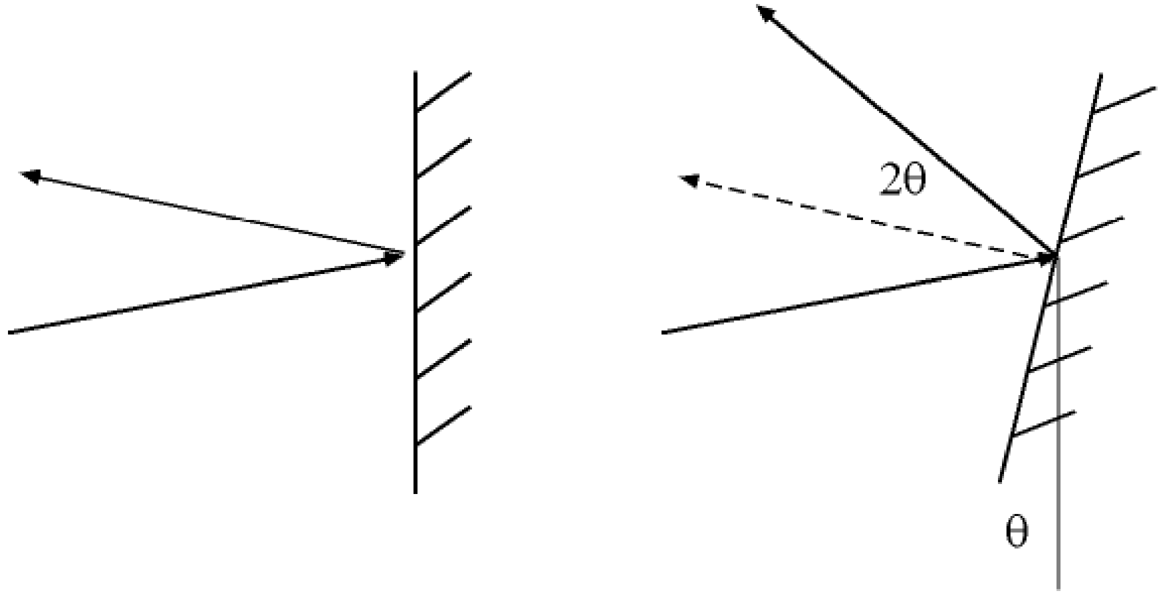


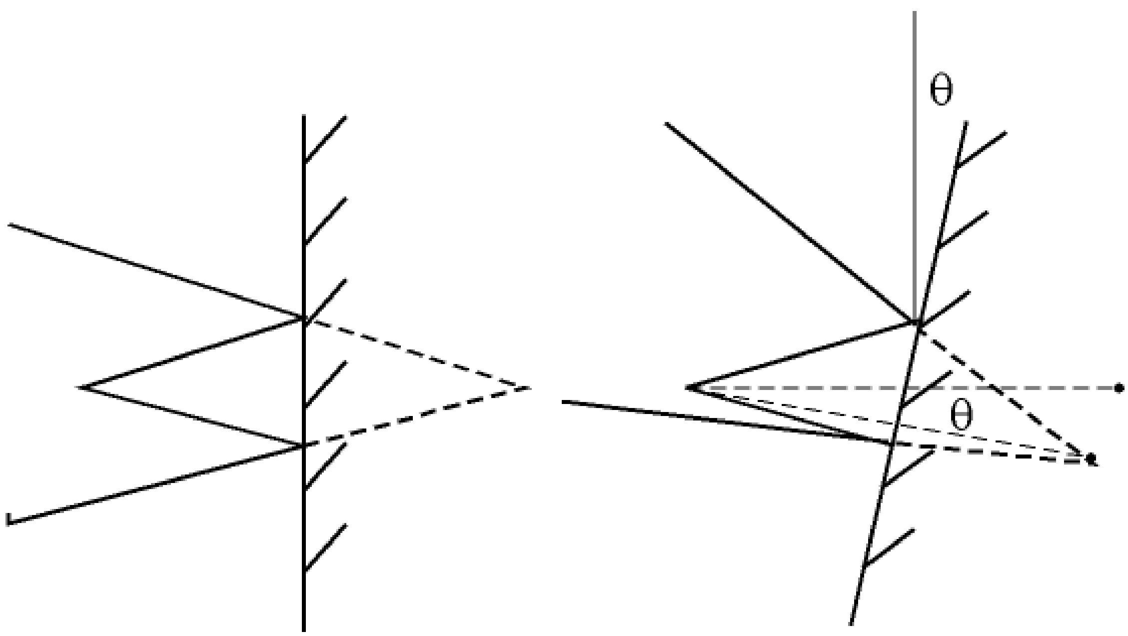




Motion of a plane mirror

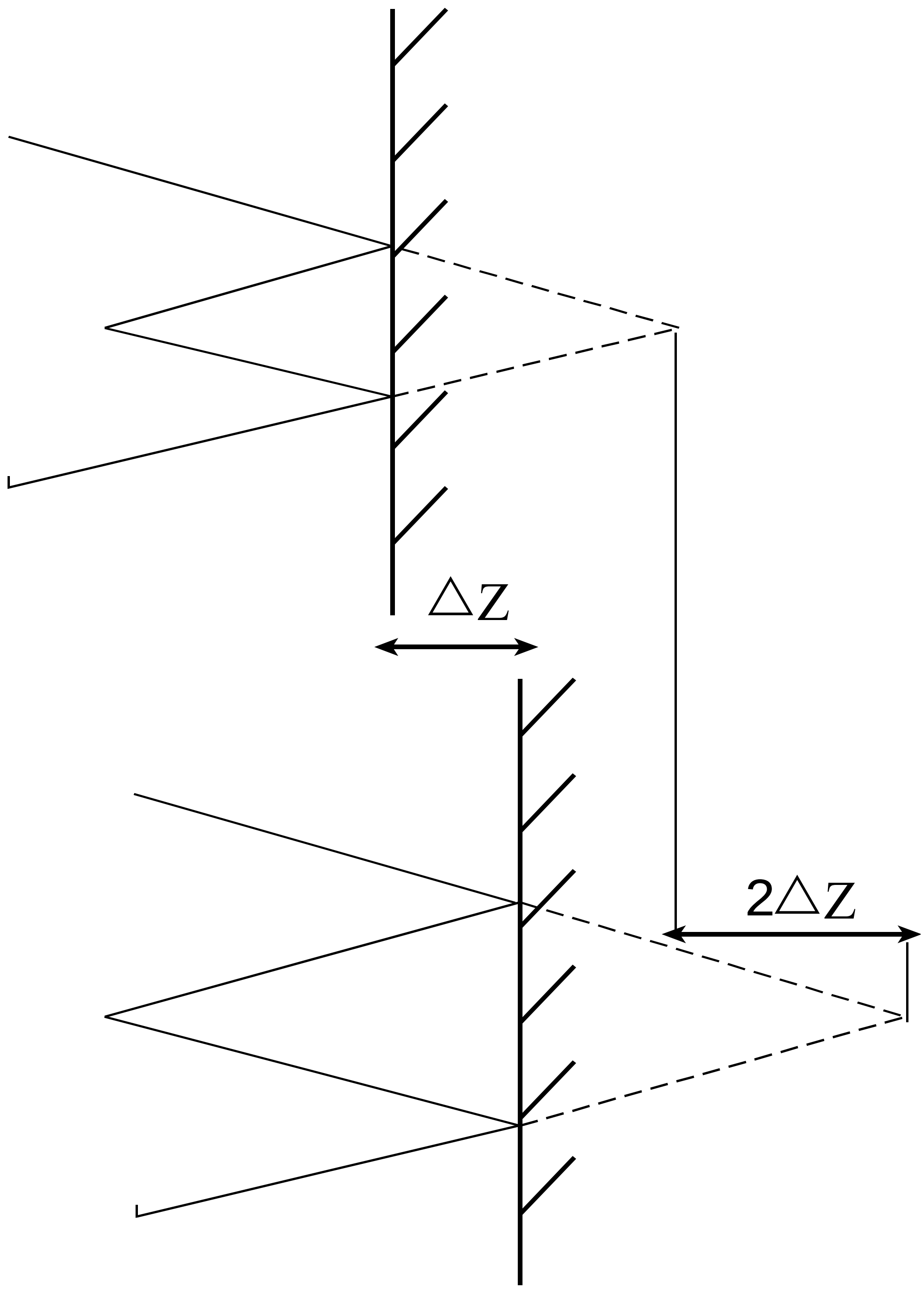
Tilt: LOS is rotated 2 times the mirror motion





Motion of a plane mirror

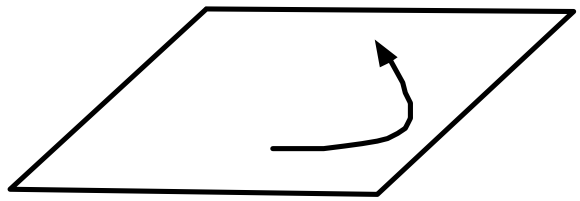
Z translation : image moves two times mirror motion



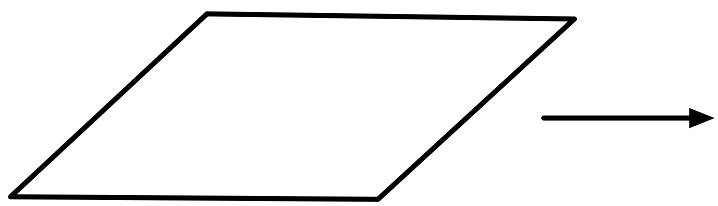
Motion of a plane mirror

Three degrees of freedom do not matter

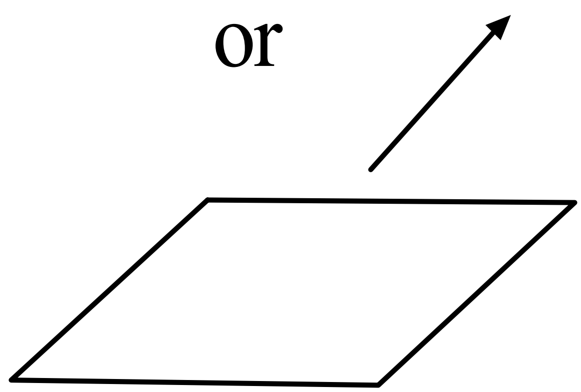
rotation



translation

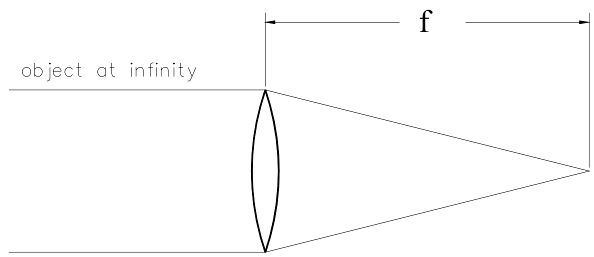


or

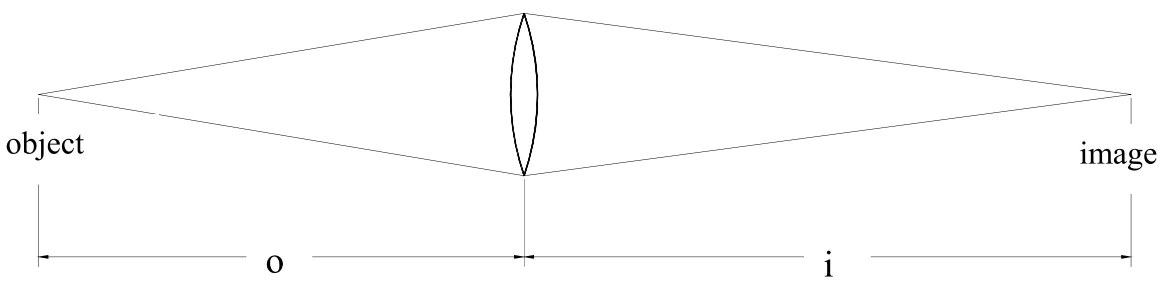


Imaging systems

Positive thin lens, creates real image



f = focal length



$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

magnification

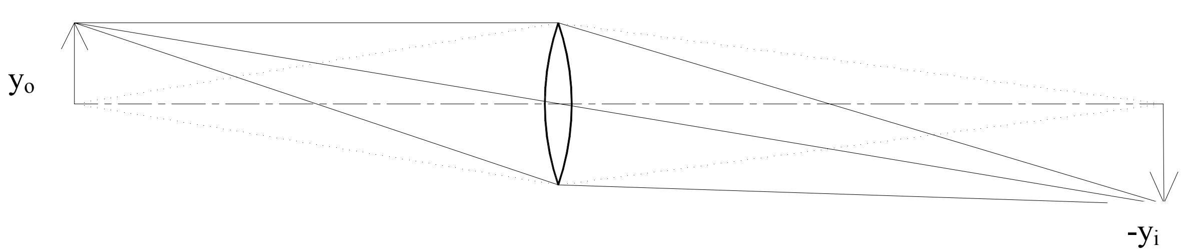


image is rotated 180°, maintains ‘handedness’

lateral magnification $$m = -\frac{y_i}{y_o} = \frac{i}{o}$$

Object at infinity, $m = 0$
Object at focal point, $m = \infty$

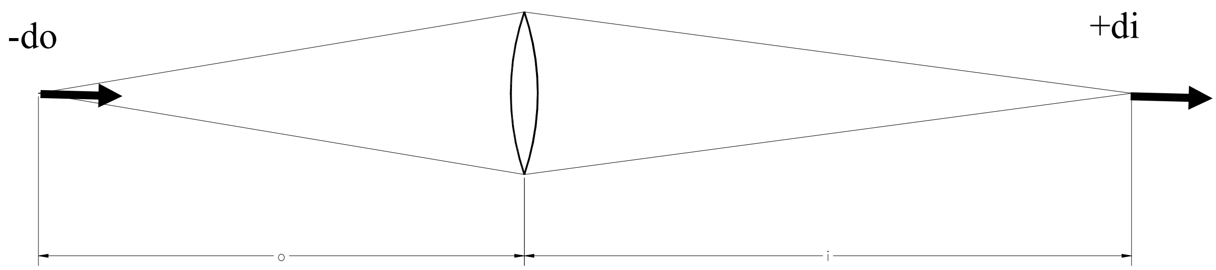
If the object moves, how much does the image move?

For lateral motion, simply scales by magnification

Motion of y_0 in object space appears as y_i in image space

$$\frac{y_i}{y_o} = -m$$

What about motion along the axis:



For axial motion, differentiate:

$$\frac{-di}{i^2} + \frac{-do}{o^2} = 0$$

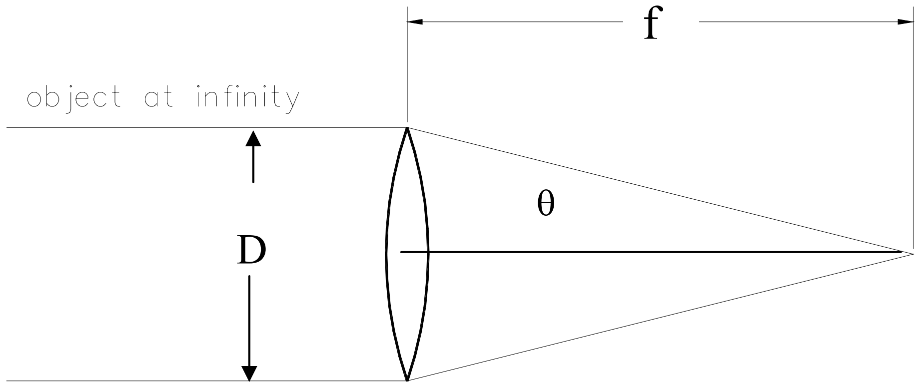
$$\frac{di}{do} = -\frac{i^2}{o^2} = -m^2$$

This is often called the axial magnification

(Object and image always move in the same direction)

Focal ratio

Simple case
stop at lens, object at infinity



f/number : $F\# = \dfrac{f}{D}$

100 mm focal length, 10 mm diameter lens -- *f*/10

<u>Numerical aperture NA</u>
(in medium with refractive index n)

u=sinθ

$$NA = n \sin\theta \quad \cong \frac{1}{2F\#}$$

<u>Diffraction limit:</u>

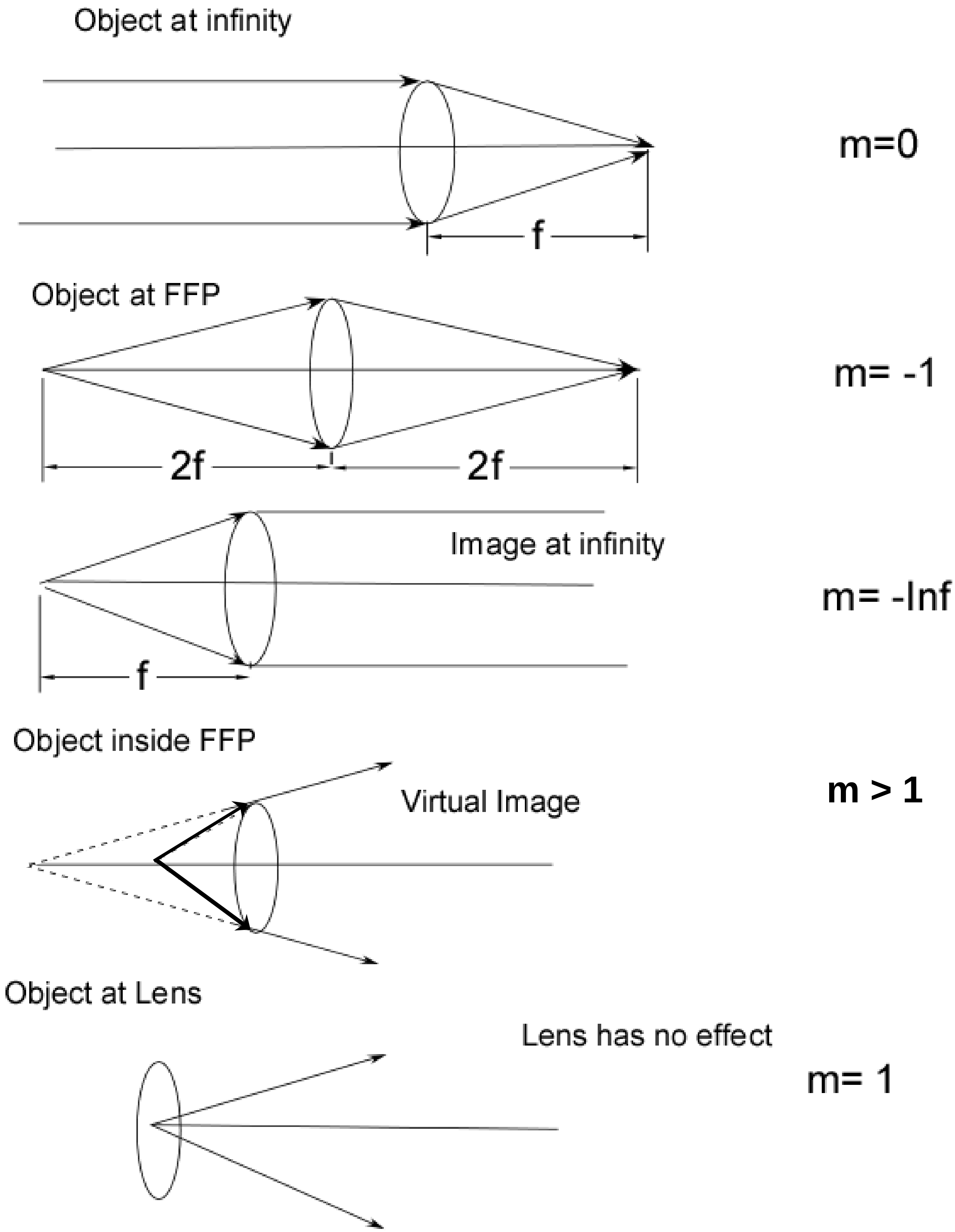
Width of Airy function = 2.44 λ F#

(FWHM = λ F#)

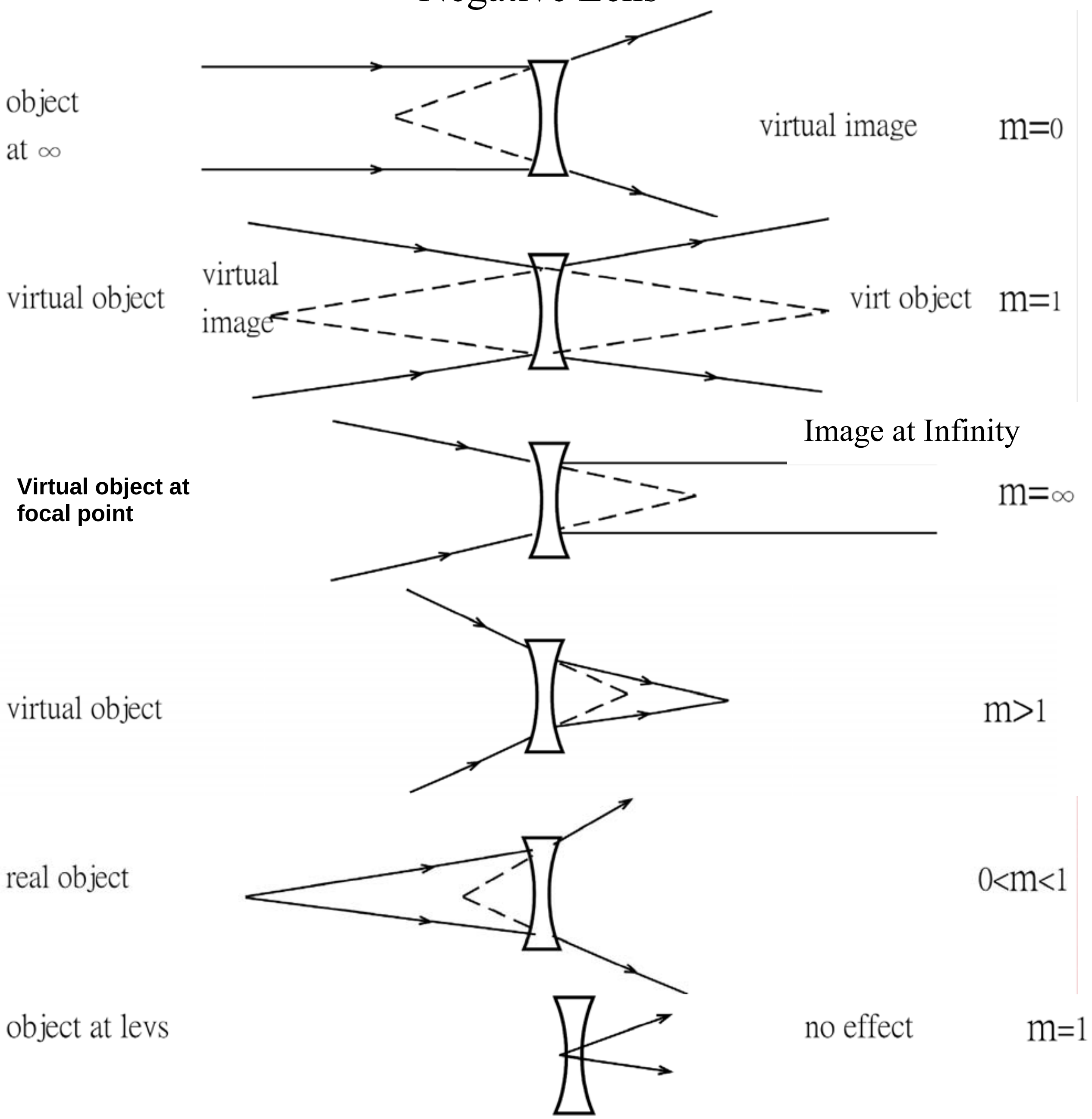
Depth of focus : $\Delta z = \pm 2\,\lambda\,(F\#)^2$

MTF cutoff : $f_c = 1/(\lambda\, F\#)$

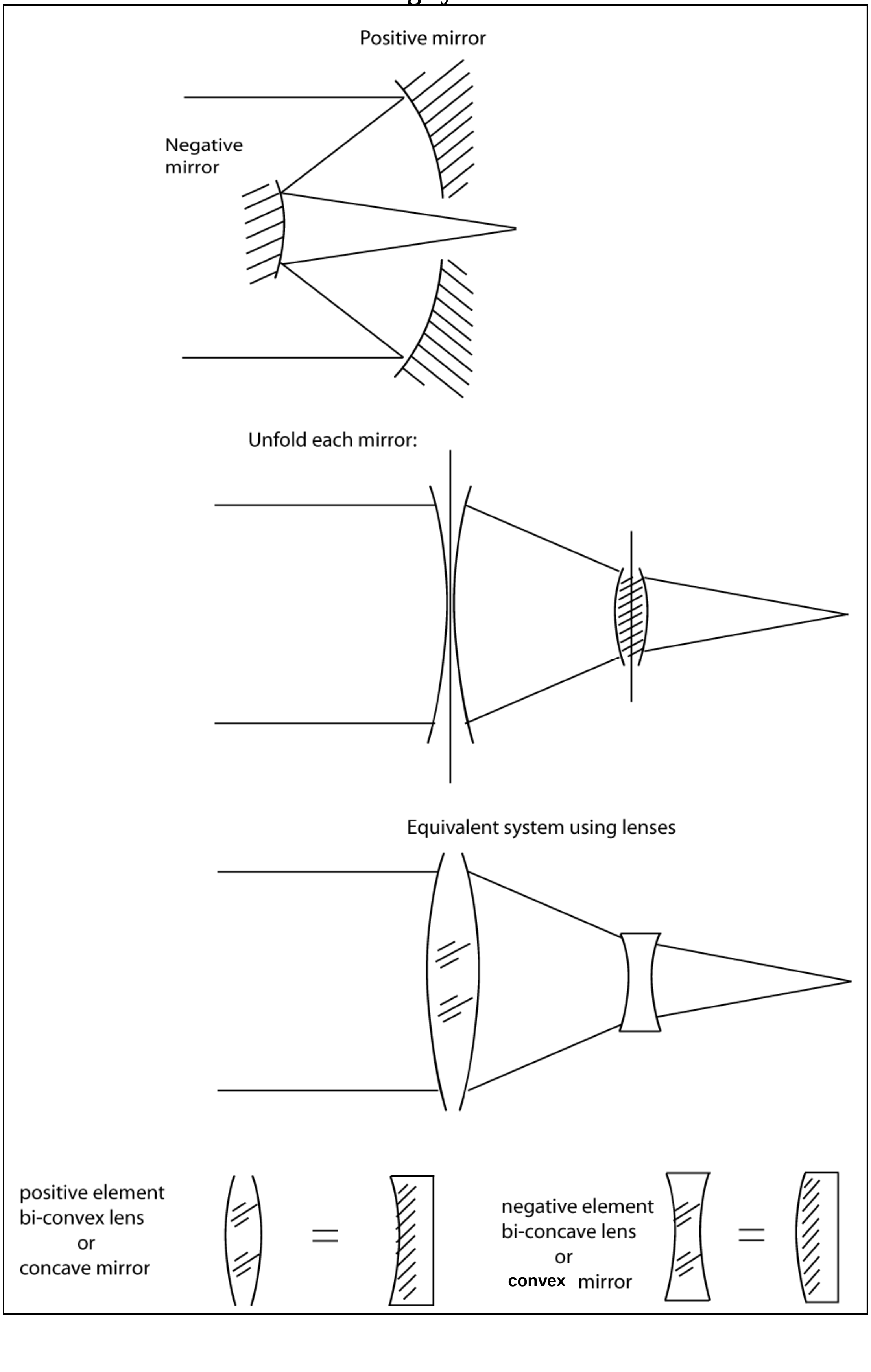
Positive lens



Negative Lens



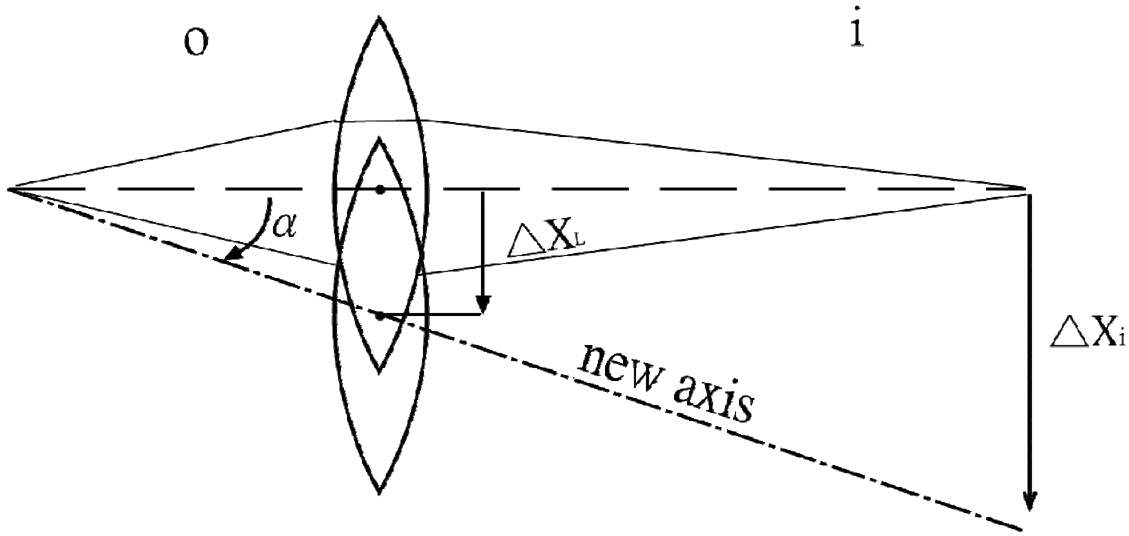
Unfolding systems with mirrors



Lateral motion of lens

We treat the case where the lens moves, yet the object and the image plane do not. To calculate the amount of image motion, simply sketch this out.

You can solve this using similar triangles



$$\frac{i}{o} = -m$$

New Axis , angle $\alpha = \frac{\Delta X_L}{O}$

Image moves $\alpha(o+i) = \Delta X_i$

$$\Delta X_i = \Delta X_L \frac{o+i}{o}$$

$$\Delta X_i = \Delta X_L (1-m)$$

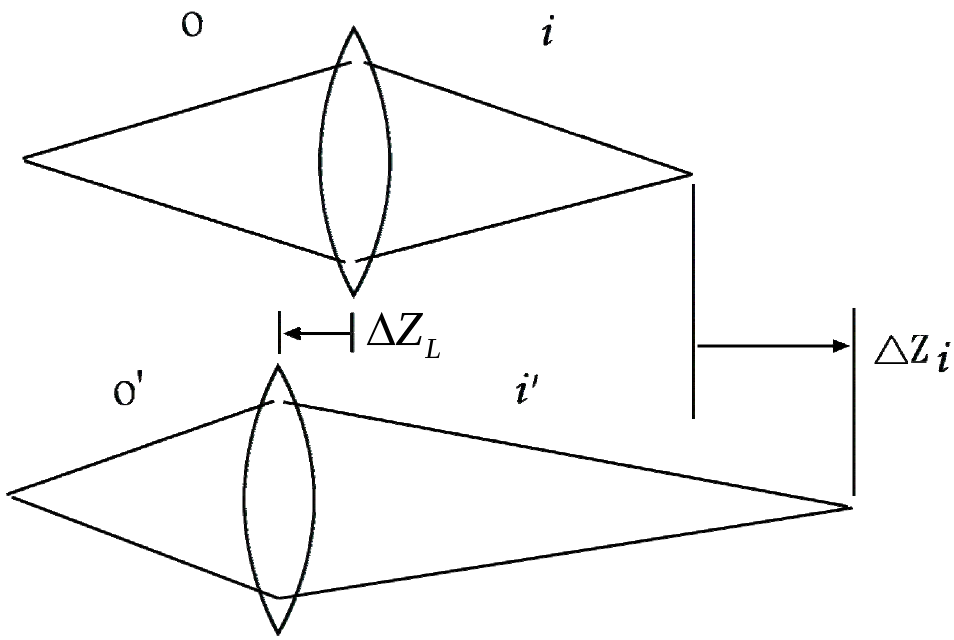
For object at infinity, $\Delta X_i = \Delta X_L$

(Mirrors behave the same way)

Axial motion of lens

We treat the case where the lens moves axially, yet the object and the image plane do not. To calculate the amount of image defocus, you need to be careful. Make a good sketch!

Absolute image motion = Lens motion + (Image motion relative to lens)



$$o' = o - \Delta Z_L \qquad o - o' = \Delta Z_L$$

$$i' = i + \Delta Z_L + \Delta Z_i \qquad i - i' = -\Delta Z_L - \Delta Z_i$$

$$\frac{i - i'}{o - o'} = \frac{\Delta i}{\Delta o} = -m^2$$

$$\frac{-\Delta Z_L - \Delta Z_i}{\Delta Z_L} = -m^2 \quad \text{so} \quad \Delta Z_i = (m^2 - 1)\Delta Z_L$$

Object at infinity, $m = 0$, $\Delta Z_i = -\Delta Z_L$

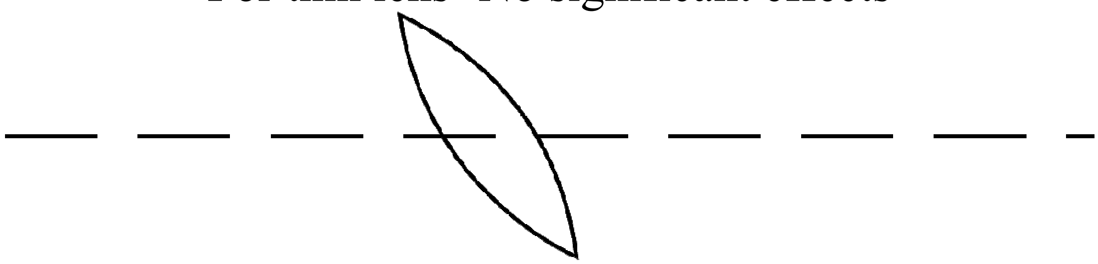
1:1 conjugate, $m = -1$, $\dfrac{\Delta Z_i}{\Delta Z_L} = 0$ (stationary point)

Be careful with mirrors!

Tilt of optical element

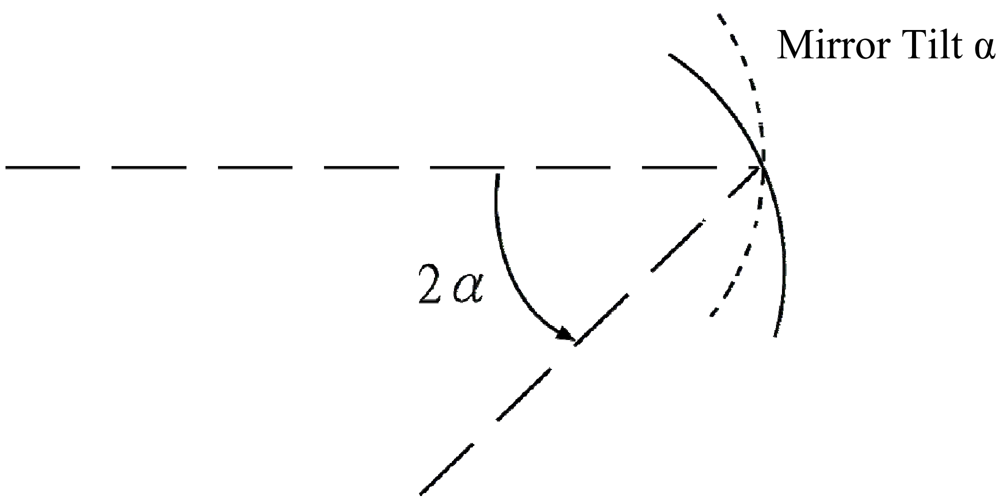
Tilt an element about its center, what happens to the image?

For thin lens- No significant effects



(Large tilt cause aberrations)

For Mirrors



Follow the chief ray!!

Motion of detector

The “detector” could be film, CCD, fiber end, …

What we care about is motion of the image *with respect to the detector*.

This motion would cause a blurred image, tracking error, or degraded coupling efficiency.

If the image and detector move together, the system performs perfectly. Motion of the detector has the same (but opposite sign) as motion of the image.

Although pointing performance is defined by image motion on the detector, it is usually not specified in image space where problem occurs, but it is referred back to object space.

You must be able to go efficiently back and forth between these two spaces:

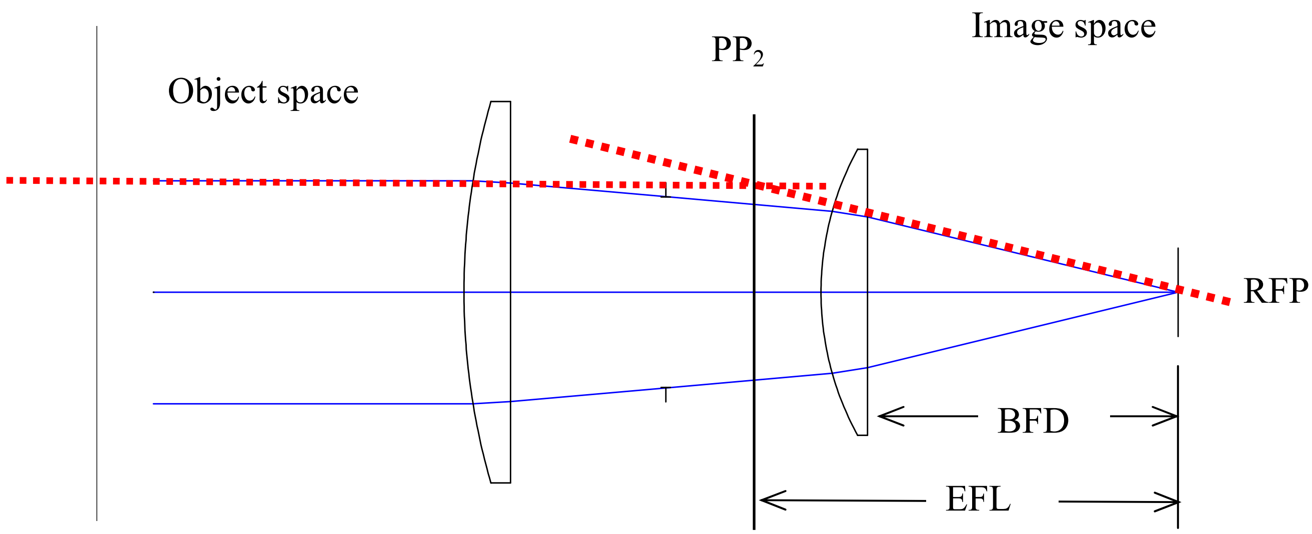
$$\Delta x_i = m \Delta x_o$$

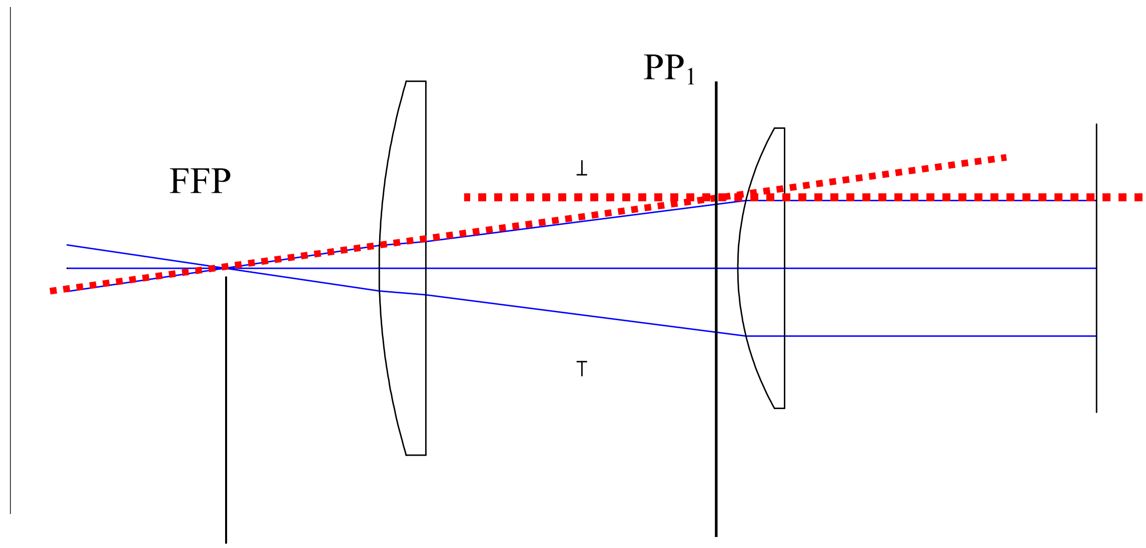
For object at infinity, $m = 0$

$$\Delta x_i = EFL \cdot \Delta \alpha_o$$

Where $\Delta\alpha_0$ gives the angle in object space.

Definition of cardinal points – project rays from object and image space



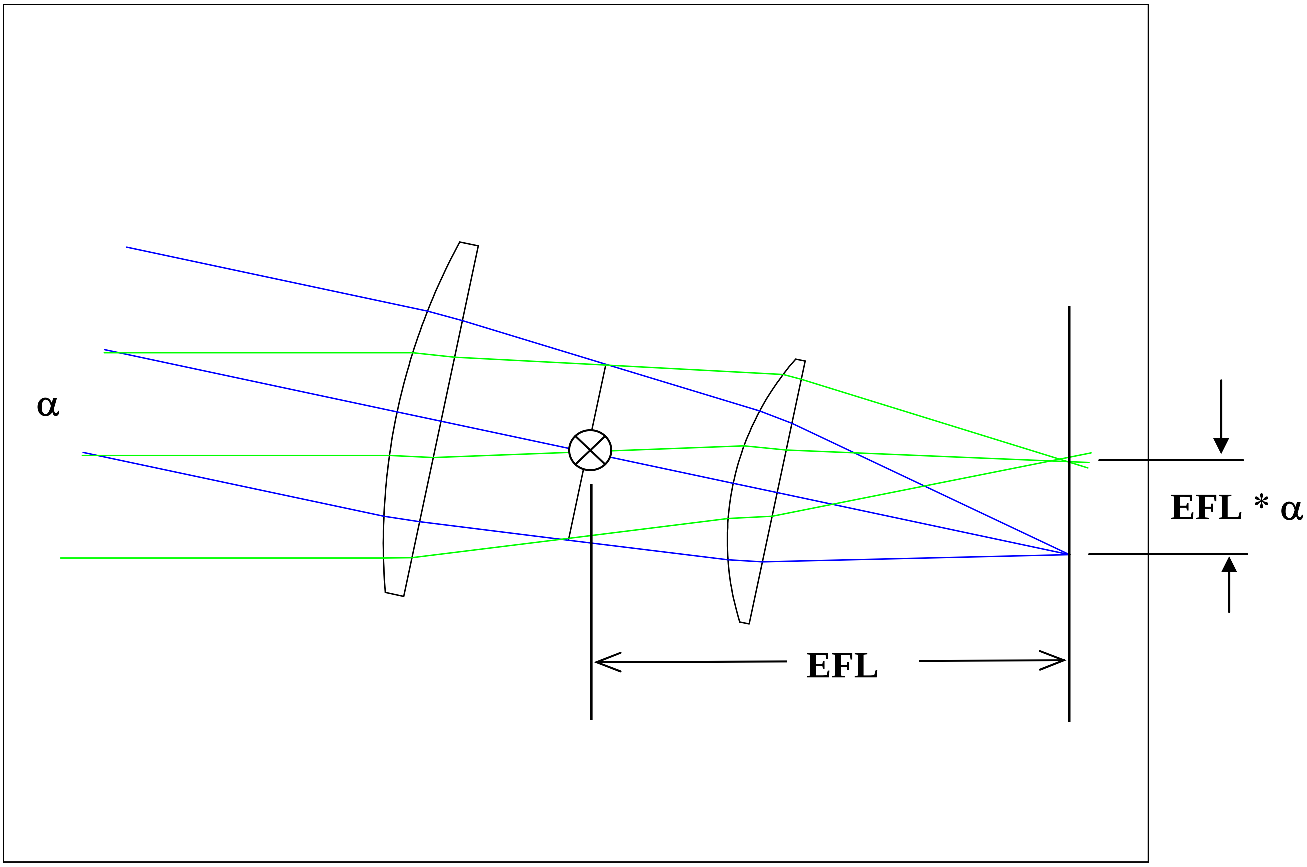


PP1: Front principal point
PP2: Rear principal point

FFP: Front focal point
RFP: Rear focal point (image of object at infinity)

EFL: Effective focal length
BFD:Back focal distance

Nodal point at rear principal plane



In air, object at infinity, nodal point is coincident with rear principal point

Rotation of lens system about nodal point does not move image

Simple proof (for images in air):
Object at field angle α has image height of EFL x α relative to axis
Lens rotation α about **PP_2** moves system axis at focal plane by EFL x α
Lens rotation α causes a fixed object to shift by angle -α relative to axis

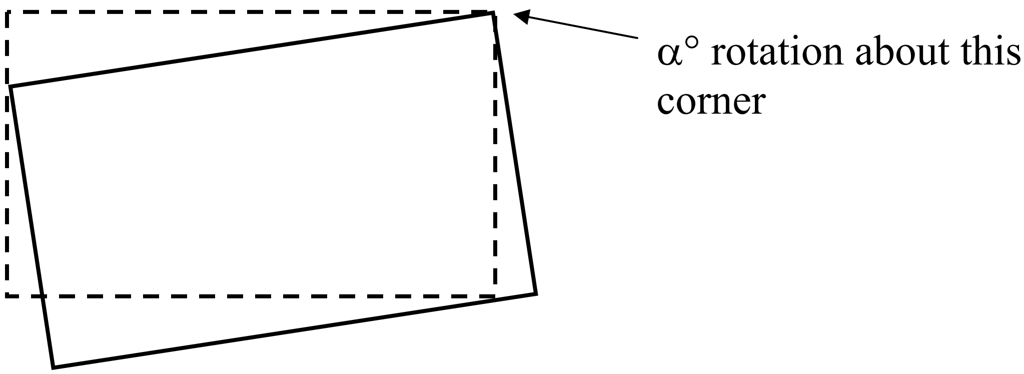
The absolute image motion is

image motion relative to lens axis	=	EFL x -α
+ motion of lens axis		+ EFL x α
————————————		————
		0, no motion

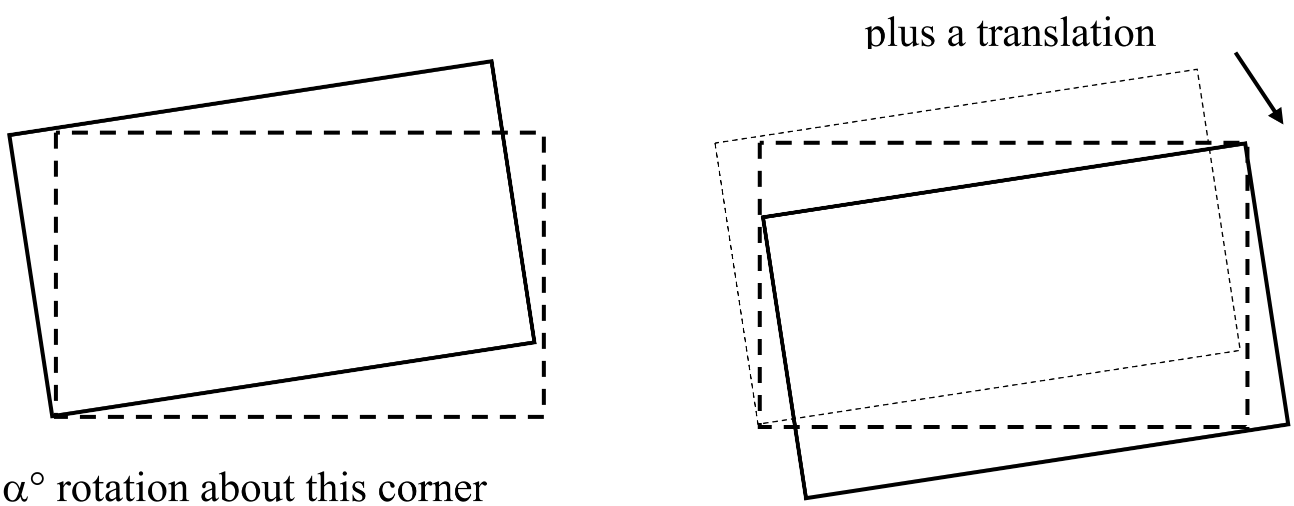
Only for the case where the system is rotated about the rear principal point.

Rigid body rotation

Rotation about one point on an object is equivalent to rotation about any other point plus a translation.



is equivalent to



α° rotation about this corner

(Calculate the magnitude of the translation using trigonometry)

You can choose any point you want to rotate about as long as you keep track of the translation

To calculate effect of rotating an optical system:

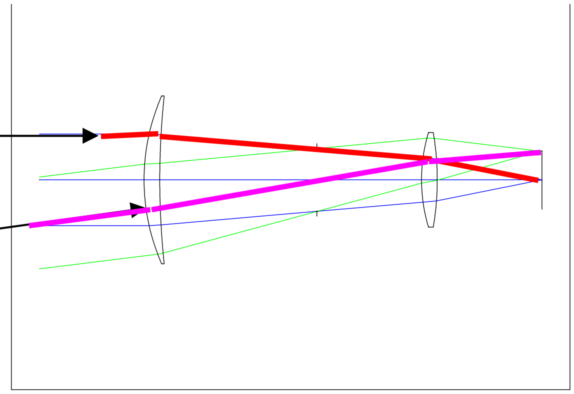
1. Decompose rotation to
   a. translation of the nodal point
   b. rotation about that point
2. Image motion will be caused only by ***translation*** of nodal point

Definition of pupils

Aperture stop
Actual "hole" that defines which rays get through the system

Marginal ray – on axis ray that goes through edge of stop

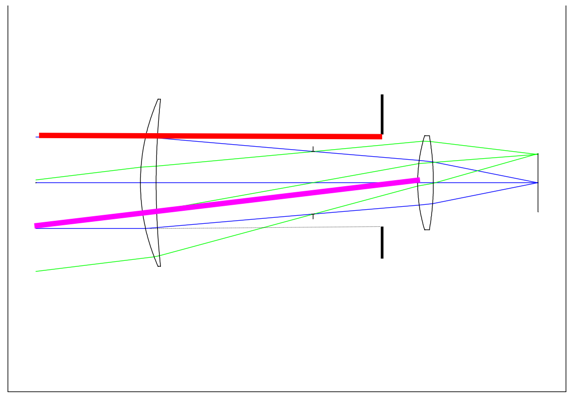
Chief ray – off axis ray that goes through center of stop



<u>Entrance pupil</u>
Image of the stop in object space

Located where chief ray cross the axis in object space

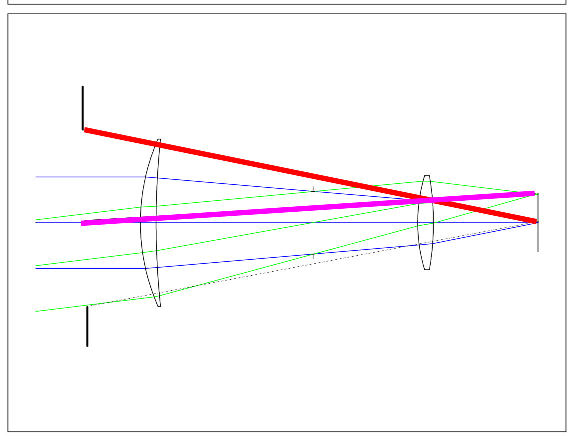
Sized by marginal ray height of pupil image in object space



<u>Exit pupil</u>
Image of the stop in image space

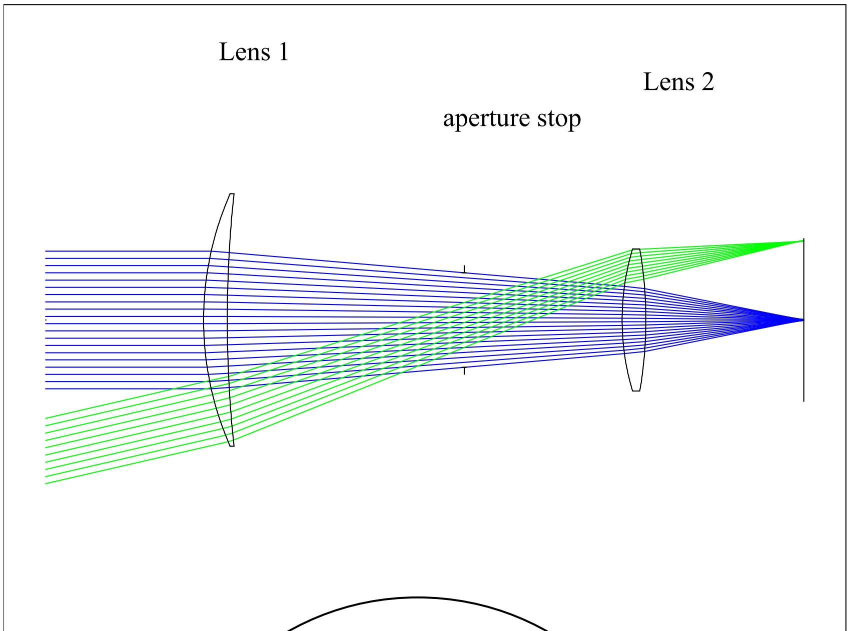
Located where chief ray cross the axis in image space

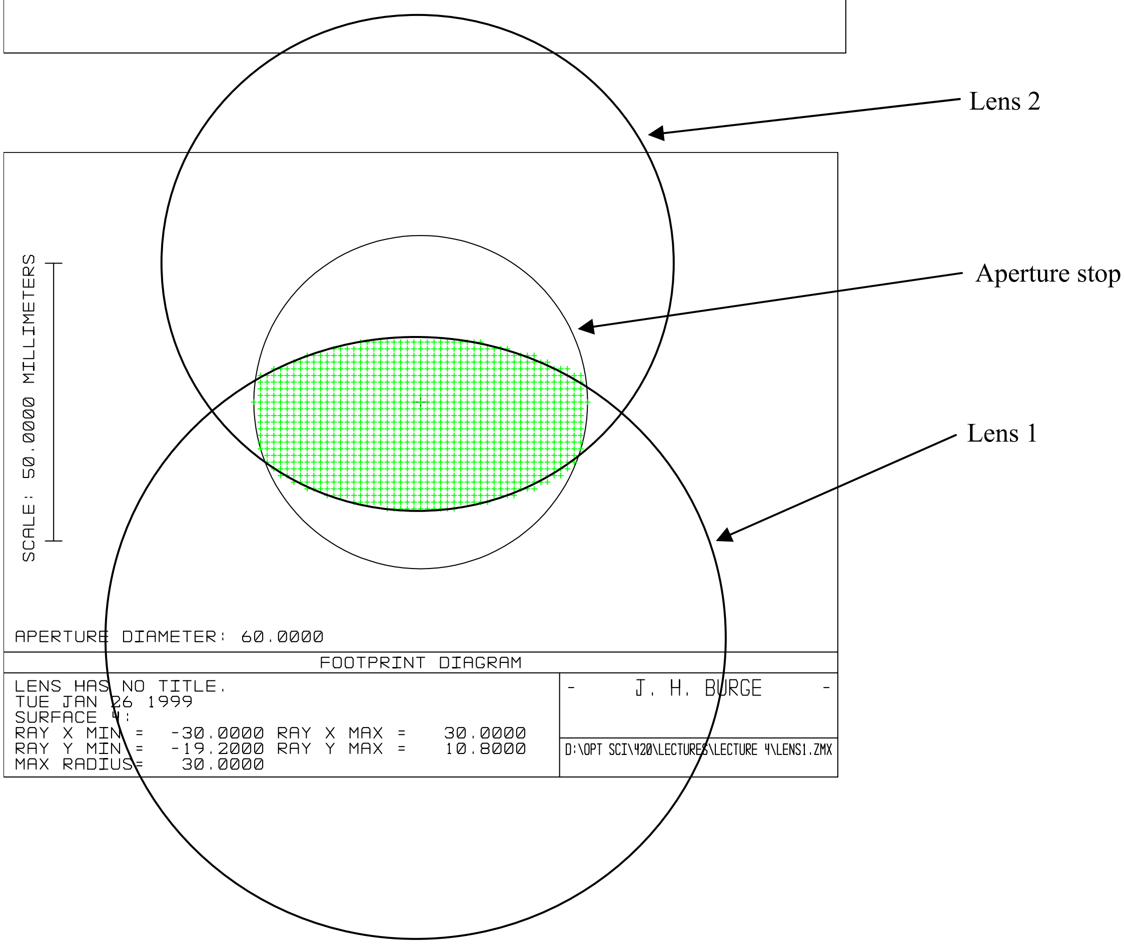
Sized by marginal ray height of pupil image in image space



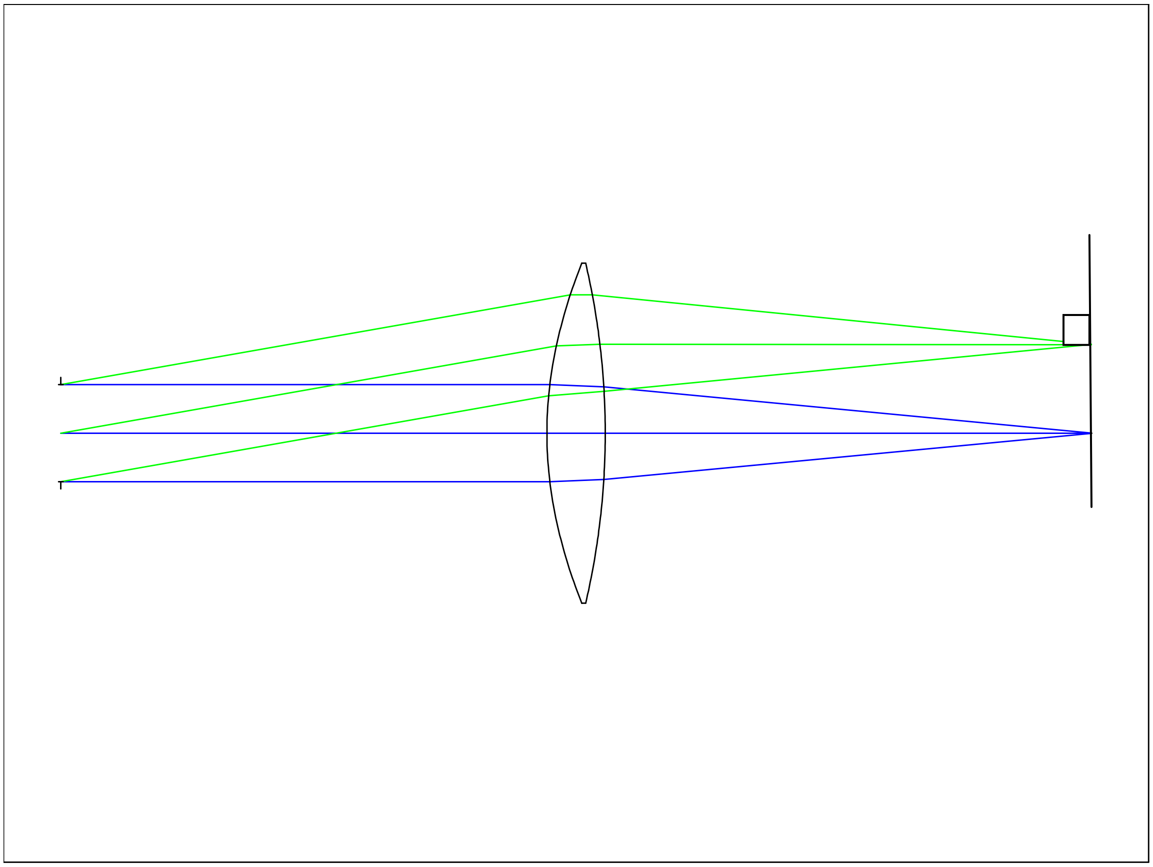
Vignetting

When something other than the aperture defines which rays get through. Leads to loss of light.





Telecentric system



<u>Telecentric in image space</u>
Exit pupil at infinity
Chief ray is normal to image plane

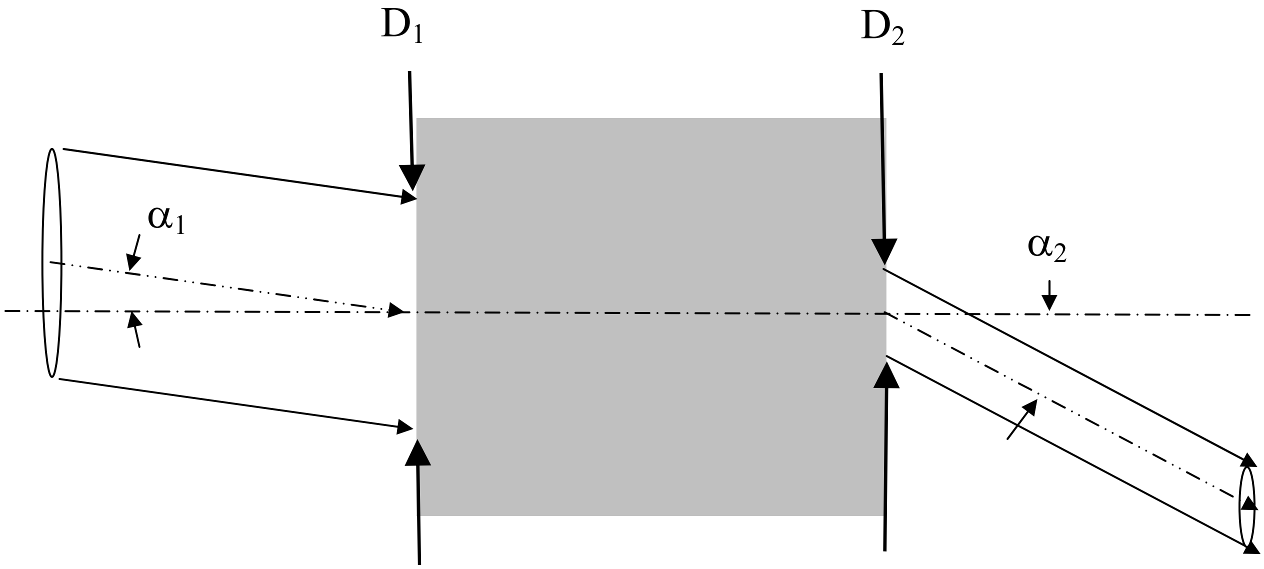
<u>Telecentric in object space,</u>
Entrance pupil at infinity
Chief ray is normal to object plane

Why do we care?

- Going through focus, image does not shift
- Provides uniform illumination across the field
- Some detectors are highly directional (fiber optic bundles)

Afocal systems

Do not create a real image -- object at infinity, image at infinity



D_1 = Entrance Pupil
D_2 = Exit pupil

It makes stuff appear larger – magnifying power

$$MP = \frac{\alpha_2}{\alpha_1}$$

LaGrange Invariant requires $D_1\alpha_1 = D_2\alpha_2$

Examples:
Galilean, Keplerian telescope, laser beam projector
Binoculars