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3. Image motion due to optical element motion

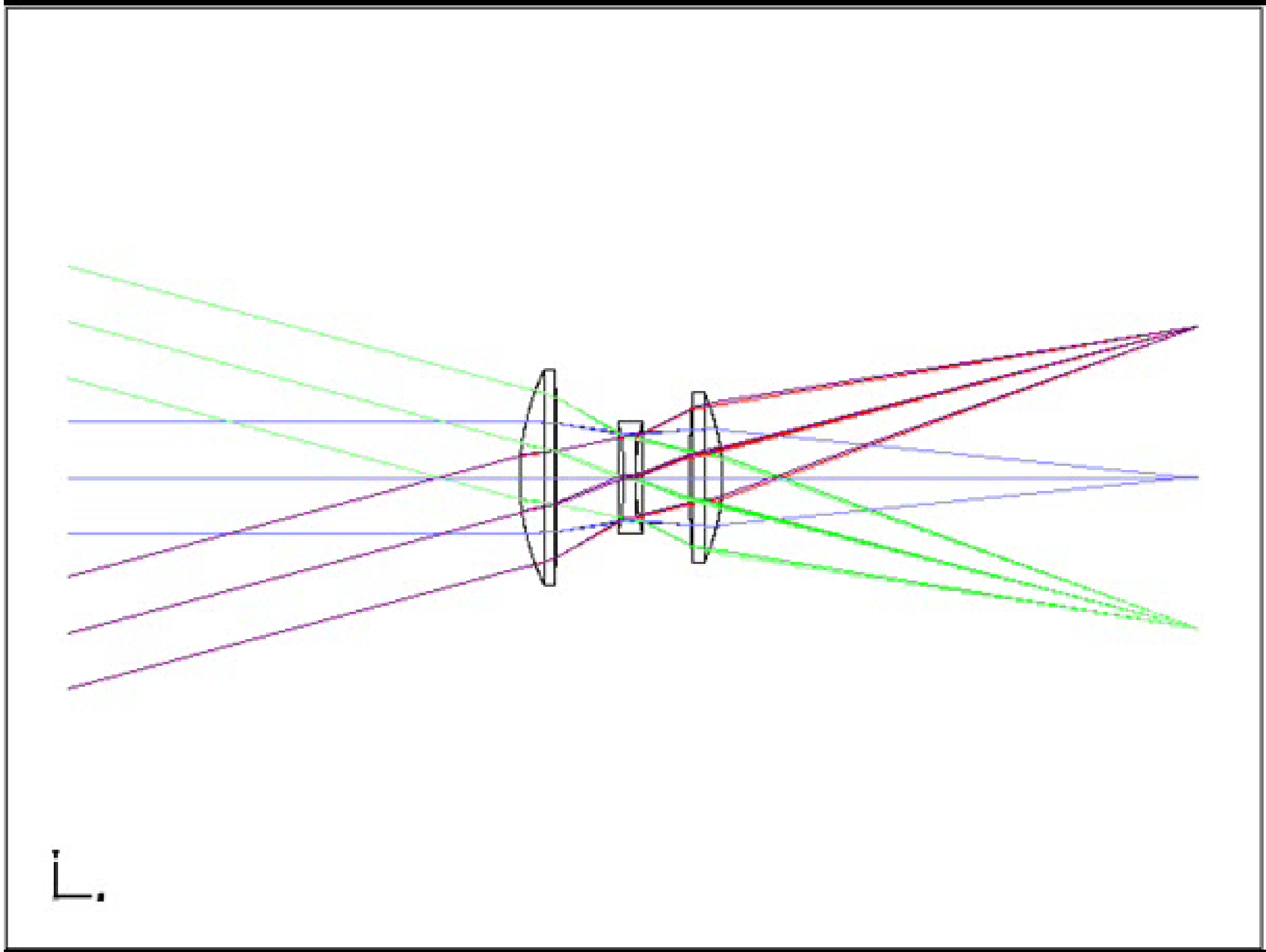
- Tilt and decenter of optical components (lenses, mirrors, prisms) will cause motion of the image
  - Element drift causes pointing instability
    - Affects boresight, alignment of co-pointed optical systems
    - Degrades performance for spectrographs
  - Element vibration causes image jitter
    - Long exposures are blurred
    - Limit performance of laser projectors

Small motions, entire field shifts (all image points move the same)

Image shift has same effect as change of line of sight direction
(defined as where the system is looking)

J. H. Burge, "An easy way to relate optical element motion to system pointing stability," in *Current Developments in Lens Design and Optical Engineering VII*, Proc. SPIE **6288** (2006).

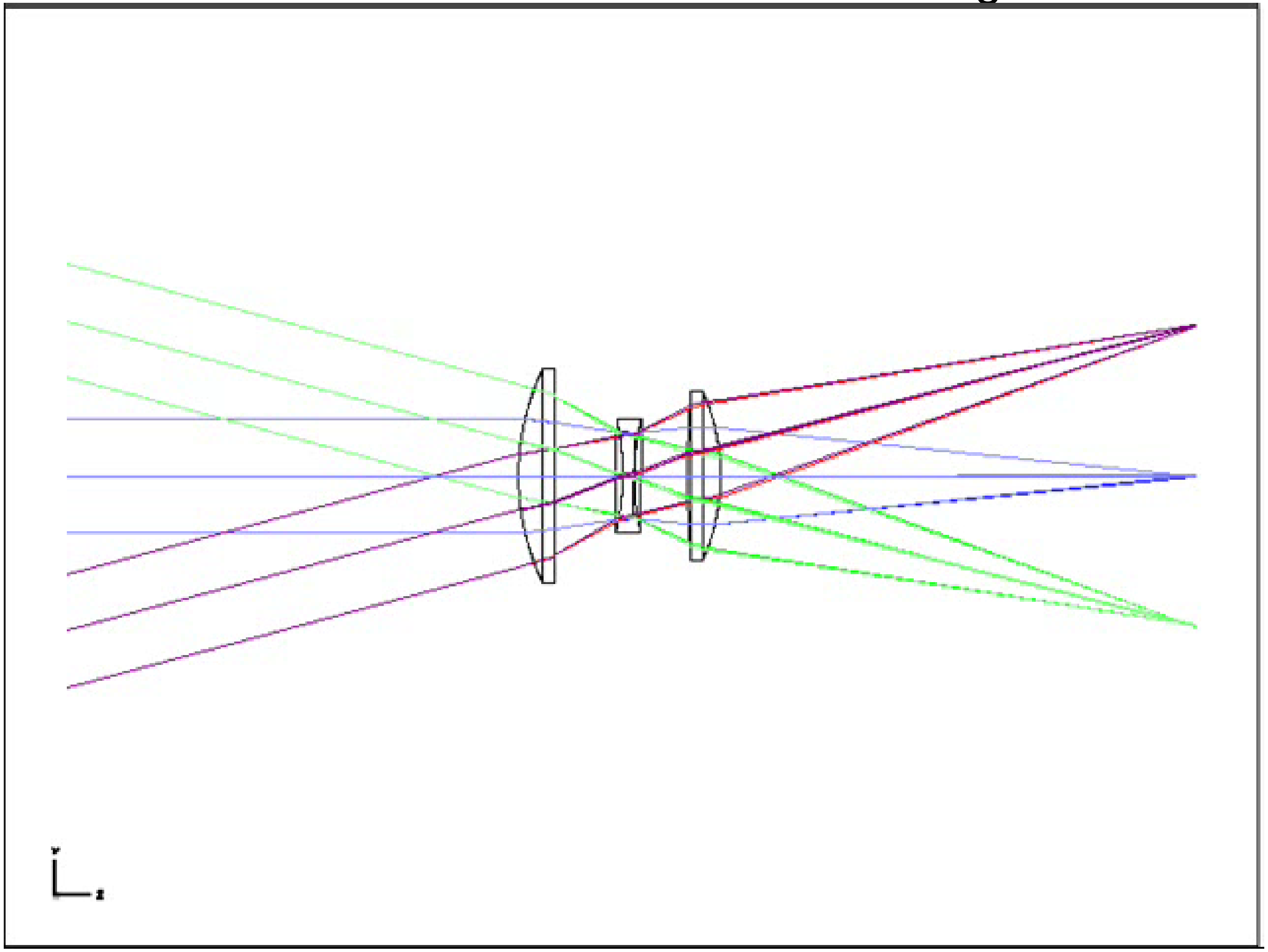
Lens decenter



- All image points move together
- Image motion is magnified

Effect for lens tilt

- Can use full principal plane relationships
- Lens tilt often causes more aberrations than image motion



What happens when an optical element is moved?

To see image motion,
follow the central ray

Generally, it changes in position and angle

<u>Element motion</u>

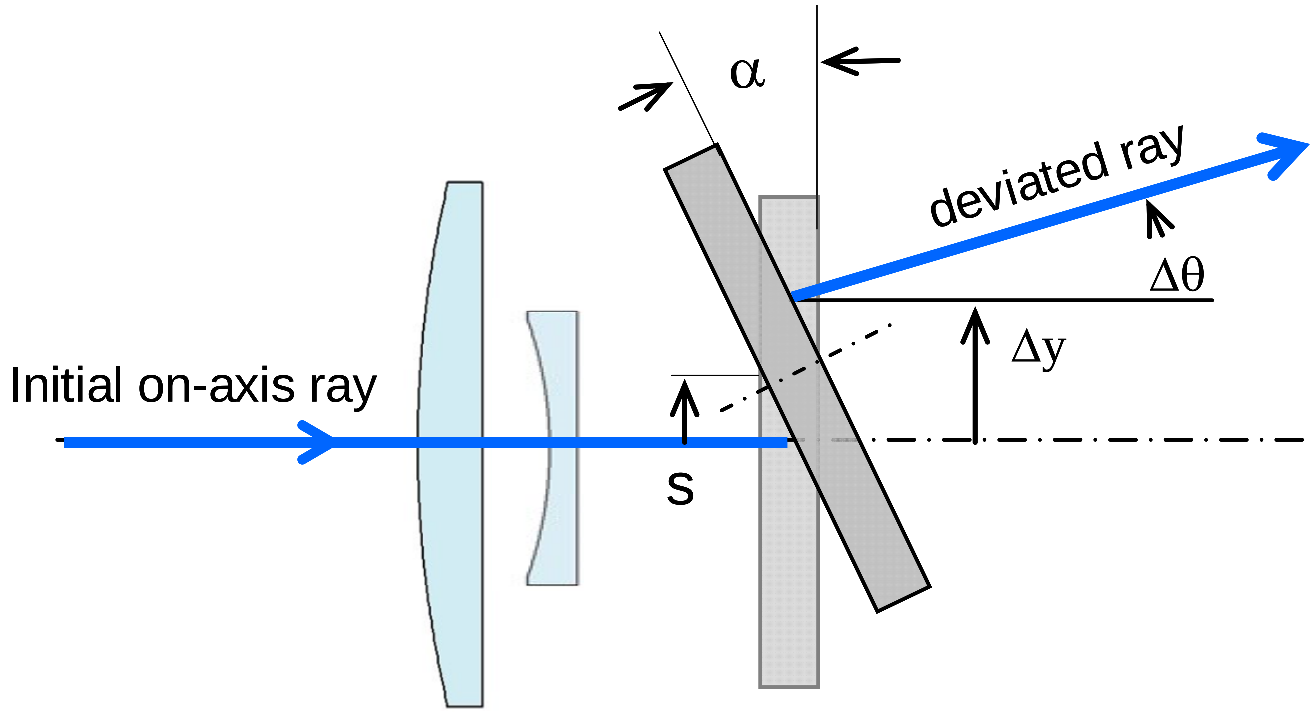
s : decenter

α : tilt

<u>Central ray deviation</u>

Δy : lateral shift

$\Delta\theta$: change in angle



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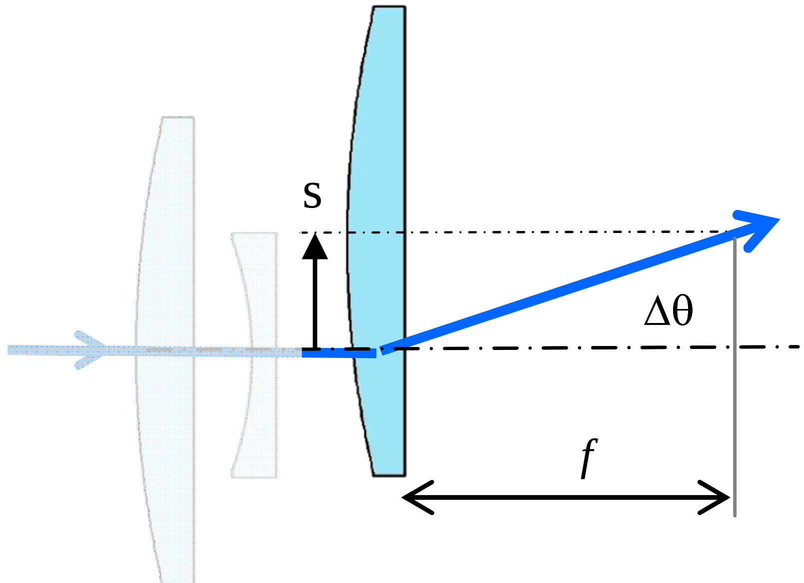
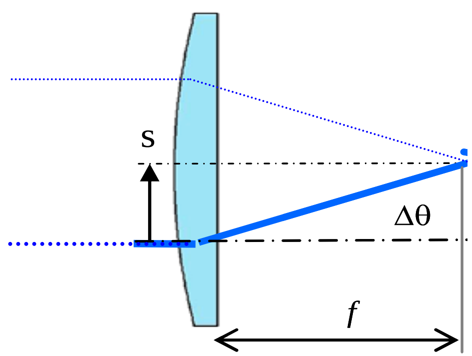
Lens motion

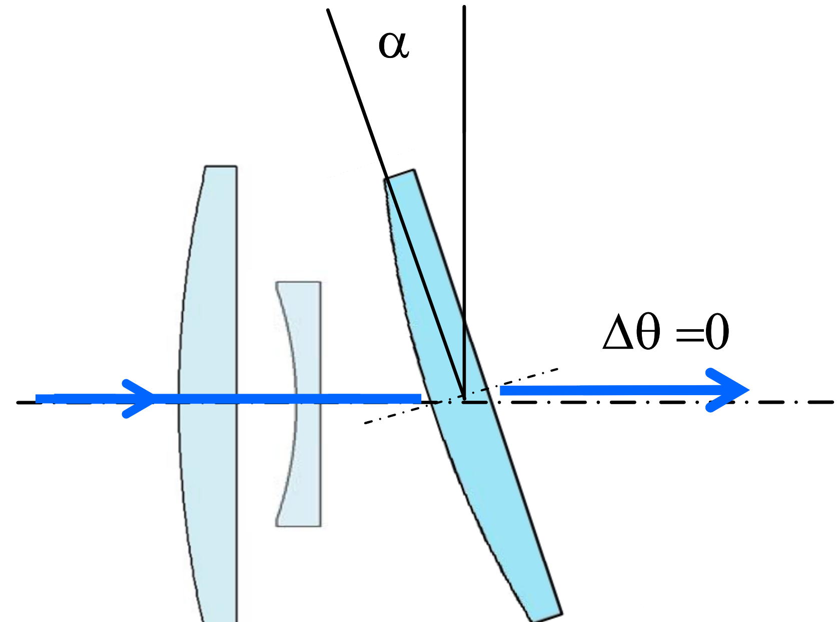
decenter



$$\Delta\theta_s \cong \frac{s}{f}$$



tilt



(Very small effect)

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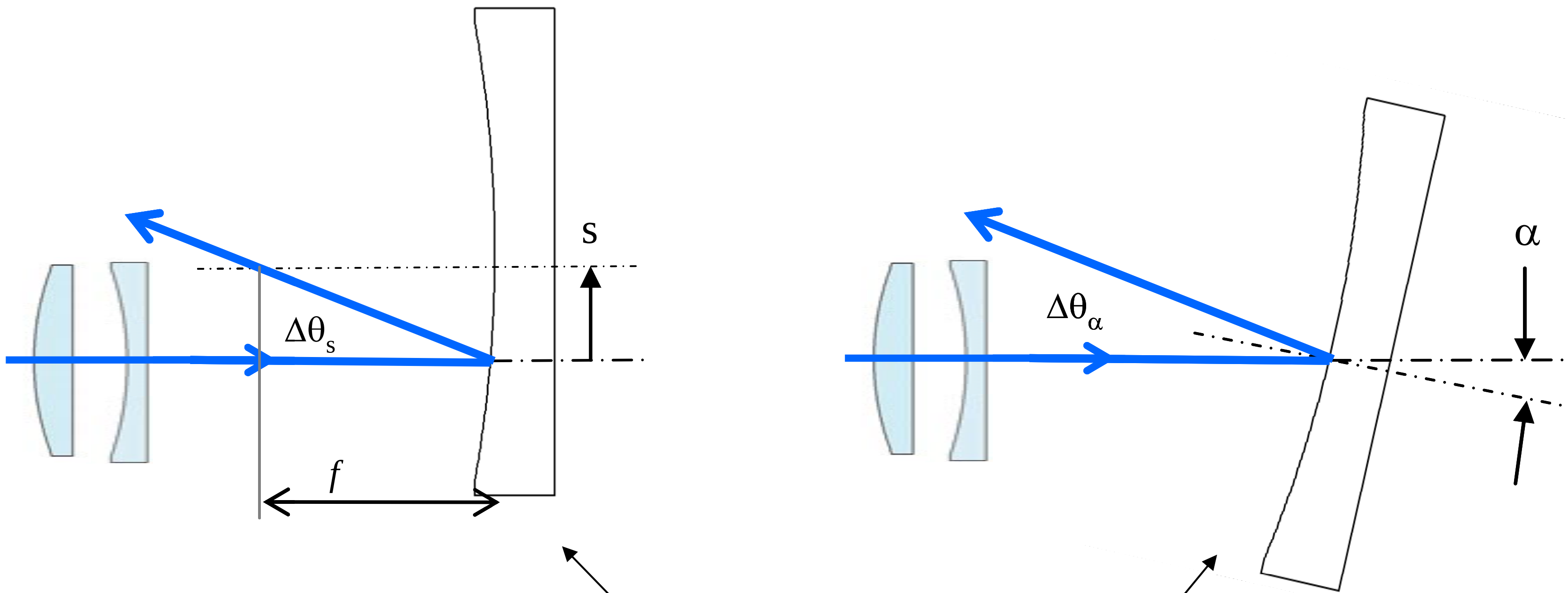
Mirror motion



like lens

$\Delta\theta_\alpha = 2\alpha$
like flat mirror

$$\Delta\theta \cong \frac{s}{f} + 2\alpha$$

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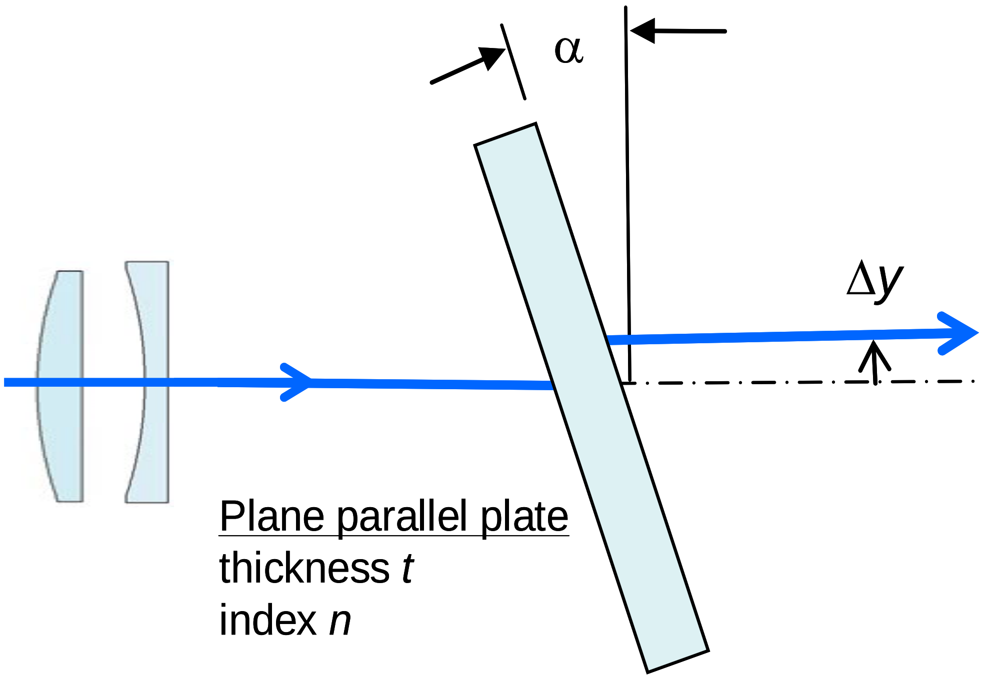
Motion for a plane parallel plate



Plane parallel plate
thickness *t*
index *n*

$$\Delta y \cong \frac{\alpha t(n-1)}{n}$$

No change in angle

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The Optical Invariant

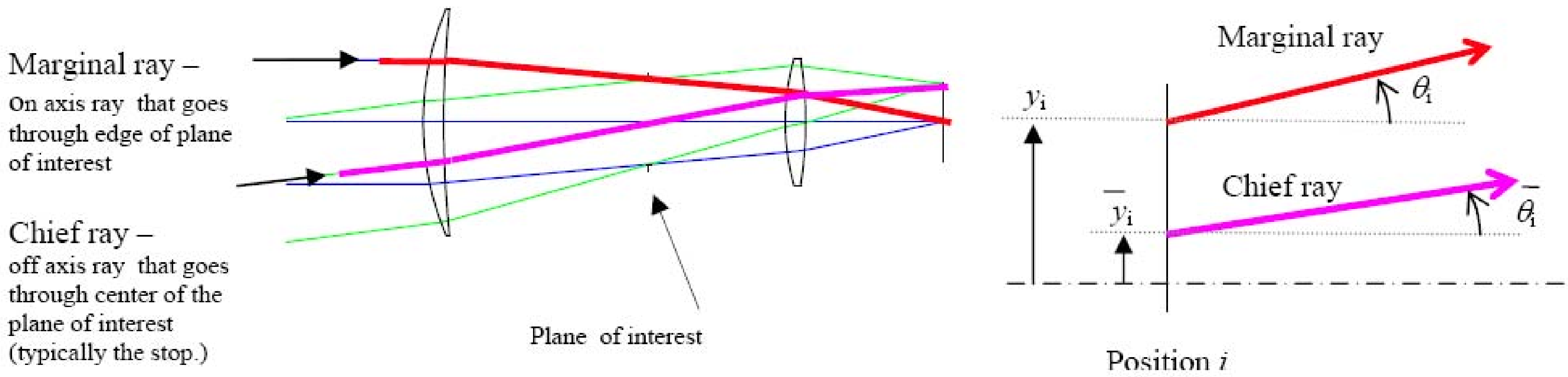


Figure 2. Definition for selection and sign convention for chief and marginal rays.

The effects of different refractive indices can be accommodated by defining the term $u_i = n_i \sin \theta_i$. This equivalent angle u will be used for subsequent analysis in this paper. It is well known that the value computed as

$$I \equiv \bar{u}_i y_i - u_i \bar{y}_i \qquad \text{Eq. 4}$$

The stop is not special. Any two independent rays can be used for this. The optical invariant will be maintained through the system

J. H. Burge

Table 1. Evaluation of optical invariant at surface i and at final image plane

	At surface i	At image plane N
u : marginal ray angle	$u_i = NA_i$ = numerical aperture at i	u_N = NA = $1/2F_n$
y : marginal ray height	$y_i = D_i / 2$ defined by beam footprint	0
$\bar{u}$: chief ray angle	$\bar{u}_i = \Delta\theta_i$ due to element motion	$\bar{u}_N$
$\bar{y}$: chief ray height	$\bar{y}_i = \Delta y_i$ due to element motion	$\bar{y} = \Delta y_N \equiv \varepsilon$, image motion
$I \equiv \bar{u}_i y_i - u_i \bar{y}_i$: optical invariant	$\frac{D_i}{2}\Delta\theta_i - NA_i \Delta y_i$	$NA \cdot \varepsilon = \frac{\varepsilon}{2F_n}$

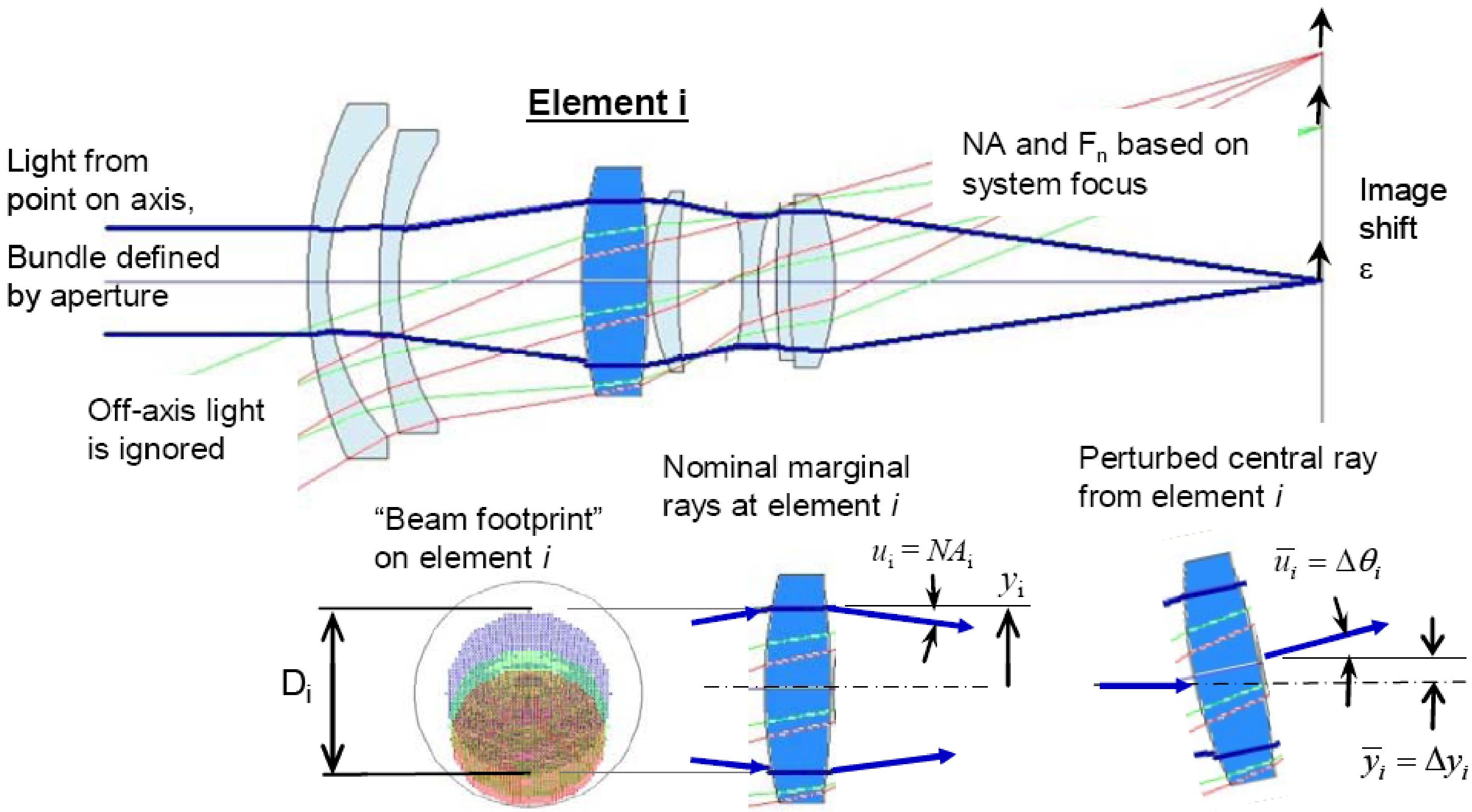
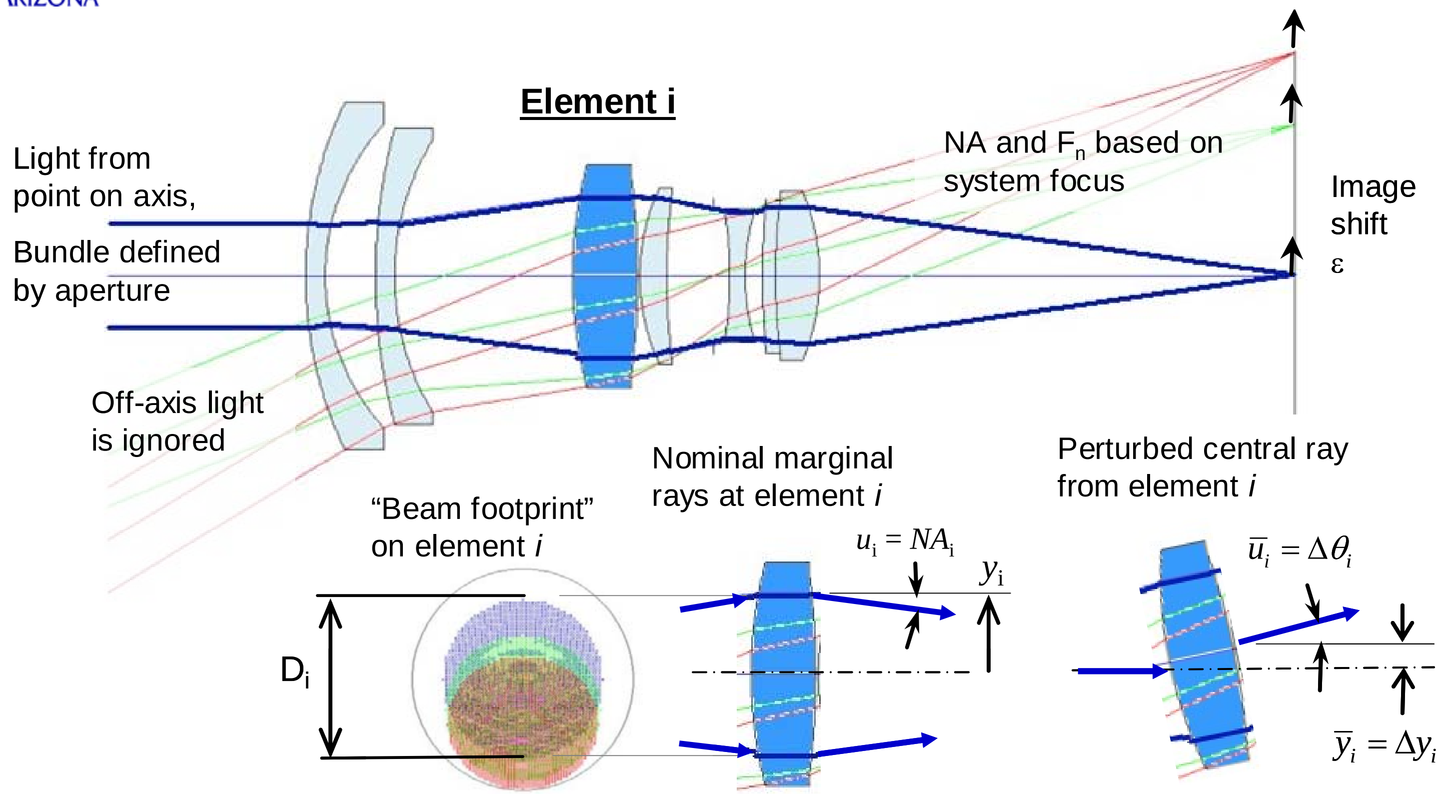


Figure 9. Definitions for system analysis relating image shift to element motion

General expression for image motion



$$\varepsilon = F_n D_i \Delta\theta_i - \frac{NA_i}{NA}\Delta y_i$$

F_n final working f-number = $\frac{1}{2NA}$

D_i beam footprint for on-axis bundle

$\Delta\theta_i$ = change in central ray angle due to motion of element i

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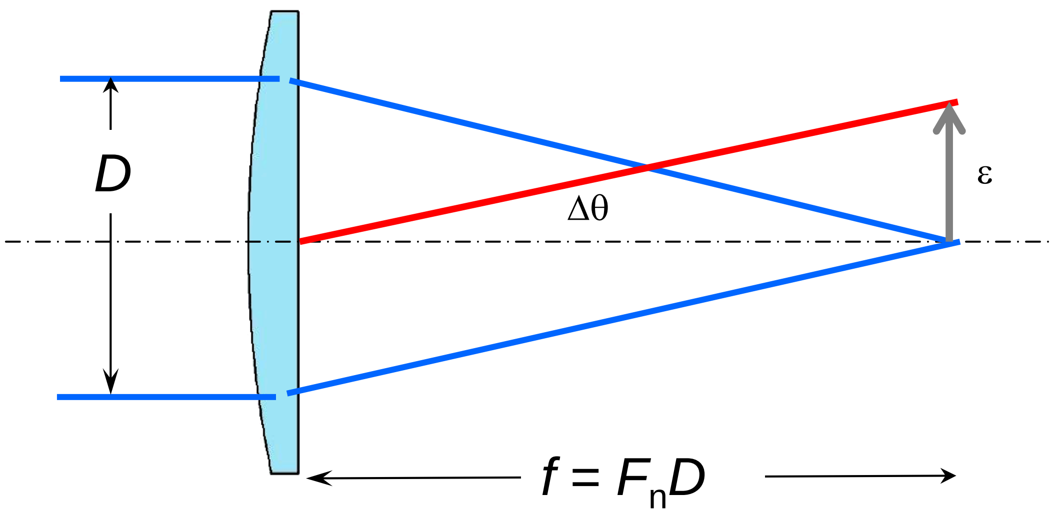
Example for change in angle

Image motion from change in ray angle $$\varepsilon = F_n D_i \Delta\theta_i$$

For single lens, this is trivial



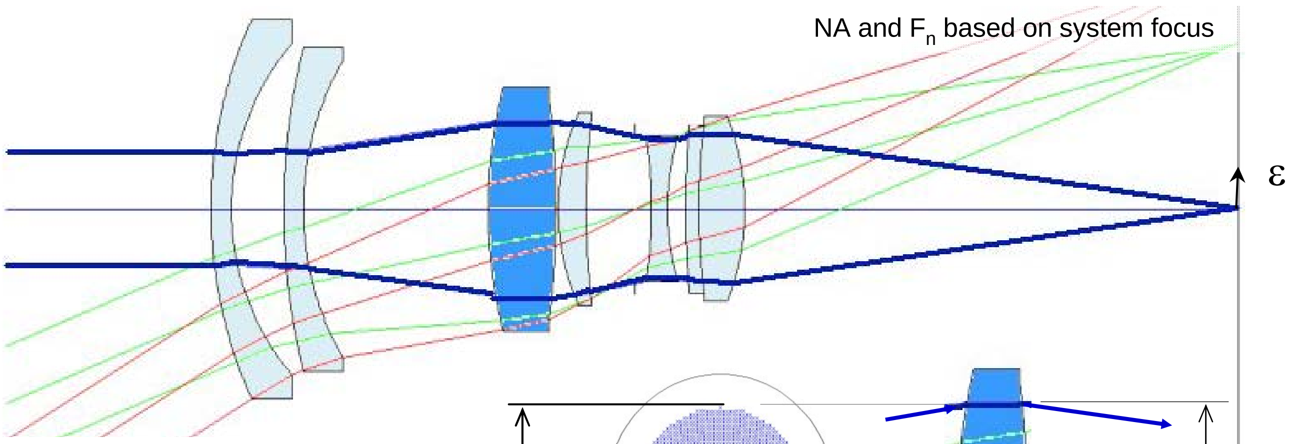
$$\varepsilon = f\Delta\theta$$
$$= F_n D \Delta\theta$$

Effect of lens decenter

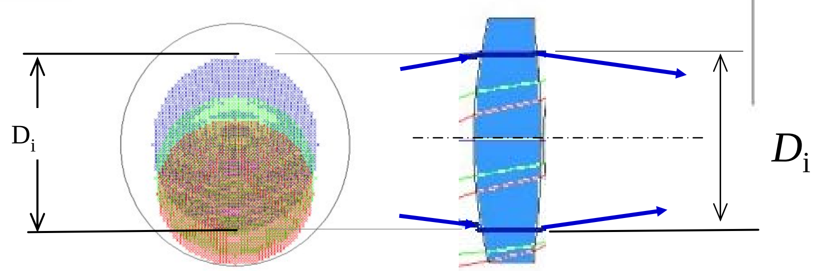
Decenter *s* causes angular change in central ray $\Delta\theta_i \cong \frac{s}{f_i}$

Which causes image motion $\varepsilon = F_n D_i \frac{s}{f_i}$

Magnification of Image / lens motion $\varepsilon/s = \frac{F_n}{f_i/D_i}$



D_i is "Beam footprint" on element *i*



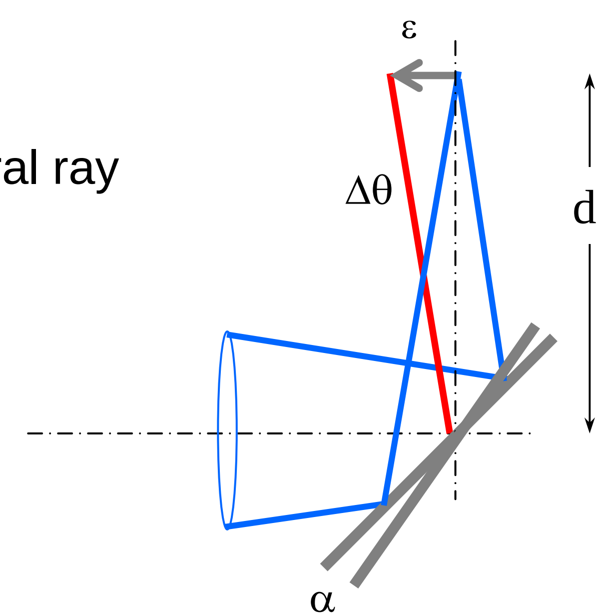
Example for mirror tilt

Tilt α causes angular change in central ray $$\Delta\theta_i = 2\alpha$$

Which causes image motion $$\varepsilon = 2F_n D_i \cdot \alpha_i$$

"Lever arm" of 2 F_n D_i (obvious for case where mirror is the last element)

Follow the central ray



$$\varepsilon = d\Delta\theta$$ Small angle approx

$$= F_n D_i \Delta\theta$$ D_i is beam size at mirror

$$= 2F_n D_i \alpha$$

This is valid for any mirror!

J. H. Burge

Afocal systems

- For system with object or image at infinity, effect of element motion is tilt in the light.

- Simply use the relationship from the optical invariant:

$$\frac{\varepsilon}{F_n} = D_0 \Delta \theta_0$$

Where

$\Delta\theta_0$ is the change in angle of the light in collimated space

D_0 is the diameter of the collimated beam

$$D_0 \Delta\theta_0 \cong D_i \Delta\theta_i$$