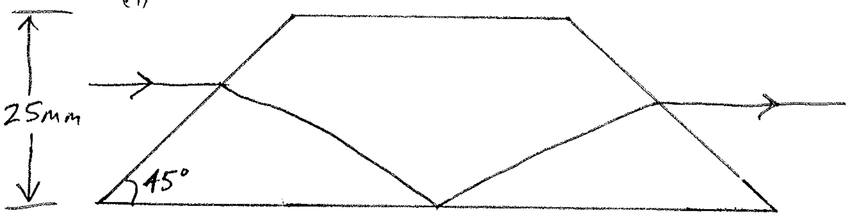
OPTI 421/521 HOMEWORK 2 SOLUTION

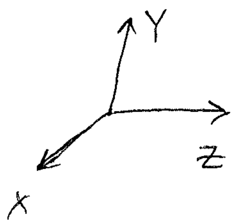
1

DOVE PRISM

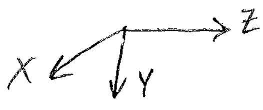
(A)



(B)



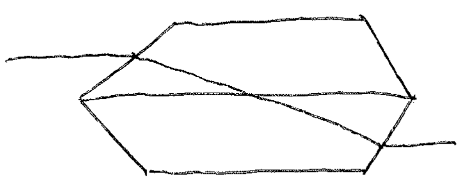
INPUT

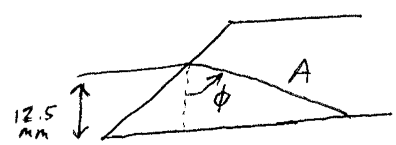


OUTPUT

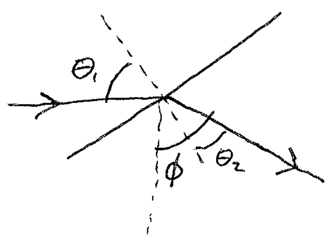
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = M$$

(C)





A = HALF LENGTH TRAVELED BY LIGHT



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_2 = \operatorname{ASIN}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$

$$\theta_2 = \operatorname{ASIN}\left(\frac{\sin 45°}{1.5}\right)$$

$$\theta_2 = 0.4909 \text{ RADIANS}$$

$$\phi = 45° + \theta_2$$

$$= .758 \text{ RAD} + \theta_2$$

$$\phi = 1.276 \text{ RADIANS}$$

$$\cos\phi = \frac{12.5}{A} \Rightarrow A = \frac{12.5}{\cos\phi} = 43.06 \text{ mm}$$

$2A$ = TOTAL LENGTH

$\frac{2A}{n}$ = REDUCED THICKNESS

$$\frac{2A}{n} = \boxed{57.4 \text{ mm}}$$

(D) DETERMINE ROLL, PITCH, YAW FROM MATRICES

FOR SMALL ANGLE: $\cos\theta = 1$, $\sin\theta = \theta$

M_R = ROTATED PRISM MATRIX

R = ROTATION MATRIX

$M_R = R \cdot M \cdot R^T$

ROTATION ABOUT Y $= R_y(\beta) = \begin{bmatrix} 1 & 0 & \beta \\ 0 & 1 & 0 \\ -\beta & 0 & 1 \end{bmatrix}$

ROTATION ABOUT X $= R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\alpha \\ 0 & \alpha & 1 \end{bmatrix}$

ROTATION ABOUT Z $= R_z(\gamma) = \begin{bmatrix} 1 & -\gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

PITCH

$$M_{RX} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\alpha \\ 0 & \alpha & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \alpha \\ 0 & -\alpha & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\alpha \\ 0 & \alpha & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -\alpha \\ 0 & -\alpha & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1-\alpha^2 & -2\alpha \\ 0 & -2\alpha & 1-\alpha^2 \end{bmatrix}$$

FOR SMALL ANGLES, $\alpha^2 = 0$

$$M_{RX} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2\alpha \\ 0 & -2\alpha & 1 \end{bmatrix}$$

LOS CHANGES WITH ANGLE 2α,

NO IMAGE ROTATION

YAW

$$M_{RY} = \begin{bmatrix} 1 & 0 & \beta \\ 0 & 1 & 0 \\ -\beta & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -\beta \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \beta \\ 0 & 1 & 0 \\ -\beta & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -\beta \\ 0 & -1 & 0 \\ \beta & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+\beta^2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -\beta^2+1 \end{bmatrix}$$

SMALL ANGLE, $\beta^2 = 0$

$$M_{RY} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

NO LOS CHANGE,

NO IMAGE ROTATION

ROLL

$$M_{RZ} = \begin{bmatrix} 1 & -\gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \gamma & 0 \\ -\gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \gamma & 0 \\ \gamma & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-\gamma^2 & 2\gamma & 0 \\ 2\gamma & \gamma^2-1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

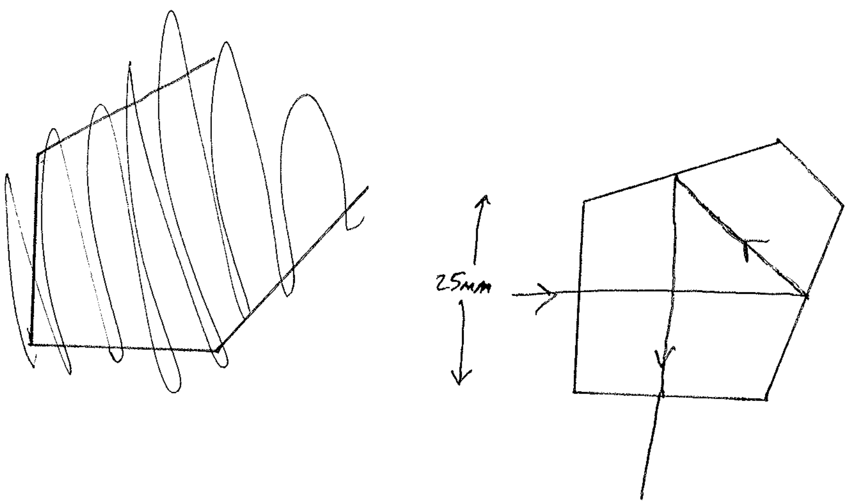
SMALL ANGLE, $\gamma^2 = 0$

$$M_{RZ} = \begin{bmatrix} 1 & 2\gamma & 0 \\ 2\gamma & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

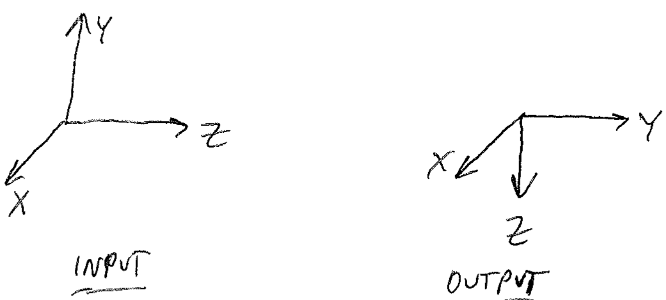
LOS DOESNT CHANGE,
IMAGE ROTATES WITH ANGLE 2γ

PENTA

(A)

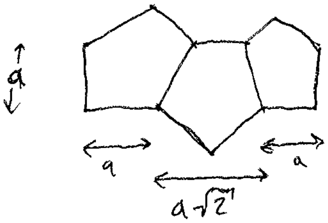


(B)



$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

(C)



TOTAL LENGTH $= 2a + a\sqrt{2} = 85.36\text{mm} = D$

$a = 25\text{mm}$

REDUCED THICKNESS $= \dfrac{D}{n} = \dfrac{85.36}{1.5} = \boxed{56.9\text{mm}}$

(D)

SMALL ANGLE, $\cos\theta \approx 1$, $\sin\theta = \theta$, $\alpha^2 = \beta^2 = \gamma^2 = 0$

PITCH

$$M_{RX} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\alpha \\ 0 & \alpha & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \alpha \\ 0 & -\alpha & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\alpha \\ 0 & \alpha & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & +\alpha & -1 \\ 0 & +1 & +\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1-\alpha^2 \\ 0 & 1+\alpha^2 & 0 \end{bmatrix}$$

$$M_{RX} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

NO LOS CHANGE,
NO IMAGE ROTATION

YAW

$$M_{RY} = \begin{bmatrix} 1 & 0 & \beta \\ 0 & 1 & 0 \\ -\beta & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\beta \\ 0 & 1 & 0 \\ \beta & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \beta \\ 0 & 1 & 0 \\ -\beta & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\beta \\ -\beta & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & \beta & -\beta \\ -\beta & 0 & -1 \\ -\beta & 1 & \beta^2 \end{bmatrix}$$

$$M_{RY} = \begin{bmatrix} 1 & \beta & -\beta \\ -\beta & 0 & -1 \\ -\beta & 1 & 0 \end{bmatrix}$$

~~NO LOS CHANGE, NO IMAGE ROTATION~~

LOS ROTATES ABOUT Z WITH ANGLE β, IMAGE ROTATION OF β

ROLL

$$M_{RZ} = \begin{bmatrix} 1 & -\gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & \gamma & 0 \\ -\gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \cancel{\begin{bmatrix} 1 & +\gamma & 0 \\ -\gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & -\gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \gamma & 0 \\ 0 & 0 & -1 \\ -\gamma & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & \gamma & \gamma \\ \gamma & \gamma^2 & -1 \\ -\gamma & 1 & 0 \end{bmatrix}$$

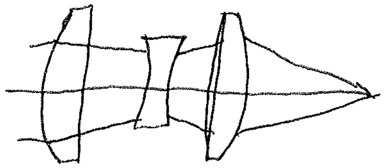
$$M_{RZ} = \begin{bmatrix} 1 & \gamma & \gamma \\ \gamma & 0 & -1 \\ -\gamma & 1 & 0 \end{bmatrix}$$

~~NO LOS CHANGE, NO IMAGE ROTATION~~

LOS ROTATES ABOUT Z WITH ANGLE γ

NO IMAGE ROTATION

2



ASSUME BEAM DIAMETER AT EACH LENS:

$D_1 = 10\,mm$

$D_2 =$

$D_3 =$

USE APPROXIMATION: $\varepsilon \approx F_n D_i \Delta\theta$, $F_n = 5$ (GIVEN)

LENS TILT IS NEGLIGIBLE AND Δz OF LENS IS ALSO NEGLIGIBLE.

$\Delta\theta = \frac{s}{f}$, s = LATERAL DEFLECTION

ONLY CONSIDER LATERAL SHIFT $\Rightarrow$ $\varepsilon \approx F_n D_i \left(\frac{s}{f}\right)$

TWO POSSIBLE METHODS: SAME IMAGE MOTION, ε, (A) OR SAME LENS MOTION, s. (B)

(A) $\varepsilon_{TOTAL} = \sqrt{\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2}$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon$

$\varepsilon_{TOTAL} = 1\,\mu m = \varepsilon\sqrt{3}$

$\varepsilon \cong .5774\,\mu m = F_n D_i \left(\frac{s}{f}\right)$

$s = \frac{.5774 \cdot f}{F_n \cdot D_i}$

LENS	D_i (mm)	f (mm)	s (μm)
1	10	34	$\pm .393$
2	8	-17	$\pm .2454$
3	9	24	$\pm .3079$

SAME IMAGE MOTION

+/- FOR TOLERANCE

(B) $\varepsilon_{TOTAL} = \pm 1\,\mu m = \sqrt{\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2}$ $s_1 = s_2 = s_3 = s$

$\varepsilon = \frac{F_n D_i s}{f}$

$\pm 1\,\mu m = \sqrt{\left(\frac{F_n D_1 s}{f_1}\right)^2 + \left(\frac{F_n D_2 s}{f_2}\right)^2 + \left(\frac{F_n D_3 s}{f_3}\right)^2}$

$\pm 1\,\mu m = \sqrt{s^2\left[\left(\frac{F_n D_1}{f_1}\right)^2 + \left(\frac{F_n D_2}{f_2}\right)^2 + \left(\frac{F_n D_3}{f_3}\right)^2\right]}$

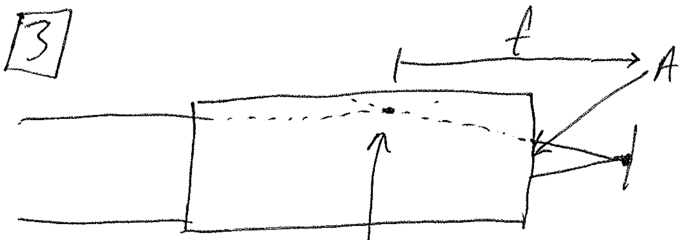
$\pm 1\,\mu m = s\sqrt{(2.163 + 5.536 + 3.516)}$

$s = \pm\frac{1\,\mu m}{\sqrt{11.215}} = \boxed{\pm 0.299\,\mu m}$

SAME LENS MOTION

LENS	F_n	D_i (mm)	f (mm)	$(F_n D/f)^2$
1	5	10	34	2.163
2	5	8	-17	5.536
3	5	9	24	3.516

3

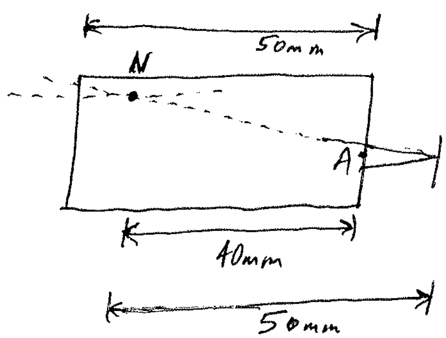


INTERSECTION OF RAYS AT ~~NODAL POINT~~ PRINCIPAL PLANE. NODAL POINT IS AT PRINCIPLE PLANE SINCE SYSTEM IS IN AIR.

$$f = (f/\#)\, D = \left(\frac{1}{2NA}\right) D = \left(\frac{1}{2(.1)}\right)(10\text{mm}) = 50\text{mm}$$

IF L_A = DISTANCE FROM POINT A TO NODAL POINT,

$$f = L_A + BFD \Rightarrow L_A = f - BFD = 50\text{mm} - 10\text{mm} = 40\text{mm}$$



CAN DECOMPOSE ANY ROTATION AS ROTATION ABOUT ANOTHER POINT PLUS A SHIFT,

$$\varepsilon = \cancelto{0}{\Delta\theta_{NODAL}} + \Delta\theta_A \cdot L_A$$

$$\varepsilon = (10\text{m RAD})(40\text{mm}) = \boxed{400\mu\text{m}}$$