OPTI 521 HW 1 SOLUTION

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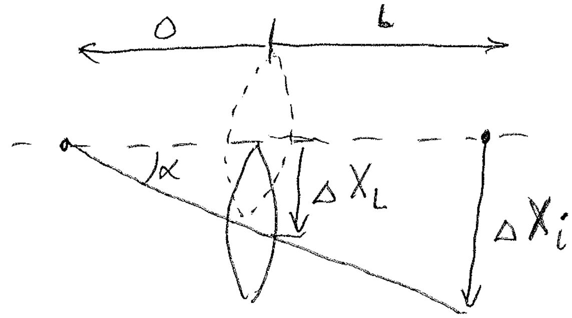
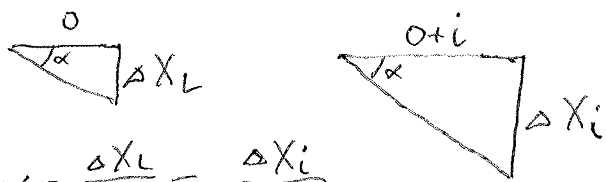
PART 2

1 LENS MOTION

(A) LENS LATERAL SHIFT, ΔX_L



SIMILAR TRIANGLES



$$\alpha = \frac{\Delta X_L}{o} = \frac{\Delta X_i}{o+i}$$

$$\Rightarrow \Delta X_L (o+i) = \Delta X_i \, o$$

$$\Delta X_i = \frac{\Delta X_L}{o}(o+i)$$

$$\Delta X_i = \Delta X_L + \frac{i}{o}\Delta X_L$$

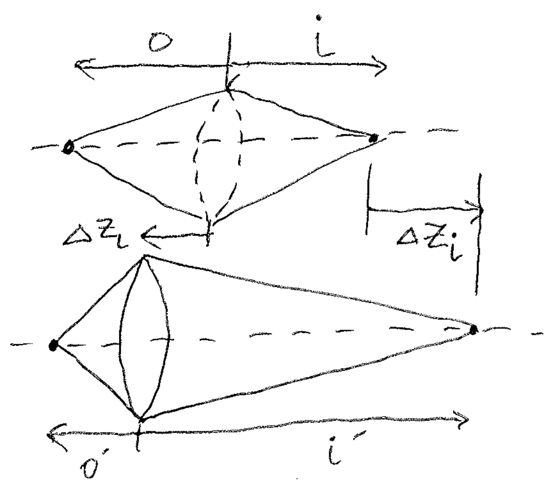
MAGNIFICATION

$$m = -\frac{i}{o}$$

SUBSTITUTE m, PULL OUT ΔX_L

$$\boxed{\Delta X_i = \Delta X_L (1-m)}$$

AXIAL SHIFT, ΔZ_L



LONGITUDINAL MAGNIFICATION $= -\bar{m} \equiv \frac{\Delta i}{\Delta o}$

IN AIR, $\bar{m} = m^2$

$$\frac{\Delta i}{\Delta o} = -m^2$$

$$\Rightarrow \frac{i - i'}{o - o'} = -m^2$$

$$o' = o - \Delta Z_L$$

$$i' = i + \Delta Z_L + \Delta Z_i$$

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CONTINUED FROM PAGE 1

$$o - o' = \Delta z_L$$

$$i - i' = -\Delta z_L - \Delta z_i$$

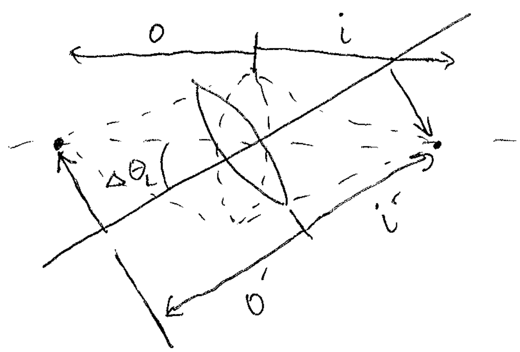
$$-m^2 = \frac{i - i'}{o - o'} = \frac{-\Delta z_L - \Delta z_i}{\Delta z_L}$$

$$m^2 = \frac{\Delta z_L + \Delta z_i}{\Delta z_L}$$

$$\Delta z_i + \Delta z_L = m^2 \cdot \Delta z_L$$

$$\boxed{\Delta z_i = \Delta z_L (m^2 - 1)}$$

TILT, $\Delta\theta_L$



FOR SMALL ANGLES,

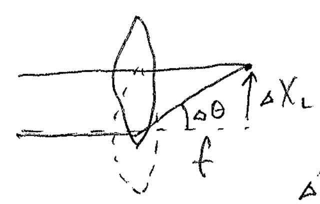
$o' \approx o$ AND $i' \approx i$

SO, $\boxed{\Delta z_i = 0}$

(B) PROVE IMAGE SHIFT ΔX_i FOLLOWS $\Delta X_i \approx F_n D_i \Delta\theta_i$

$$\cancel{F_n D_i \Delta\theta_i = \frac{f}{D_i} D_i \Delta\theta_i = f \Delta\theta_i}$$

FOR SINGLE LENS, $F_n D_i \Delta\theta = \frac{i}{D} D \Delta\theta = i \Delta\theta$



$$\Delta\theta = \frac{\Delta X_L}{f}$$

IMAGE EQN $\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$

FROM (1A) $m = -\frac{i}{o} \Rightarrow \frac{1}{o} = -\frac{m}{i}$

SUBSTITUTE

$$\frac{1}{i} - \frac{m}{i} = \frac{1}{f}$$

$$i = f(1 - m)$$

$$\Delta X_i = i \Delta\theta = f(1-m)\Delta\theta$$

$$= f(1-m)\frac{\Delta X_L}{f}$$

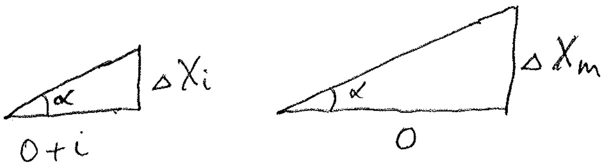
$$\boxed{\Delta X_i = \Delta X_L (1 - m)} \checkmark$$

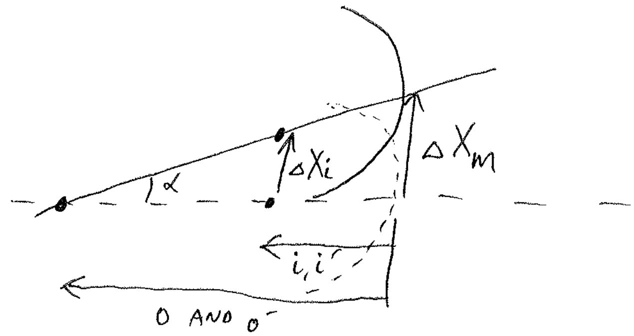
2 Mirror Motion

(A)

Mirror lateral shift, ΔX_m

Similar triangles





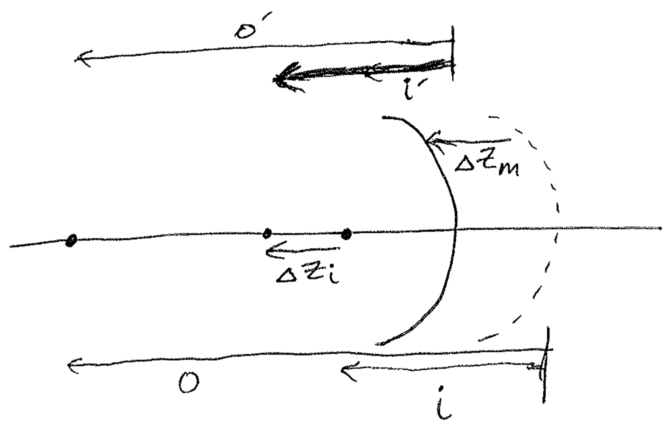
$$\alpha = \frac{\Delta X_i}{o+i} = \frac{\Delta X_m}{o}$$

$$\Delta X_i = \frac{\Delta X_m}{o}(o+i)$$

$$\Delta X_i = \Delta X_m\left(1+\frac{i}{o}\right) \qquad m = -\frac{i}{o}$$

$$\boxed{\Delta X_i = \Delta X_m (1-m)}$$

Axial shift, ΔZ_m



$$\Delta Z_i = \Delta i + \Delta o$$

$$i = i' + m^2 \Delta Z_m$$

$$o = o' + \Delta Z_m$$

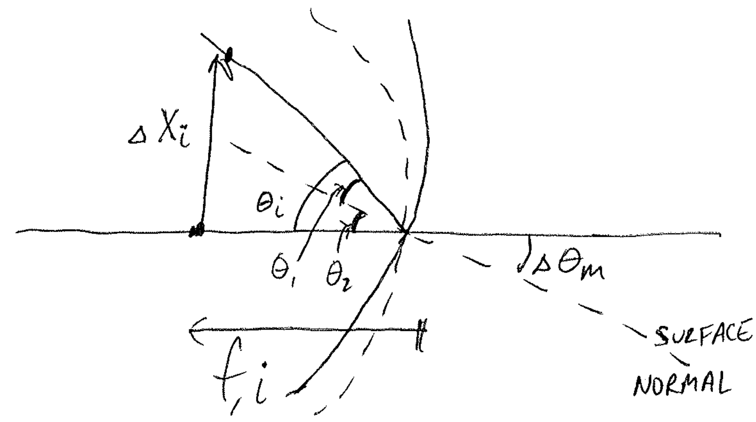
$$\Delta i = i - i' = m^2 \Delta Z_m$$

$$\Delta o = o - o' = \Delta Z_m$$

$$\Delta Z_i = m^2 \Delta Z_m + \Delta Z_m$$

$$\boxed{\Delta Z_i = \Delta Z_m (m^2+1)}$$

TILT, $\Delta\theta_m$



$\theta_i = \theta_1 + \theta_2$

$\theta_1 = \theta_2$ LAW OF REFLECTION

$\theta_1 = \Delta\theta_m$

$\theta_i = 2\Delta\theta_m$

$$\theta_i = \frac{\Delta X_i}{i}$$

$\Delta X_i = i \cdot \theta_i$

$$\boxed{\Delta X_i = 2M \cdot O \cdot \Delta\theta_m}$$

$M = \frac{i}{O}$

$i = M \cdot O$

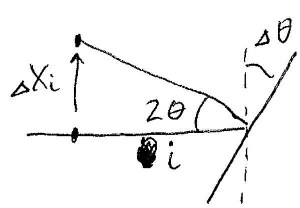
(B) PROVE THAT IMAGE SHIFT, ΔX_i FOLLOWS $\Delta X_i \approx F_n D_i \Delta\theta_i$

$$\Delta X_i = \frac{f}{D} D \Delta\theta_i$$

$$= i \cdot \Delta\theta_i$$

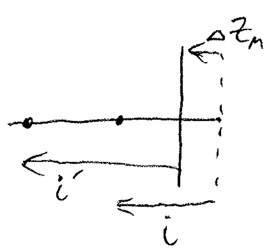
$$\boxed{\Delta X_i = 2m \cdot O \cdot \Delta\theta_m}$$ ✓

MAGNIFICATION CHANGES FOR FLAT MIRROR ($m=1$)



$\Delta X_i = m \cdot O \cdot 2\Delta\theta_m$

$= i \cdot 2\Delta\theta_m$ ✓



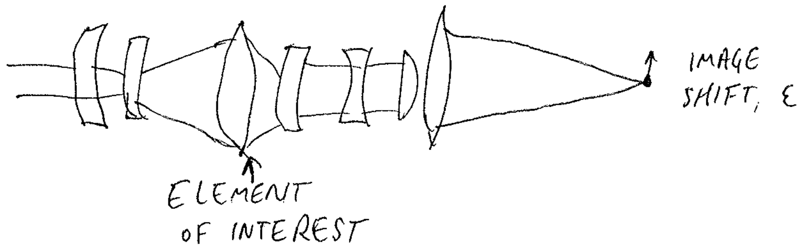
$\Delta Z_i = \Delta Z_m (m^2 + 1)$

$\Delta Z_i = \Delta Z_m (2)$

MAGNIFICATION ALSO CHANGES TO WORK FOR CONCAVE OR CONVEX MIRRORS.

PART 3 - 521 STUDENTS ONLY (421 EXTRA CREDIT)

APPLY OPTICAL INVARIANT TO DERIVE EQN 10: $\varepsilon = F_n D_i \Delta\theta_i - \frac{NA_i}{NA}\Delta Y_i$



CALCULATE OPTICAL INVARIANT AT ELEMENT

$$I_i \equiv \bar{u}Y - u\bar{Y}$$

$$I_i = \frac{D_i}{2}\Delta\theta_i - NA_i \Delta Y_i$$

CALCULATE OPTICAL INVARIANT AT IMAGE PLANE

$$\cancel{I = NA \cdot \theta}$$

$$I_N = \bar{u}_N \cdot 0 - NA\,\varepsilon$$

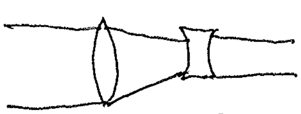
SET EQUAL

$$I_i = I_N$$

$$\frac{D_i}{2}\Delta\theta_i - NA_i \Delta Y_i = -NA\,\varepsilon$$

$$-\varepsilon = \frac{D_i}{2NA}\Delta\theta_i - \frac{NA_i}{NA}\Delta Y_i$$

$$\boxed{-\varepsilon = F_n D_i \Delta\theta_i - \frac{NA_i}{NA}\Delta Y_i}$$



FOR AN AFOCAL SYSTEM, IT CANNOT BE DESCRIBED IN TERMS OF IMAGE DISPLACEMENT. THE PROPER WAY TO DESCRIBE IT WOULD BE ANGULAR DISPLACEMENT.