HW 7 - SOLUTIONS

HAND CALCULATIONS FOR PROBLEMS

1 $F = 10N,\ L = 10cm,\ A_{xy} = 1cm^2,\ E = 7\times10^{10}\, N/m^2$

(A) $\Delta L = \frac{FL}{EA} = \frac{(10N)(.1m)}{(7\times10^{10}\frac{N}{m})(10^{-4}m^2)} = \boxed{1.4\times10^{-7}m}$ $\quad \sigma = \frac{F}{A} = \frac{10N}{10^{-4}m^2} = \boxed{1\times10^{5}\,Pa}$ CONSTANT THROUGHOUT BEAM

(B) $I = \frac{1}{12}bh^3 = \frac{1}{12}(10^{-2})^4 m^4 = 8.3\times10^{-10}m^4$

$$\delta = \frac{FL^3}{3EI} = \frac{(10N)(0.1)^3 m^3}{3(7\times10^{10}\frac{N}{m})(8.3\times10^{-10}m^4)} = \boxed{5.7\times10^{-5}m}$$

$$\sigma_{MAX} = \frac{My_{MAX}}{I} = \frac{(10N\cdot0.1m)(5\times10^{-3}m)}{8.3\times10^{-10}m^4} = \boxed{6\times10^{6}\,Pa}$$ MAX AT FIXED POSITION

(C) $\delta = \frac{FL^3}{48EI} = \frac{(10N)(0.1)^3m^3}{48(7\times10^{10}\frac{N}{m})(8.3\times10^{-10}m^4)} = \boxed{3.57\times10^{-6}m}$

$$\sigma_{MAX} = \frac{My_{MAX}}{I} = \frac{\left(\frac{FL}{4}\right)(5\times10^{-3}m)}{I} = \frac{\left(\frac{10N\cdot0.1m}{4}\right)(5\times10^{-3}m)}{8.3\times10^{-10}m^4} = \boxed{1.5\times10^{6}\,Pa}$$ MAX AT CENTER OF BEAM

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2 $\alpha = 24\ ppm/^\circ C$

(A) $\Delta L = \alpha L \Delta T = (24\times10^{-6}/c)(0.1m)(1^\circ c) = \boxed{2.4\times10^{-6}m}$ $\quad \boxed{\sigma = 0}$

(B) $\sigma = \frac{F}{A} = \frac{\Delta L E}{L} = \frac{(2.4\times10^{-6}m)(7\times10^{10}\frac{N}{m})}{0.1m} = \boxed{1.68\times10^{6}\,Pa}$ $\quad \boxed{\delta = 0}$

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3 (A) $\Delta L = \alpha \Delta x \Delta T$
$= \alpha \Delta x(\Delta T x)$ NEED TO INTEGRATE OVER BEAM

$$\Delta L = \int_0^L \alpha \Delta T x\, dx = \frac{1}{2}\alpha\Delta T(x^2)\Big|_{x=0}^{x=L} = \frac{1}{2}\alpha\Delta T L^2 = \frac{1}{2}(24\times10^{-6}/c)(0.1\tfrac{c}{cm})(0.1m)^2$$

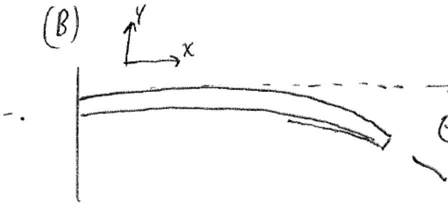
$\Delta L = \boxed{1.2\times10^{-6}m}$

HEAT FOR GRADIENT:

$$H = \frac{A\lambda\Delta T}{L} = \frac{(1cm^2)(170\frac{W}{m\cdot K})(1^\circ C)}{0.1m} = \boxed{0.17W}$$

$\boxed{\sigma = 0}$

(B)



$$\theta = \frac{\alpha L \Delta T}{t} = \frac{23\,ppm \cdot 100\,mm \cdot 1^\circ C}{10\,mm} = 230\,\mu rad$$

$$\theta(x) = \frac{\alpha \Delta T}{t} \cdot x + \cancel{C}^{\,0}$$

$$\delta(x) = \int \theta\, dx = \frac{1}{2}\frac{\alpha \Delta T}{t} x^2 + \cancel{C}^{\,0} = \frac{230\,\mu rad}{2} \cdot 100\,mm$$

$$= 115\,\cancel{rad} \cdot 0.1\ \mu m = 11.5\,\mu m$$

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HEAT FOR GRADIENT:

$$H = \frac{A \lambda \Delta T}{L} = \frac{(10\,cm^2)(170\,\frac{W}{m\cdot K})(0.1^\circ C)}{0.01\,m} = \boxed{1.7W}$$

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(4) $K_t = \frac{\sigma_{MAX}}{\sigma_{NORMAL}} \Rightarrow \sigma_{MAX} = K_t \cdot \sigma_{NORMAL}$, $D = 1.05"$, $d = 1.0"$

$\sigma_{NORMAL} = \frac{P}{D \cdot t}$, ASSUME $P = 1lb$, FROM TABLE: $B = 1.091$, $a = -0.242$

$$\sigma_{MAX} = \left[1.091 \cdot \left(\frac{r}{1.05"}\right)^{-.242}\right] \frac{1lb}{1.05" \cdot 0.2"} = 5.195 \left(\frac{r}{1.05"}\right)^{-.242}$$

(A) $r = 0.2" \Rightarrow \sigma_{MAX} = 5.195 \left(\frac{0.2"}{1.05"}\right)^{-.242} = \boxed{7.76\,lb}$

(B) $r = 0.1" \Rightarrow \sigma_{MAX} = 5.195 \left(\frac{0.1"}{1.05"}\right)^{-.242} = \boxed{9.18\,lb}$

(C) $r = 0.025" \Rightarrow \sigma_{MAX} = 5.195 \left(\frac{0.025"}{1.05"}\right)^{-.242} = \boxed{12.84\,lb}$