Image analysis for LIDAR telescope

James Scholl

College of Optical Sciences
University of Arizona
September 2, 2006

The coupling between optical element motion and image motion for a LIDAR telescope was analyzed using estimates of the element stability that were provided by Dr. Burge. By RSS combination of all of the effects, we expect the boresight to be maintained to 160 µrad, or 110 µm image motion at the detector. The dominant sources of error are the 100 µrad system pointing and the 50 µrad tilt of the primary mirror.

The optical system is shown in Figure 1 and the expected component errors are listed in Table 1.

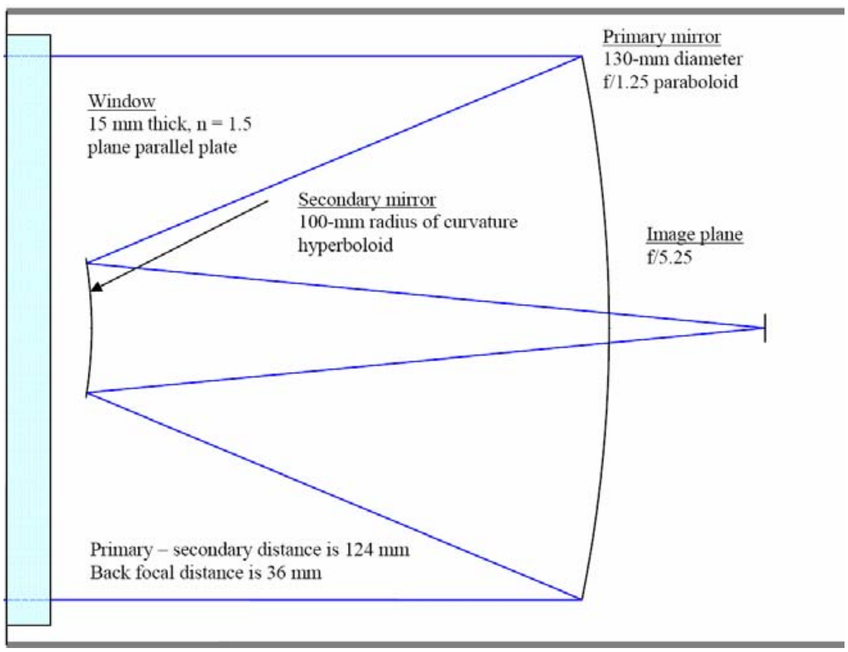


Figure 1: Cassegrain LIDAR telescope under study

Table 1. Expected component motion, as provided by Dr. Burge

Component	**Expected motion**
Primary Mirror	5 µm rms lateral motion, 50 µrad rms tilt
Secondary Mirror	10 µm rms lateral motion, 100 µrad rms tilt
Window	20 µm rms lateral motion, 200 µrad rms tilt
Detector motion	5 µm rms lateral motion, 50 µrad rms tilt
System as a unit (entire optical assembly)	100 µm rms lateral motion, 100 µrad rms tilt

The calculations between the component shift and the system line of sight are provided in the Appendix. The results are summarized in Table 2 and are shown graphically in Figure 2.

The motion of the window has no effect on pointing since this element is in collimated space. Detector tilt also does not shift the relative image position, so it does not affect pointing.

Table 2. System line of sight motion for LIDAR telescope

	units	element motion (rms)	Line of sight motion (μrad rms)	Image motion (μm rms)
Primary Mirror				
Decenter	μm	5	31	21
Tilt	μrad	50	100	68
Secondary Mirror				
Decenter	μm	10	47	32
Tilt	μrad	100	47	32
Detector				
Decenter	μm	5	7	5
System as a unit				
Tilt	μrad	100	100	68
System performance (RSS)			**159**	**108**

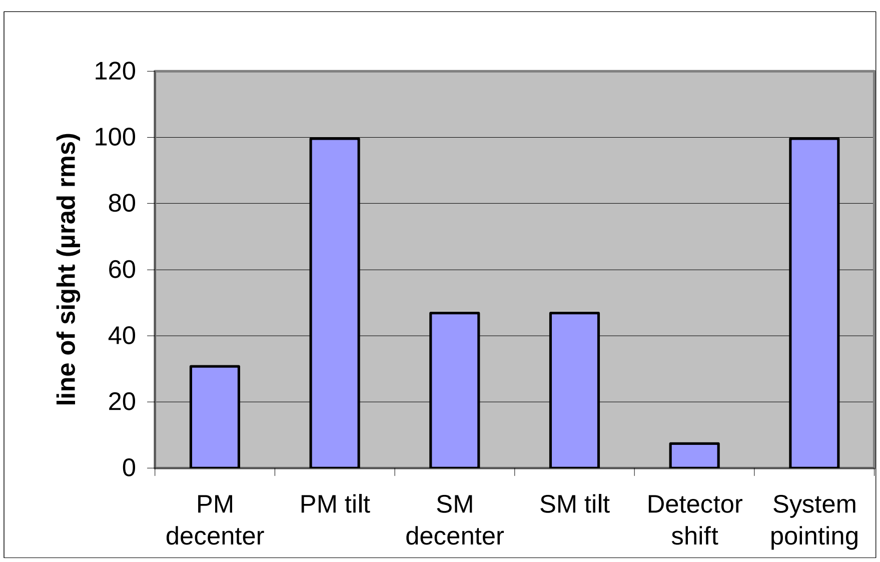


Figure 2. Effect of component motion on the system line of sight. To convert to image motion, multiply the LOS in μrad times the system EFL of 0.683 m.

Appendix

Supporting calculations

The System EFL was not given. Assuming the system aperture to match the 130 mm primary, the EFL can be calculated from the aperture size and the system focal ratio

EFL = 130 mm * 5.25 = 683 mm

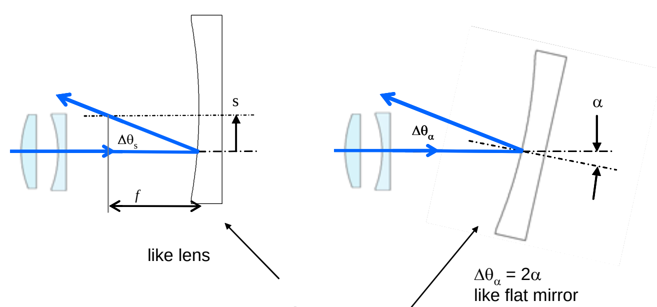
Also the primary mirror focal length can be inferred from its diameter and focal ratio.

f_p = 130 mm * 1.25 = 163 mm

The focal length of the secondary mirror can be determined from its radius of curvature f_s = 100 mm/2 = 50 mm

When the mirrors are moved in translation or tilt, they affect the angle of the light reflected as:

Mirror motion



$$\Delta\theta \cong \frac{s}{f} + 2\alpha$$

The change in angle affects the image position according to

$$\varepsilon = F_n D_i \Delta\theta_i$$

Where

F_n is the system focal ratio = 5.25
D_i is the beam footprint on mirror i
$\Delta\theta_i$ is the change in angle given above

To convert to LOS angle, $\theta_{LOS} = \varepsilon/EFL$,

The following subsidiary calculations are needed for all the calculations:

$f_{sec} = R_{sec} / 2$ = 100mm / 2 = ***50 mm***

Primary Mirror

5 µm PM Lateral Motion:

$\Delta\theta_l = \Delta s_{prim} / f_{prim}$ = 5 µm / 162.5 mm = 30.7 µrad
$\varepsilon_{pl} = F_{sys} * D_{prim} * \Delta\theta_l$ = 5.25 * 130 mm * 30.7 µrad = ***21 µm***

$\Delta\theta_{LOS} = \varepsilon$ / EFL = 21 µm/0.683 m = 29 µrad

50 µrad PM tilt:

$\Delta\theta_t = 2\ \Delta\alpha_{prim}$ = 2 * 50 µrad =100 µrad
$\varepsilon_{pt} = F_{sys} * D_{prim} * \Delta\theta_t$ = 5.25 * 130 mm * 100 µrad = ***68 µm***
$\Delta\theta_{LOS} = \varepsilon$ / EFL = 68 µm/0.683 m = 100 µrad

Or The mirror is in object space, so
$\Delta\theta_{LOS} = 2\ \Delta\alpha_{prim}$ = 2 * 50 µrad =100 µrad

Secondary Mirror

distance from sec, mirror to image = $d_{si} = d_{primary\text{-}secondary}$ + bfd = 124 mm + 36 mm = ***160mm***

Use system focal ratio to determine beam footprint at secondary
D_{sec} = from geometry = d_{si} /5.25 = 30.8 mm

SM Lateral Motion of 10 µm:

$\Delta\theta_l = \Delta s_{sec} / f_{sec}$ = 10 µm / 50.0 mm = ***200 µrad***
$\varepsilon_l = F_{sys} * D_{sec} * \Delta\theta_l$ = 5.25 * 30.8 mm * 200 µrad = ***32 µm***

Or, use the fact that the SM is in image space.
$\varepsilon_l = d_{si} * \Delta\theta_l$ = 160 mm * 300 µrad = ***32 µm***

Or use the relationship derived in Part 1
m = 5.25/1.25 = 4.3 (magnification of secondary is ratio of the focal ratios)
$\varepsilon_l = (1 - m)\ \Delta s_{sec}$
ε_l = (1 – 4.2) 10 µm = 32 µm

$\Delta\theta_{LOS,l} = \varepsilon_l$ / EFLl = 32 µm / 682.5 mm = ***47 µrad***

SM Tilt of 100 µrad:

$\Delta\theta_t = 2\ \Delta\alpha_{sec}$ = 2 * 100 µrad = ***200 µrad***
$\varepsilon_t = F_{sys} * D_{sec} * \Delta\theta_t$ = 5.25 * 30.8 mm * 200 µrad = ***32 µm***

Or, using fact that SM is in image space
$\varepsilon_t = d_{si} * \Delta\theta_t$ = 160 mm * 200 µm = ***32 µm***
$\Delta\theta_{LOS,t} = \varepsilon_t$ / efl = 32 µm / 682.5 mm = ***47 µrad***

Window

Lateral Motion:

$\Delta\theta_{LOS,l} = 0$ ***μrad*** (no effect)
$\varepsilon_l = efl * \Delta\theta_{LOS,l} = 682.5\text{ mm} * 0\ \mu\text{rad} =$ ***0 μm*** (no effect)

Tilt:

$\Delta\theta_{LOS,t}$ ***= 0 μrad*** (no effect)
$\varepsilon_t = efl * \Delta\theta_{LOS,t} = 682.5\text{ mm} * 0\ \mu\text{rad} =$ ***0 μm*** (no effect)

Detector

Lateral Motion:

$\varepsilon_l =$ ***5 μm***
$\Delta\theta_{LOS,l} = \varepsilon_{det,l} / efl = 5\ \mu\text{m} / 682.5\text{ mm} = 7.3\ \mu\text{rad} =$ ***7 μrad***

Tilt:

$\Delta\theta_{LOS,t} =$ ***0 μrad*** (no effect)
$\varepsilon_t = efl * \Delta\theta_{LOS,t} = 682.5\text{ mm} * 0\ \mu\text{rad} =$ ***0 μm*** (no effect)

System as Unit

Lateral Motion:
$\Delta\theta_{LOS,l} =$ ***0 μrad*** (no effect)
$\varepsilon_l = efl * \Delta\theta_{LOS,l} = 682.5\text{ mm} * 0\ \mu\text{rad} =$ ***0 μm*** (no effect)

Tilt:
$\Delta\theta_{LOS,t} = \alpha_{sys} =$ ***100 μrad***
$\varepsilon_t = efl * \Delta\theta_{LOS,t} = 682.5\text{ mm} * 100\ \mu\text{rad} = 68.3\ \mu\text{m} =$ ***68 μm***

Reference:

1: J. H. Burge, “An easy way to relate optical element motion to system pointing ability,” *Proc. SPIE*, **6288,** (2006).