A.12 Vibrating beams, tubes, and disks

Any undamped system vibrating at one of its natural frequencies can be reduced to the simple problem of a mass m attached to a spring of stiffness K. The lowest natural frequency of such a system is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

Specific cases require specific values for m and K. They can often be estimated with sufficient accuracy to be useful in approximate modelling. Higher natural frequencies are simple multiples of the lowest.

The first box on the facing page gives the lowest natural frequencies of the flexural modes of uniform beams with various end-constraints. As an example, the first can be estimated by assuming that the effective mass of the beam is one quarter of its real mass, so that

$$m = \frac{m_0 L}{4}$$

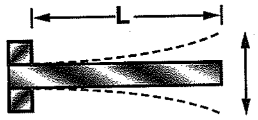
where m_0 is the mass per unit length of the beam and that K is the bending stiffness (given by F/δ from Section A.3); the estimate differs from the exact value by 2 percent. Vibrations of a tube have a similar form, using I and m_0 for the tube. Circumferential vibrations can be found approximately by "unwrapping" the tube and treating it as a vibrating plate, simply supported at two of its four edges.

The second box gives the lowest natural frequencies for flat circular disks with simply-supported and clamped edges. Disks with doubly-curved faces are stiffer and have higher natural frequencies.

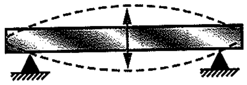
A.12 Vibrating beams, tubes and disks

C_1 Beams and tubes

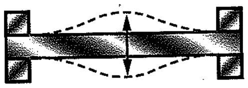
$$\boxed{f_1 = \frac{C_1}{2\pi}\sqrt{\frac{E I}{m_o L^4}}}$$



3.52

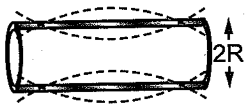


9.87

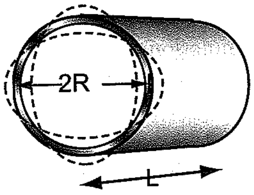


22.4

f = Natural frequency (s^{-1})
m_o = ρA = Mass / length (kg/m)
ρ = Density (kg/m^3)
A = Section area (m^2)
I = See Table A.2



9.87 $\left\{ \begin{array}{l} \text{With } A = 2\pi R t \\ \text{and } I = \pi R^3 t \end{array} \right.$



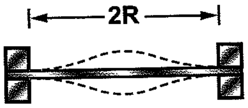
2.68 $\left\{ \begin{array}{l} \text{With } A = L t \\ \text{and } I = \dfrac{L t^3}{12} \end{array} \right.$

L = Length of tube

C_2 Disks

$$\boxed{f_1 = \frac{C_2}{2\pi}\sqrt{\frac{E t^3}{m_1 R^4 (1 - \nu^2)}}}$$

1.44



2.94

m_1 = ρt = Mass / area (kg/m^2)
t = Thickness (m)
R = Radius (m)
ν = Poisson's ratio