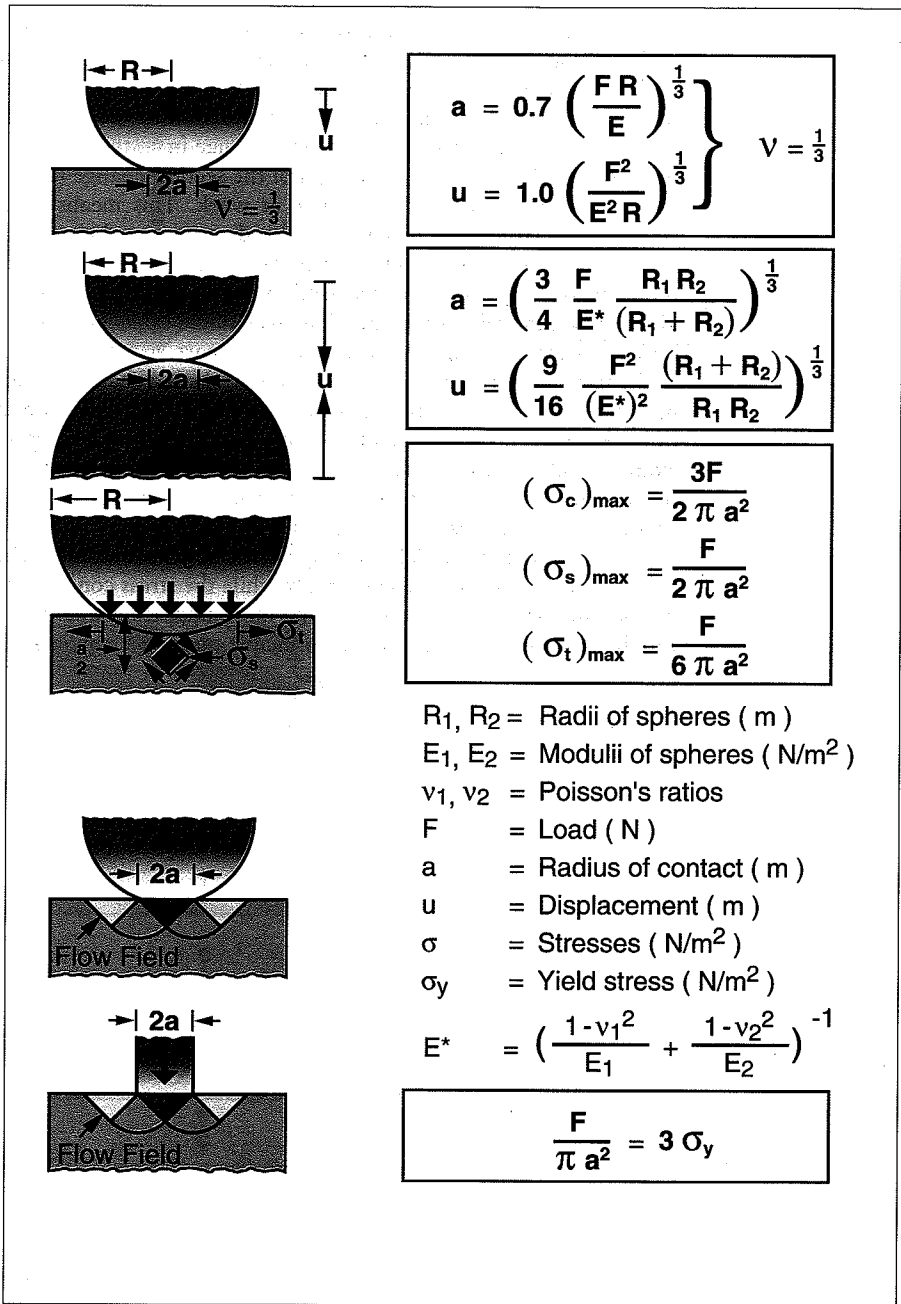


A.8 Contact stresses



$$\left. \begin{aligned} a &= 0.7 \left(\frac{FR}{E} \right)^{\frac{1}{3}} \\ u &= 1.0 \left(\frac{F^2}{E^2 R} \right)^{\frac{1}{3}} \end{aligned} \right\} \nu = \frac{1}{3}$$

$$\left. \begin{aligned} a &= \left(\frac{3}{4} \frac{F}{E^*} \frac{R_1 R_2}{(R_1 + R_2)} \right)^{\frac{1}{3}} \\ u &= \left(\frac{9}{16} \frac{F^2}{(E^*)^2} \frac{(R_1 + R_2)}{R_1 R_2} \right)^{\frac{1}{3}} \end{aligned} \right\}$$

$$\begin{aligned} (\sigma_c)_{\max} &= \frac{3F}{2\pi a^2} \\ (\sigma_s)_{\max} &= \frac{F}{2\pi a^2} \\ (\sigma_t)_{\max} &= \frac{F}{6\pi a^2} \end{aligned}$$

- R_1, R_2 = Radii of spheres (m)
- E_1, E_2 = Moduli of spheres (N/m^2)
- ν_1, ν_2 = Poisson's ratios
- F = Load (N)
- a = Radius of contact (m)
- u = Displacement (m)
- σ = Stresses (N/m^2)
- σ_y = Yield stress (N/m^2)
- $E^* = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)^{-1}$

$$\frac{F}{\pi a^2} = 3 \sigma_y$$

A.8 Contact stresses

When surfaces are placed in contact they touch at one or a few discrete points. If the surfaces are loaded, the contacts flatten elastically and the contact areas grow until failure of some sort occurs: failure by crushing (caused by the compressive stress, σ_c), tensile fracture (caused by the tensile stress, σ_t), or yielding (caused by the shear stress σ_s). The boxes on the facing page summarize the important results for the radius, a , of the contact zone, the center-to-center displacement u and the peak values of σ_c , σ_s , and σ_t .

The first box shows results for a sphere on a flat, when both have the same moduli and Poisson's ratio has the value $1/3$. Results for the more general problem (the "Hertzian indentation" problem) are shown in the second box: two elastic spheres (radii R_1 and R_2 , moduli and Poisson's ratios E_1, ν_1 and E_2, ν_2) are pressed together by a force F .

If the shear stress σ_s exceeds the shear yield strength $\sigma_y/2$, a plastic zone appears beneath the center of the contact at a depth of about $a/2$ and spreads to form the fully-plastic field shown in the two lower figures. When this state is reached, the contact pressure is approximately three times the yield stress, as shown in the bottom box.