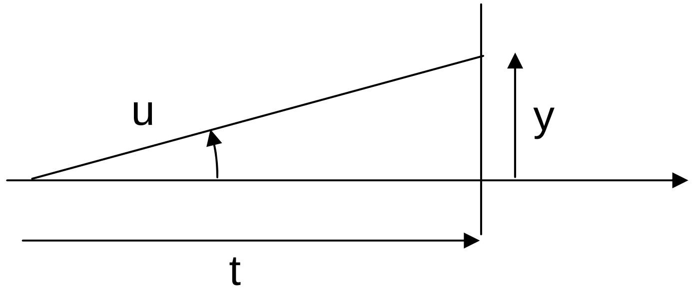
Paraxial Ray Angles

While they are referred to as angles, paraxial ray angles are not angles at all. They measure an angle-like quantity, but these paraxial angles are actually the slope of the ray or the ratio of a height to a distance. As a result, paraxial angles are unitless. If the physical angle in degrees or radians is θ, then the paraxial angle is given by the tangent of θ.



$$u = \frac{y}{t}$$

$$u = \tan\theta = \frac{y}{t}$$

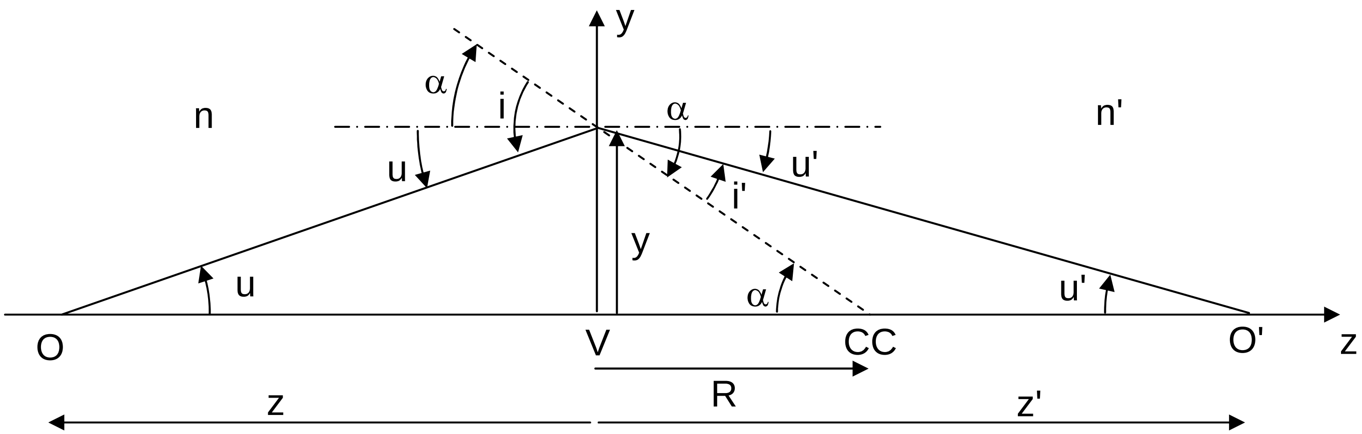
The use of ray slopes is critical for paraxial raytracing as it results in the linearity of paraxial raytracing. This is easy to see from the transfer equation:

$$y' = y + ut$$

This linear equation for the paraxial ray is the equation of a line, and the constant of proportionality is the ray slope. The need to use the ray slope is also apparent in the above figure. As the physical angle goes from 0 to 90 degrees (or 0 to π/2 radians), the ray height at the following surface goes from 0 to infinity. Since the ray slope also goes from 0 to infinity, the paraxial raytrace equation correctly gives the correct result without any approximations. The use of a physical angle in radians instead of the paraxial ray angle in the transfer equation is only valid by approximation for small angles or by the use of trig functions.

Origin of Paraxial Ray Angles

Paraxial angles first show up in the derivation of the paraxial raytrace equations. It should now be clear that the slope is required for the transfer equation, but the ray slopes are also used in finding the refraction equation. Returning to the anamorphic figure used to derive the paraxial refraction equation, the paraxial ray angles u and u′ are initially defined by the ray slopes.



$$u = \frac{y}{-z}$$

$$u' = -\frac{y}{z'}$$

The paraxial approximations that are made in this derivation:

- Small angle approximation for Snell's Law (ni = n′i′)
- Ignoring the surface sag and assuming refraction occurs in the vertex plane.

There are no small angle assumptions made for the paraxial ray angles.

Small Angle Approximations

While it is true that for small angles the tangent of an angle (in radians) is approximately equal to the angle, this is only an approximation – even here the angle loses its units of radians in this conversion to obtain the unitless ray slope.

$$u \approx \theta \quad \text{for small angles } (u \text{ unitless; } \theta \text{ radians})$$

Care must be used in making this approximation as paraxial angles are often used that exceed the small angle approximation. Since the raytrace equations are linear in ray slope and not in ray angle, the ray slope must be used for the paraxial ray angles.

In general, a tangent is required to convert between paraxial angles (or more accurately slopes) and physical angles in degrees or radians.

$$u = \tan\theta = \frac{y}{t}$$

Since the paraxial ray angle is a slope, it is incorrect to determine the paraxial ray angle as if it were a physical angle.

$$u \neq \tan^{-1}\frac{y}{t}$$