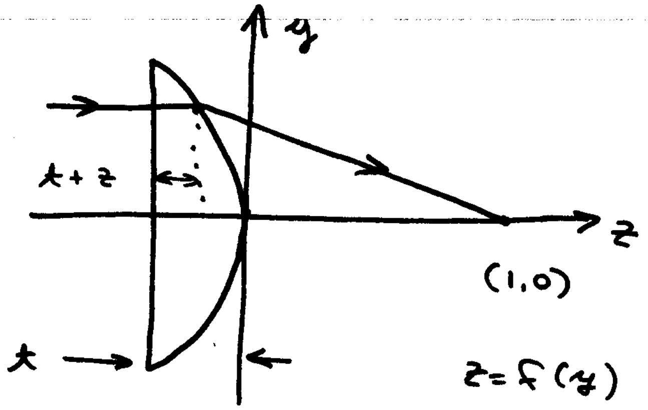


Perfect Plano-Convex Lens

a) By Fermat's Principle : All OPL's are equal

for $y = 0$ $\quad OPL = nt + 1$

for y: $\quad OPL = n(t+z) + \sqrt{y^2 + (1-z)^2}$

$$nt + 1 = nt + nz + \sqrt{y^2 + (1-z)^2}$$

$$(1 - nz)^2 = y^2 + (1-z)^2$$

$$1 - 2nz + n^2 z^2 = y^2 + 1 - 2z + z^2$$

$$\boxed{z^2(n^2 - 1) - 2z(n-1) - y^2 = 0}$$

Of the form: $Az^2 + Cy^2 + Dz + Ey + F = 0 \; ; \quad AC < 0$

$\longrightarrow$ A Hyperbola about the z-axis.

Can also be put in the form:

$$z^2(n+1)^2 - 2z(n+1) - y^2\left(\frac{n+1}{n-1}\right) = 0$$

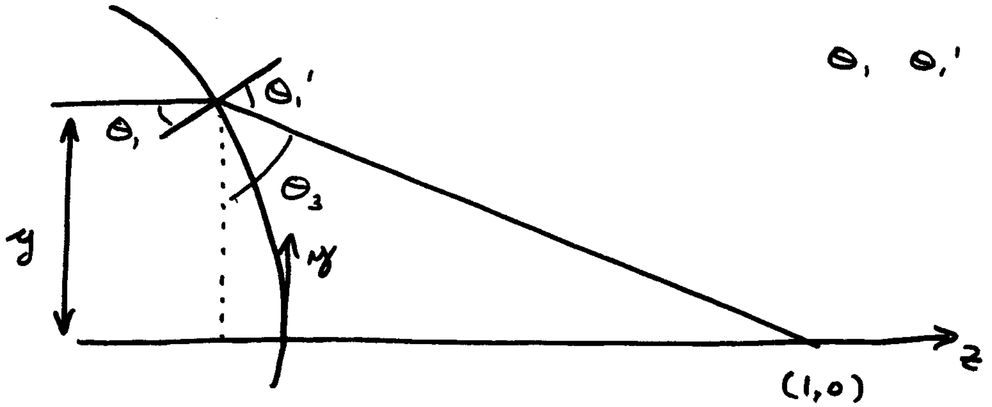
$$(z(n+1) - 1)^2 - y^2\left(\frac{n+1}{n-1}\right) = 1$$

$$\boxed{(n+1)^2\left(z - \frac{1}{n+1}\right)^2 - y^2\left(\frac{n+1}{n-1}\right) = 1}$$

Hyperbola with center at

$$z = \frac{1}{n+1} \qquad y = 0$$

b) By Snell's Law



θ_1, θ_1' are neg

$$\theta_3 - \theta_1' = 90 - \theta_1 \quad \Rightarrow \quad \theta_1' = \theta_1 - 90 + \theta_3$$

$$\tan\theta_3 = \frac{1-z}{y}$$

Slope of surface normal : $m_\perp = -\dfrac{dz}{dy}$

$$\tan\theta_1 = -m_\perp = \frac{dz}{dy}$$

$$\sin\theta_1 = \sin\left[\tan^{-1}\left(\frac{dz}{dy}\right)\right] = \frac{dz/dy}{\sqrt{1+(dz/dy)^2}} \quad \text{(A)}$$

$$\sin\theta_1' = \sin(\theta_1 - 90 + \theta_3) = -\cos(\theta_1 + \theta_3)$$

$$\sin\theta_1' = -\cos\theta_1\cos\theta_3 + \sin\theta_1\sin\theta_3$$

By Trig ID and above expressions: $\tan\theta_3$; $\sin\theta_1$

$$\sin\theta_1' = -\frac{1}{\sqrt{1+(dz/dy)^2}}\cdot\frac{1}{\sqrt{1+(1-z)^2/y^2}} + \frac{dz/dy}{\sqrt{1+(dz/dy)^2}}\cdot\frac{(1-z)/y}{\sqrt{1+(1-z)^2/y^2}} \quad \text{(B)}$$

b) Continued...

Use Snell's Law $\qquad n \sin \theta_1 = \sin \theta_1'$

Using Ⓐ and Ⓑ, equate and simplify

$$\boxed{n\sqrt{y^2 + (1-z)^2} = -\frac{dy}{dz}\,y + 1 - z}$$

Use the "Assume a solution of the form..." method of solving DEQ's — the obvious solution to use is that of part (a):

$$z^2(n^2-1) - 2z(n-1) - y^2 = 0$$

$$\frac{dy}{dz} = \left[z(n^2-1) - (n-1)\right] / y$$

Plugging these two eq's into the DEQ shows that this result satisfies the DEQ and is therefore a valid solution

$$\boxed{z^2(n^2-1) - 2z(n-1) - y^2 = 0}$$

<u>Note</u>: a rigorous solution of the DEQ is possible.

It uses 2 or 3 variable transformations.