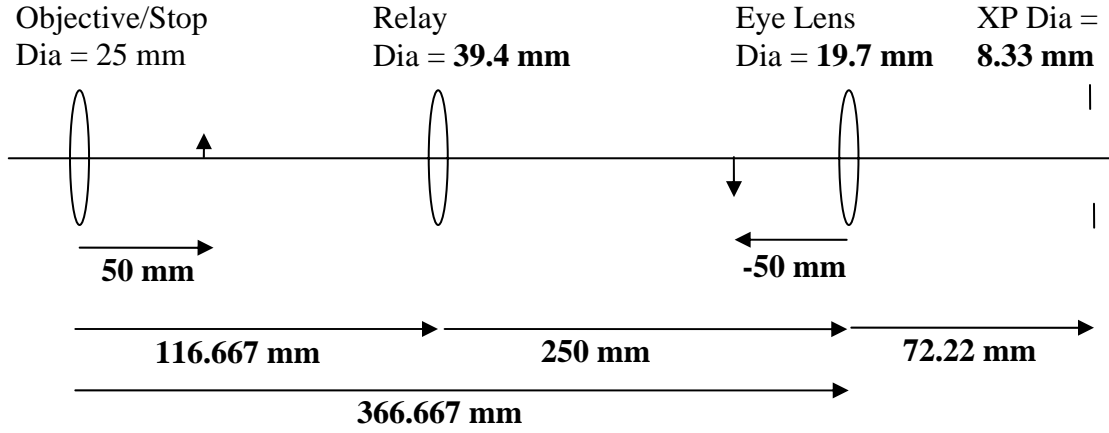


Riflescope with Existing Lenses

Section A and B (also show intermediate image planes)



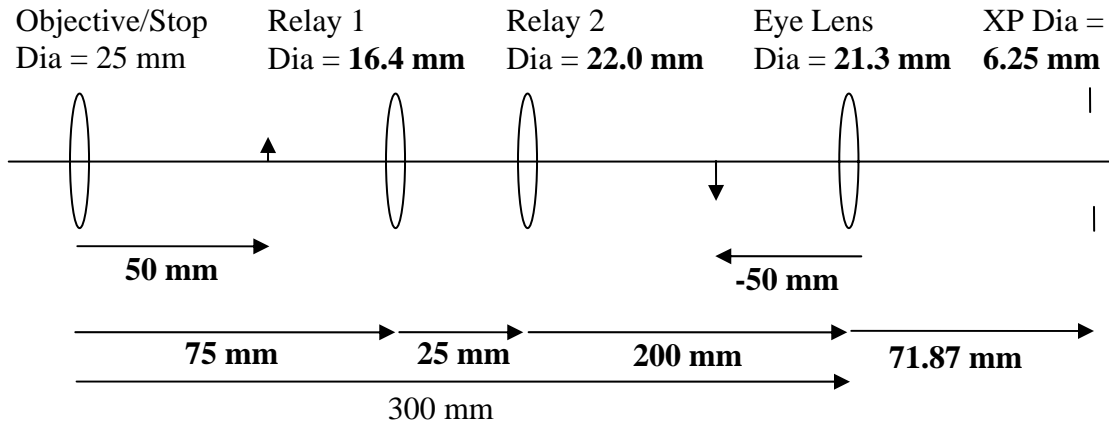
Section C – Summary comments

The relay lens becomes the stop of the system, changing the EP, the XP and the ER:

$D_{XP} = 6.25\text{ mm}$	$D_{EP} = 18.75\text{ mm}$
$ER = 62.5\text{ mm}$	EP is 87.5 mm in front of the objective

A 3x18.75 riflescope results, and you cannot meet the requirement of 3X25.

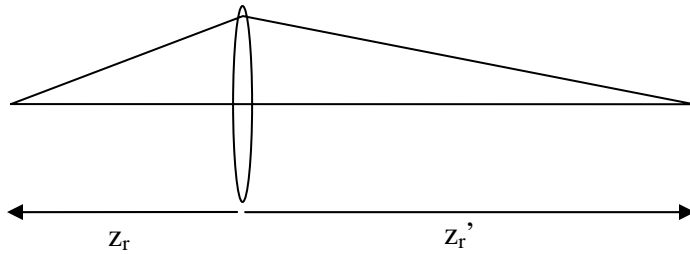
Section D (also show intermediate image planes)



Riflescope with Existing Lenses

Section A – 3X25 Riflescope with three 50 mm focal length lenses

The relayed Keplerian requires three elements (objective, relay and eye lens), and the base telescope consists of just the objective and the eye lens. Since these two lenses both have the same focal length, the base MP is -1. All of the magnification (and the image erection) in the system must come from the relay stage.



$$MP = m_r MP_{Base\ Telescope} = 3$$

$$m_r = -3 = \frac{z_r'}{z_r}$$

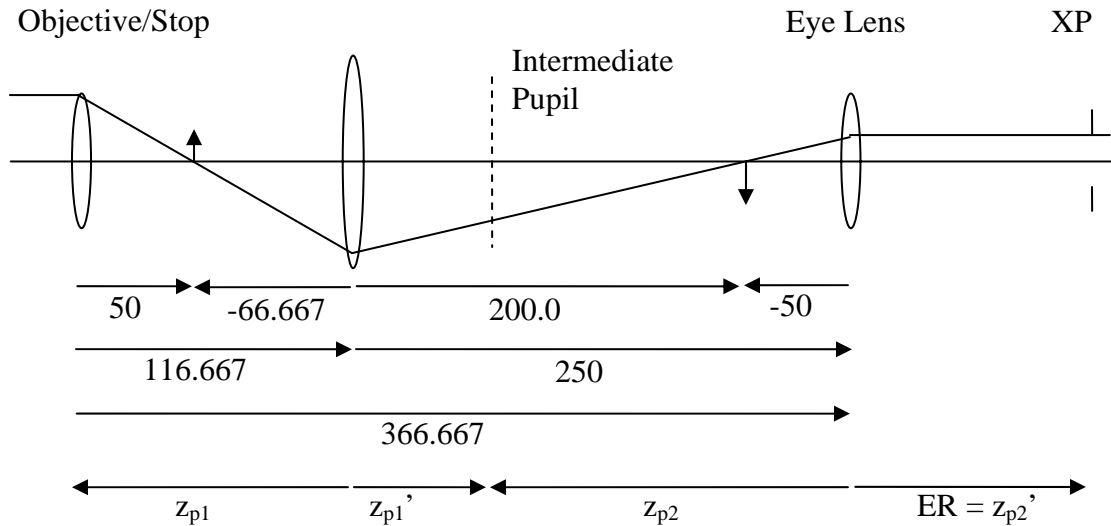
$$\frac{1}{z_r'} = \frac{1}{z_r} + \frac{1}{f}$$

$$f = 50$$

$$z_r' = 200.0$$

$$z_r = -66.667$$

The resulting riflescope (with the marginal ray and intermediate images sketched):



The XP location is found by imaging the stop first through the relay lens to an intermediate pupil, then re-imaging this pupil through the eye lens to the XP:

$$\frac{1}{z'_{p1}} = \frac{1}{z_{p1}} + \frac{1}{f}$$

$$\frac{1}{z'_{p2}} = \frac{1}{z_{p2}} + \frac{1}{f}$$

Relay:

$$f = 50$$

$$z_{p1} = z_r - f = -116.667$$

$$z'_{p1} = 87.50$$

$$m_1 = \frac{z'_{p1}}{z_{p1}} = -0.750$$

Eye Lens:

$$f = 50$$

$$z_{p2} = -z'_r - f + z'_{p1}$$

$$z_{p2} = -162.5$$

$$z'_{p2} = 72.22$$

$$m_2 = \frac{z'_{p2}}{z_{p2}} = -0.444$$

The XP magnification is the product of these two magnifications:

$$m_{XP} = m_1 m_2 = 0.3333$$

$$D_{XP} = m_{XP} D_{STOP} = 8.33 \text{ mm} \quad D_{STOP} = D_{EP} = 25 \text{ mm}$$

This result can also be obtained directly using the riflescope MP.

Summary:

Overall length = 366.667 mm = 14.4 in

Separations: 116.667 mm

250.0 mm

ER = 72.22 mm

D_{XP} = 8.33 mm

Intermediate images: Rear focal point of the objective

Front focal point of the eye lens

Section B – Required diameters for the 3X25 system

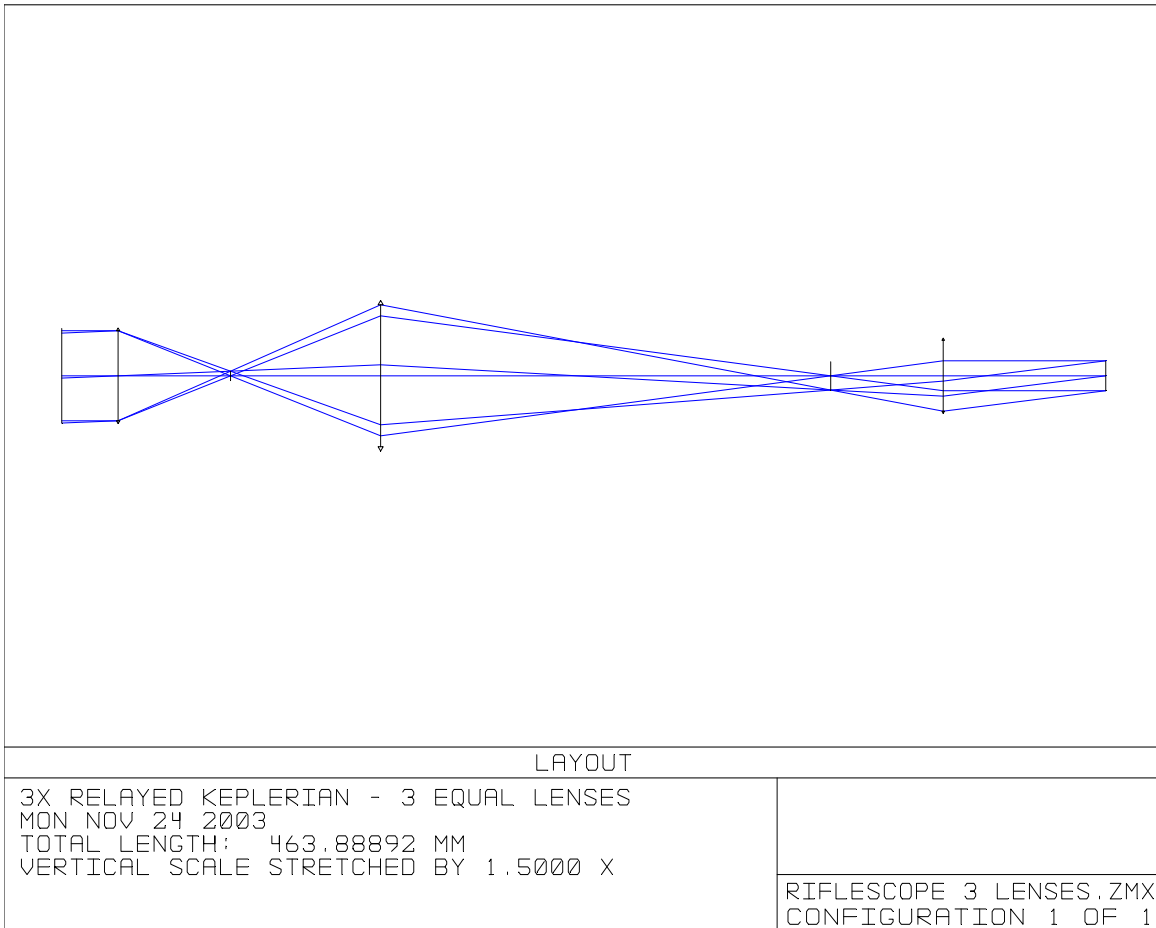
$$HFOV = 1.5 \text{ deg} \quad \bar{u} = \tan(HFOV) = 0.02619$$

The raytrace of the marginal and chief rays is attached. The required element radius for the system to be unvignetted is

$$a \geq |y| + |\bar{y}|$$

Objective/Stop	a = 12.5 mm	D _{STOP} = 25.0 mm
Relay	a = 19.7 mm	D _{RELAY} = 39.4 mm
Eye Lens	a = 9.84 mm	D _{EYE LENS} = 19.7 mm

The system layout for this three element riflescope:



Section C – 25 mm Diameter elements

At first glance, the requirement to use 25 mm diameter elements would seem to introduce vignetting because the required element diameter for the relay lens exceeds 25 mm. However, closer examination of the raytrace data shows that the 25 mm diameter relay lens will not pass the system marginal ray (required radius is 16.667 mm). As a result, the relay lens becomes the system stop. With this change, the element spacings and the system MP do not change, but the marginal and chief rays do change. Both the EP and XP also change location and size.

$$D_{RELAY} = D_{STOP} = 25mm$$

XP and ER: Image the Relay/Stop through the eye lens

$$\frac{1}{z'_{XP}} = \frac{1}{z_{XP}} + \frac{1}{f}$$

$$m_{XP} = \frac{z'_{XP}}{z_{XP}} = -0.25$$

$$f = 50$$

$$z_{XP} = -250$$

$$D_{XP} = 6.25mm$$

$$z'_{XP} = ER = 62.5$$

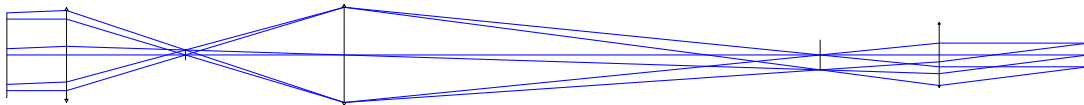
$$ER = 62.5mm$$

EP: Image the relay lens back through the objective to get the location. The EP is now located 87.5 mm in front of the objective. The diameter of the EP can be found using the MP of the riflescope:

$$D_{EP} = MP D_{XP} = 18.75mm$$

The limitation of using 25 mm diameter lenses reduces the eye relief and also the entrance pupil diameter. A 3X18.75 riflescope results, and you cannot build a 3X25 riflescope with just these three equal lenses.

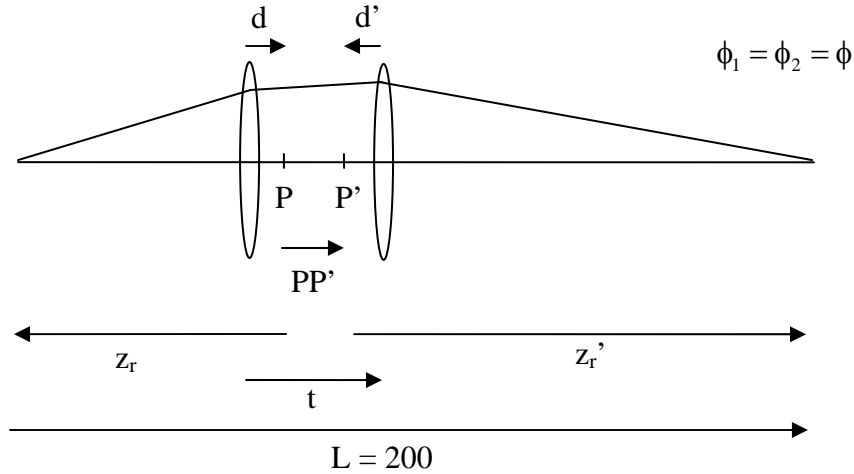
System layout with the stop at the relay lens:



Vignetting cannot occur at the system stop. This system (with the stop at the relay lens) covers the entire field of view without vignetting. However it does so with reduced EP and XP diameters and a shorter ER.

Section D – 4X25 Riflescope with a length of 300 mm using 4 elements

The relay portion of the telescope is to be redesigned using two 50 mm focal length elements instead of one. The Base telescope remains the same with a MP of -1 and a length of 100 mm. This implies that the relay stage must have a length of 200 mm (physical separation of the two intermediate images) and a magnification of -4. The three unknowns are the separation and the object and image distances.



1)

$$m_r = \frac{z_r'}{z_r} \quad z_r' = m_r z_r$$

$$-z_r + PP' + z_r' = L$$

$$PP' + (m_r - 1)z_r = L$$

2)

$$\frac{1}{z_r'} = \frac{1}{z_r} + \phi_r$$

$$\frac{1}{m_r z_r} = \frac{1}{z_r} + \phi_r$$

$$z_r = \frac{1 - m_r}{m_r \phi_r}$$

3)

$$d = \frac{\phi_2}{\phi_r} t \quad d' = -\frac{\phi_1}{\phi_r} t$$

$$\phi_r = \phi_1 + \phi_2 - \phi_1 \phi_2 t$$

$$t = \frac{\phi_1}{\phi_r} t + \frac{\phi_2}{\phi_r} t - \frac{\phi_1 \phi_2 t^2}{\phi_r}$$

$$t = -d' + d - \frac{\phi_1 \phi_2 t^2}{\phi_r}$$

$$PP' = t - d + d' = -\frac{\phi_1 \phi_2 t^2}{\phi_r}$$

$$PP' = -\frac{\phi^2 t^2}{\phi_r}$$

4)

$$\phi_r = \phi_1 + \phi_2 - \phi_1 \phi_2 t$$

$$\phi_r = 2\phi - \phi^2 t$$

Combining 1) and 2):

$$PP' + (m_r - 1)z_r = L \quad z_r = \frac{1 - m_r}{m_r \phi_r}$$

$$PP' + (m_r - 1) \frac{(1 - m_r)}{m_r \phi_r} = L$$

$$m_r \phi_r PP' - (1 - m_r)^2 = m_r \phi_r L$$

Adding 3):

$$\phi_r PP' = -\phi^2 t^2$$

$$-m_r \phi^2 t^2 - (1 - m_r)^2 = m_r \phi_r L$$

And substituting for the system power from 4):

$$-m_r \phi^2 t^2 - (1 - m_r)^2 = m_r (2\phi - \phi^2 t)L$$

$$m_r \phi^2 t^2 - m_r \phi^2 Lt + 2m_r \phi L + (1 - m_r)^2 = 0$$

$$t = \frac{m_r \phi^2 L \pm \sqrt{(m_r \phi^2 L)^2 - 4(m_r \phi^2)(2m_r \phi L + (1 - m_r)^2)}}{2m_r \phi^2}$$

$$t = \frac{L}{2} \pm \sqrt{\frac{L^2}{4} - \frac{2L}{\phi} - \frac{(1 - m_r)^2}{m_r \phi^2}}$$

Here: $L = 200$ $m_r = -4$ $\phi = 1/f = 0.02$

$$t = 25.0 \quad \text{or} \quad 175.0$$

The correct solution is $t = 25.0$ mm. The other solution leads to a net negative value for ϕ_r and requires virtual intermediate objects and images (not to mention a nonsensical arrangement of lenses). Note that the above equation is equivalent to the result in the class notes for “Thin Lens Design – Two Given Lenses.”

The remaining design parameters based upon this solution and the intermediate equations:

$$\begin{aligned} \phi_r &= 0.0333/\text{mm} \\ f_r &= 33.333 \text{ mm} \\ z_r &= -41.666 \text{ mm} \\ z_r' &= 166.666 \text{ mm} \\ PP' &= -8.333 \text{ mm} \\ d &= 16.667 \text{ mm} \\ d' &= -16.667 \text{ mm} \end{aligned}$$

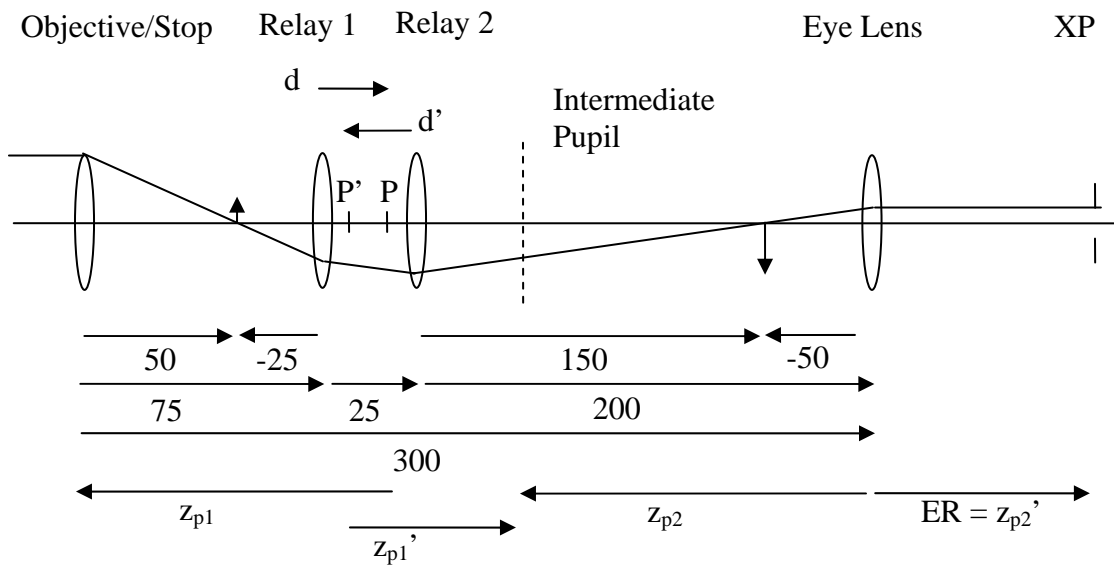
The intermediate image locations relative to the two relay elements:

$$\begin{aligned} s &= z_r + d = -25.0 \text{ mm} \\ s' &= z_r' + d' = 150 \text{ mm} \end{aligned}$$

And the required element separations are

$$\begin{aligned} t_2 &= t = 25 \text{ mm} \\ t_1 &= f + s = 75 \text{ mm} && \text{Objective to Relay 1} \\ t_3 &= s' + f = 200 \text{ mm} && \text{Relay 2 to Eye Lens} \end{aligned}$$

The layout of this four element riflescope with the intermediate image locations noted:



The principal planes shown are associated with the compound relay lens.

As before, the XP location is found by imaging the stop first through the compound relay lens to an intermediate pupil, then re-imaging this pupil through the eye lens to the XP:

$$\frac{1}{z'_{p1}} = \frac{1}{z_{p1}} + \frac{1}{f_r}$$

$$\frac{1}{z'_{p2}} = \frac{1}{z_{p2}} + \frac{1}{f}$$

	$f_r = 33.333$		$f = 50$
Relay:	$z_{p1} = z_r - f = -s - d$	Eye Lens:	$z_{p2} = -z'_r - f + z'_{p1}$
	$z_{p1} = -91.667$		$z_{p2} = -164.3$
	$z'_{p1} = 52.38$		$z'_{p2} = 71.87$
	$m_1 = \frac{z'_{p1}}{z_{p1}} = -0.571$		$m_2 = \frac{z'_{p2}}{z_{p2}} = -0.437$

The XP magnification is the product of these two magnifications:

$$m_{XP} = m_1 m_2 = 0.250$$

$$D_{XP} = m_{XP} D_{STOP} = 6.25mm \quad D_{STOP} = D_{EP} = 25mm$$

Once again, this result can also be obtained directly using the riflescope MP.

Summary:

- Overall length = 300 mm = 12 in
- Separations: 75 mm
- 25 mm
- 200 mm
- ER = 71.87 mm
- D_{XP} = 6.25 mm
- Intermediate images: Rear focal point of the objective
- Front focal point of the eye lens

The required diameters for the 4X25 system must be determined for the same FOV as before:

$$HFOF = 1.5 \text{ deg} \quad \bar{u} = \tan(HFOV) = 0.02619$$

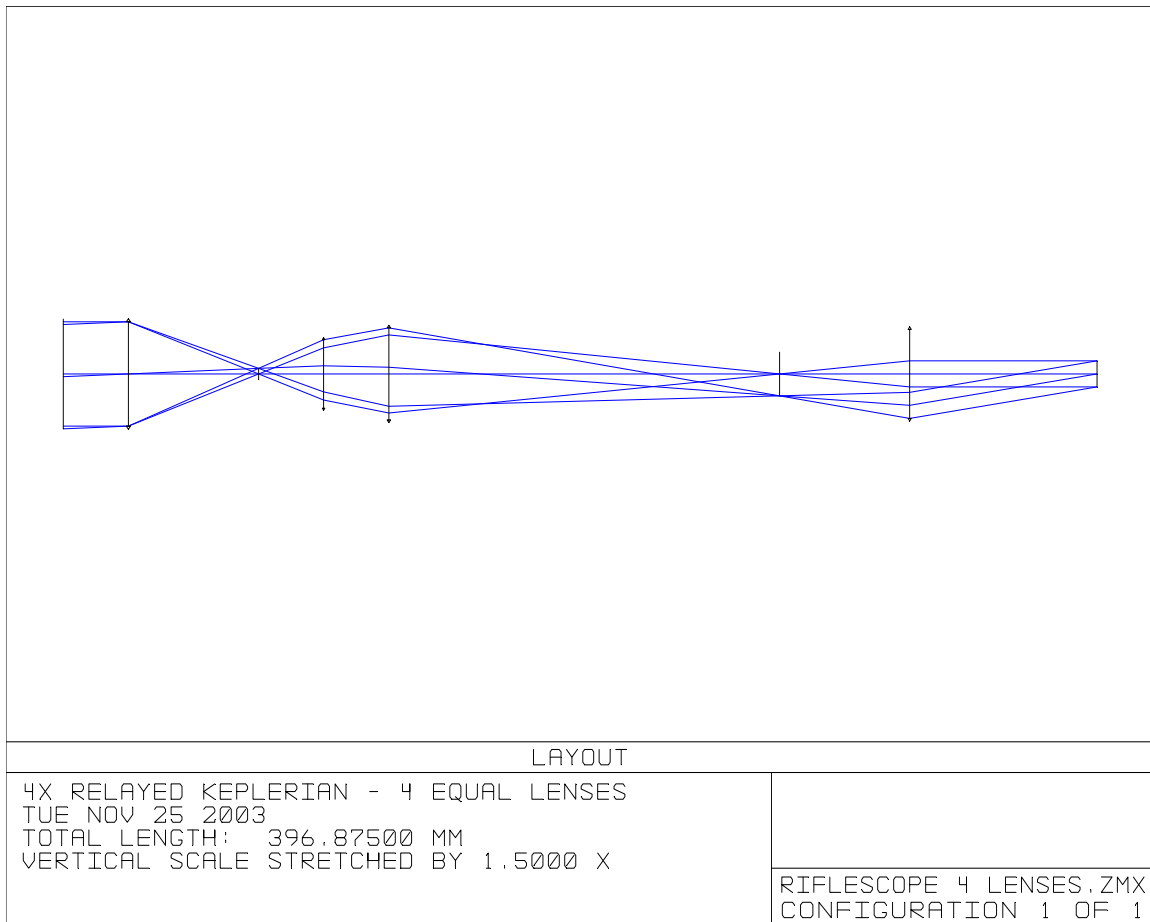
The raytrace of the marginal and chief rays is attached.

The required element radius for the system to be unvignetted is

$$a \geq |y| + |\bar{y}|$$

Objective/Stop	a = 12.5 mm	D _{STOP} = 25.0 mm
Relay 1	a = 8.2 mm	D _{RELAY1} = 16.4 mm
Relay 2	a = 11.0 mm	D _{RELAY2} = 22.0 mm
Eye Lens	a = 10.65 mm	D _{EYE LENS} = 21.3 mm

The system layout for this four-element riflescope:



Using four equal lenses produces a good riflescope design. All of the lenses are used at reasonable conjugates, and the required diameters are all less than that of the objective. An eye relief of almost 3 inches is obtained.

4-Element Rifle Scope

YNU Method

Objective Strip 1 Int. Image 2 Relay 1 Int. Image 3 Relay 2 Int. Pupil 4 Int. Image 5 Int. Image 6 Eye Lens 7 XP 8 9

f	50	-	50	50	-	50	-	50	-
t	50	25	25	35.71	114.3	50	71.87		
n									

ϕ	-0.2	-	-0.2	-	-	-	-0.2	-
u_n	50	25	25	35.71	114.3	50	71.87	

Marginal Ray

y	12.5	0	-6.25	-9.375	-7.14	0	3.125	3.125
ny	0	-25	-125	.0625	.0625	.0625	0	
u								

Chief Ray

\bar{y}	0	1.309	1.904	1.637	0	-5.237	-7.528	0
$n\bar{y}$.02619	.02619	.02619	-.01309	-.04582	-.04582	.1047	.1047
\bar{u}								

$a = 1.71 + 1.71$

a	12.5	8.2	11.0				10.65	
ny								
u								

Checks: $y_{STOP} = 4 \cdot \bar{y}_{XP}$ $u_{XP} = 0$
 $\bar{u}_{XP} = 4 \cdot \bar{u}_{STOP}$ $\bar{y}_{XP} = 0$