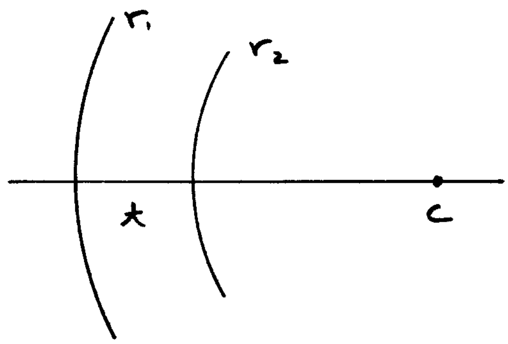
Thick Lens

Determine the Gaussian properties of a thick lens of radius r_1 and thickness t.

a) r_2 is concentric with r_1



$$r_2 = r_1 - t$$

$$\phi_1 = (n-1)/r_1$$

$$\phi_2 = -(n-1)/r_2$$

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t/n$$

$$\phi = (n-1)\left(1/r_1 - 1/r_2 + \frac{n-1}{n} t/r_1 r_2\right)$$

$$\phi = (n-1)\left(\frac{r_2 - r_1 + t - t/n}{r_1 r_2}\right)$$

$$r_2 - r_1 = -t$$

$$\phi = \frac{-(n-1)\,t}{n r_1 r_2}$$

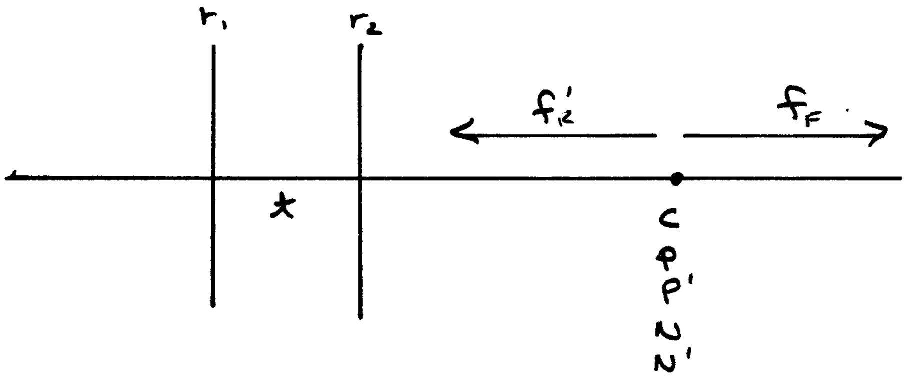
$$f_e = f_R' = \frac{1}{\phi} = \frac{-n r_1 r_2}{(n-1)\,t} \qquad f_F = -\frac{1}{\phi} = \frac{n r_1 r_2}{(n-1)\,t}$$

$$d = \delta = \frac{\phi_2}{\phi}\frac{t}{n} = +\frac{(n-1)}{r_2}\frac{n r_1 r_2}{(n-1)\,t}\frac{t}{n} = r_1$$

$$d' = \delta' = -\frac{\phi_1}{\phi}\frac{t}{n} = \frac{(n-1)}{r_1}\frac{n r_1 r_2}{(n-1)\,t}\frac{t}{n} = r_2$$

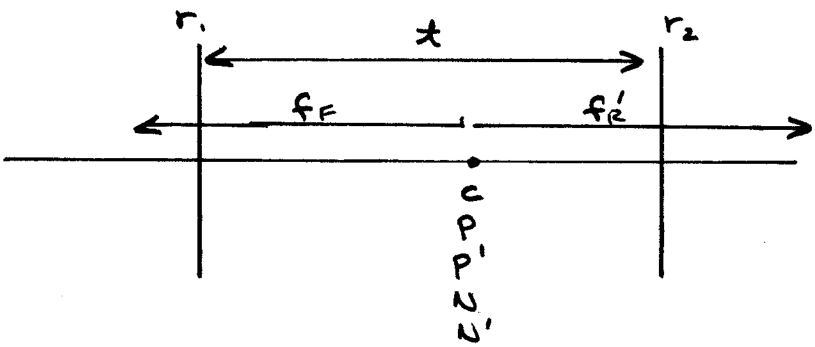
P, P', N and N' are all located at $\underline{\underline{c}}$!

For r_1 and $r_2 > 0$, this is a negative lens.

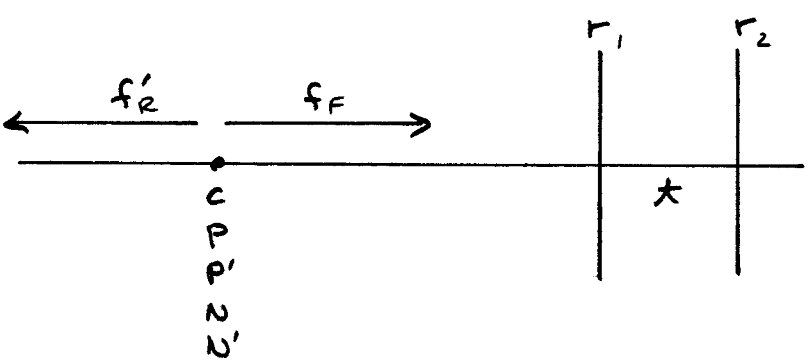


Note that these equations also work for the other two conditions:

$r_1 > 0$ $r_2 < 0$ – a positive lens



$r_1 < 0$ $r_2 < 0$ – a negative lens



b) Surface 2 has equal but opposite power.

$$\phi_1 = (n-1)/r_1 \qquad \phi_2 = -(n-1)/r_2$$

$$\phi_1 = -\phi_2 \qquad r_1 = r_2$$

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t/n$$

$$\phi = \phi_1^2 t/n$$

$$\phi = \frac{(n-1)^2 t}{n r_1^2}$$

for any r_1, this is a positive lens. $(t>0)$

$$f_e = f_R' = \frac{1}{\phi} = \frac{n r_1^2}{(n-1)^2 t}$$

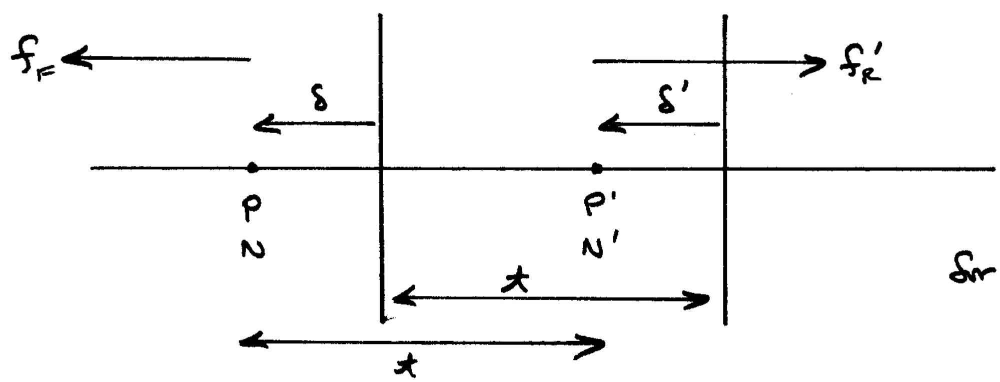
$$f_F = -\frac{1}{\phi} = -\frac{n r_1^2}{(n-1)^2 t}$$

$$d = \delta = \frac{\phi_2}{\phi}\frac{t}{n} = -\frac{(n-1)}{r_1}\frac{n r_1^2}{(n-1)^2 t}\frac{t}{n} = \frac{-r_1}{(n-1)}$$

$\delta = \delta'$

$$d' = \delta' = -\frac{\phi_1}{\phi}\frac{t}{n} = -\frac{(n-1)}{r_1}\frac{n r_1^2}{(n-1)^2 t}\frac{t}{n} = \frac{-r_1}{(n-1)}$$

P, N: δ from V_1 $\qquad$ P', N' δ' from V_2



for $r_1 > 0$

c) The lens has zero power.

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 \tau = 0$$

$$(n-1)\left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{n-1}{n}\frac{t}{r_1 r_2}\right) = 0$$

$$r_2 - r_1 + \frac{n-1}{n} t = 0$$

$$r_2 = r_1 - \frac{n-1}{n} t$$

$$f_F = -\frac{1}{\phi} = \infty$$

$$f_R' = \frac{1}{\phi} = \infty$$

$$\delta = \frac{\phi_2}{\phi}\frac{t}{n} = \infty$$

$$\delta' = -\frac{\phi_1}{\phi}\frac{t}{n} = \infty$$

Note: A better answer is that the principal planes and nodal points are undefined.

Reason: For an afocal system, the magnification m is constant. Therefore $m = 1$ everywhere or nowhere. Since in general $m \neq 1$, there are no principal planes.

Similarly, the angular magnification of an afocal system is also constant. The nodal points (angular magnification = 1) are therefore also not defined.