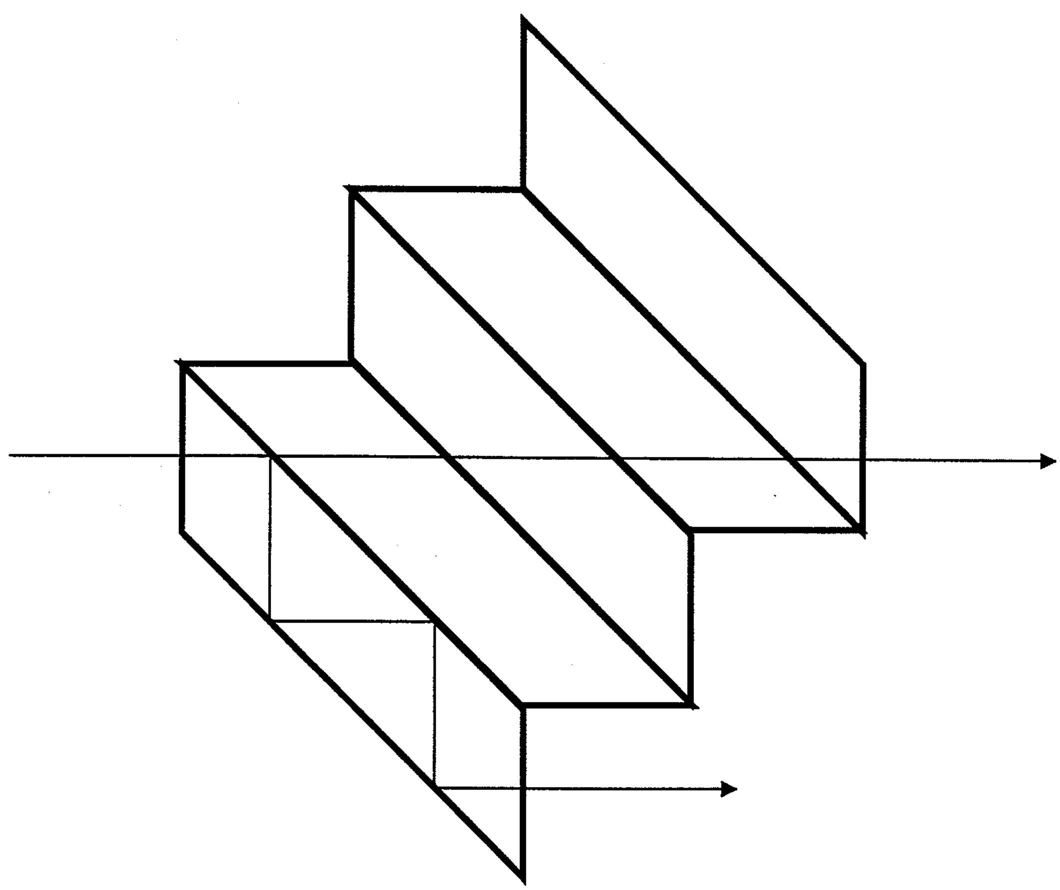
October 18, 2005 Lecture 17

Name Solutions

Closed book; closed notes. An equation sheet is attached and can be removed.
Use the back sides if required.
Do not use any pre-stored information or programs in your calculator.
Note any assumptions you make in solving the problems.
Show your work. Present it in a neat and logical fashion.
If a method of solution is specified, that method must be used.

Distance Students: Please return the original exam only; do not FAX an additional copy.

1) (10 points) Draw the tunnel diagram for this prism with the ray path shown.



2) (10 points) A 2 m tall object is to be imaged onto an image sensor that is 1 cm x 1 cm. The object is located 20 m away from the camera, and the image should fill the sensor. Approximately what focal length lens is required?

2 m is imaged onto 1 cm = .01 m

$$m = -\frac{.01\,m}{2\,m} = \frac{-1}{200}$$

$$m = \frac{z'}{z} \qquad z' = m z$$

$$z' \approx f$$

$$z = z'/m \approx -200 f$$

Object - Image Distance:

$$L = z' - z = 20\,m$$

$$f + 200f = 20\,m$$

$$f \approx .0995\,m = \underline{99.5\,mm}$$

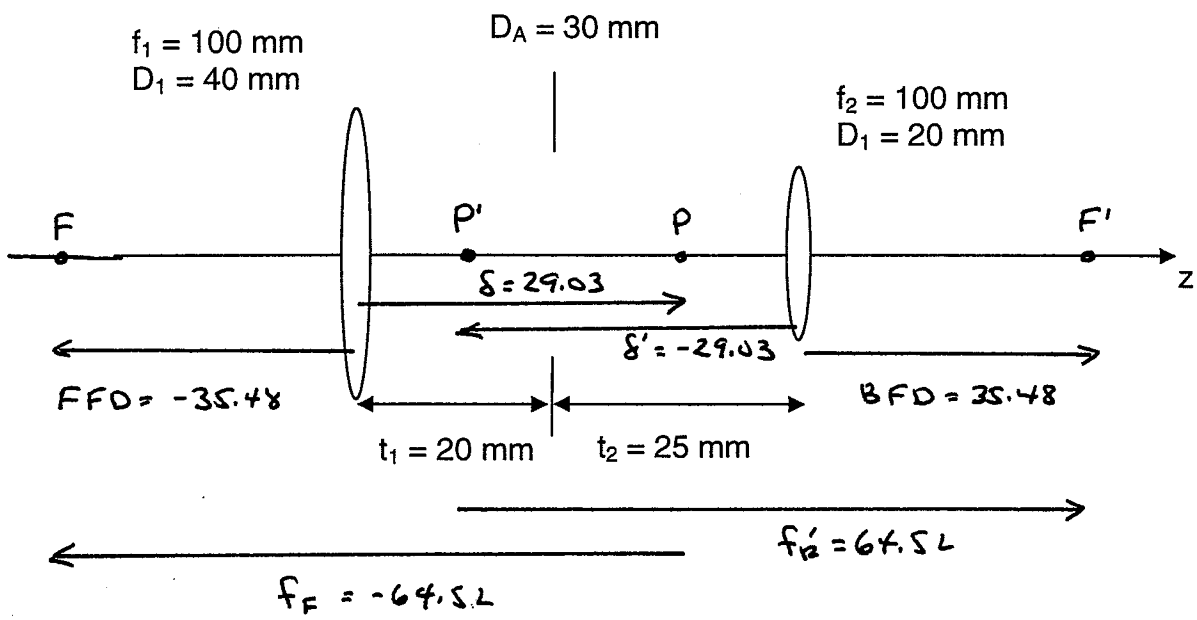
or

$$z' - z \approx -z = 20\,m$$

$$200 f = 20\,m$$

$$f = .100\,m = \underline{100\,mm}$$

3) The following imaging system consisting of two thin lenses and an aperture is used for all parts of this problem. The spacings and the focal lengths and diameters of the lenses are specified. The diameter of the aperture is also given.



3a) (15 points) Use Gaussian reduction to determine System Focal Length, the Front Focal Distance, the Back Focal Distance and the locations of the Front and Rear Principal Planes. Sketch the locations on the system diagram.

Apertures not required here.

$\phi_1 = \phi_2 = .01/mm$

$t = t_1 + t_2 = 45mm$

$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t = .0155/mm$

$f = f_E = \underline{64.52\ mm}$ $\qquad$ $f_R' = 64.52\ mm$

$f_F = -64.52\ mm$

Symmetric System:

$\delta = -\delta' = \frac{\phi_2}{\phi} t$

$\delta = 29.03\ mm$ $\qquad$ $d = \delta$

$\delta' = -29.03mm$ $\qquad$ $d' = \delta'$

$BFD = f_R' + \delta' = 35.48\ mm$

$FFD = f_F + \delta = -35.48\ mm$

3b) (15 points) For an object at infinity, use a paraxial raytrace to determine the Aperture Stop.

Scale this ray so that it goes through the edge of the Aperture Stop. Determine the beam diameter for an on-axis object at the other two elements.

			Lens 1		Aperture		Lens2		F'				
	Surface	0		1		2		3		4		5	6
	f			100		~		100					
	$-\phi$			-.01		-		-.01					
	t		∞		20		25		BFD 35.48				
①	$\tilde{y}$	1		1		.8		.55		0			
	$\tilde{u}$		0		-.01		-.01		-.0155				
②	y	18.18		18.18		14.54		10.0		0			
	u		0		-.1818		-.1818		-.2818				
	y												
	u												

① Trace a ray parallel to the axis at an arbitrary height.

At each aperture, form the ratio of the aperture radius a_i to this ray height $\tilde{y}_i$

Lens 1: $a_1 = 20$ $\tilde{y}_1 = 1$ $a_1/\tilde{y}_1 = 20$

Aperture: $a_A = 15$ $\tilde{y}_A = .8$ $a_A/\tilde{y}_A = 18.75$

Lens 2: $a_2 = 10$ $\tilde{y}_2 = .55$ $a_2/\tilde{y}_2 = 18.18$

The minimum ratio is at Lens 2. blank page follows…

Lens 2 is the aperture stop.

(2) Scale the ray by $a_2/\bar{y}_2 = 18.18$ to obtain the system marginal ray for an object at infinity.

Beam Diameter at Lens 1 = 36.36 mm < D_1

Beam Diameter at Aperture = 29.08 mm < D_A

Beam Diameter at Lens 2 = 20 mm = D_2

The aperture has no effect on the system for an on-axis object.

3c) (10 points) Use the scaled ray from Part (b) to verify the Focal Length and Back Focal Distance of the system. Also determine the Entrance Pupil Diameter from this ray.

BFD = 35.48 mm from ray trace

$$\phi = -\frac{u'}{y_1} = \frac{-.2818}{18.18} = .0155/\text{mm}$$

$f = \underline{64.52 \text{ mm}}$ as before

Since this marginal ray is parallel to the axis in object space

$D_{EP} = 2y_1 = \underline{36.36 \text{ mm}}$

3d) (15 points) Use Gaussian methods to determine the locations and sizes of the Entrance Pupil and the Exit Pupil.

The stop is at Lens 2. The XP is also at Lens 2.

XP: $D_{XP} = 20\,mm$

Located at Lens 2.

EP: Image the stop through Lens 1 to object space.

Light from R → L $\quad n = n' = -1$

$z_S = 45\,mm$

$$\frac{n'}{z'_{EP}} = \frac{n}{z_S} + \frac{1}{f_1} \qquad f_1 = 100\,mm$$

$\underline{z'_{EP} = 81.81\,mm}$ to the right of Lens 1

$$m_{EP} = \frac{z'_{EP}/n'}{z_S/n} = \frac{81.81}{45} = 1.818$$

$D_{EP} = m_{EP}\, D_{STOP} = m_{EP}\, D_2 \qquad D_2 = 20\,mm$

$\underline{D_{EP} = 36.36\,mm}$

(as from the raytrace)

3e) (5 points) Determine the f/# and image space Numerical Aperture of the system for an object at infinity.

$f = 64.52$ mm

$D_{EP} = 36.36$ mm

$f/\# = f/D_{EP} = 64.52\text{ mm}/36.36\text{ mm}$

$\underline{f/1.774}$

$NA = n \sin|U'| \approx n'|u'|$

u' is found from the second ray of Part (b)

$u' = -.2818$ $n' = 1.0$

$NA = \underline{.2818}$

3f) (10 points) An object is located 200 mm to the left of the first lens. Where the image relative to the second lens? Use Gaussian methods.

$S = -200 = z + d$ $\delta = 29.03$ mm

$z = -229.03$ mm

$f = 64.52$

$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$

$z' = 89.82$ mm

$S' = z' + d' = \underline{60.79\text{ mm}}$

$m = z'/z = -.392$

3g) (10 points) Determine the image space Numerical Aperture and the Working f/# for this finite conjugate system (object 200 mm to the left of the first lens). While you are free to do so, it is not necessary to trace a ray.

Is the usual approximation relating NA and f/#$_W$ ($f/\#_W \approx (1-m)\, f/\#$) valid for this system? Why?

Tracing a marginal ray:

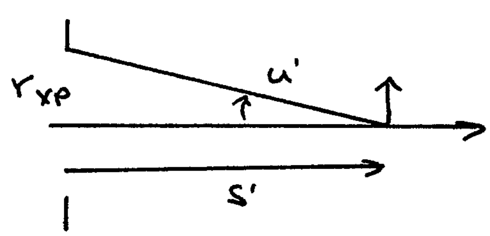
	Obj	EP	Lens 1	Lens 2	Image		
Surface	0	1	2	3	4	5	6
f		–	100	100			
$-\phi$		–	-.01	-.01			
t	281.81	-81.81	45	60.79			
	$-S+z'_{EP}$	$-z'_{EP}$		S'			
y	0	18.18*	12.902	10.0	0		
u	.06451	.06451	-.06451	-.1645			
		* = $D_{EP}/2$					
y							
u							

$u' = -.1645$

$NA = n'|u'| = .1645$

$f/\#_w = 1/2NA = 3.04$

Alternate: Since the stop is the final element, S' is the stop - image distance:



$u' = -r_{XP}/S' = -10/60.79$

$u' = -.1645$

$NA = .1645$ $f/\#_w = 3.04$

Approximation: $f/\#_w = (1-m)\, f/\#$ $f/1.774$

$m = -.392$

$f/\#_w = 2.46$ (from part (f))

The approximation fails because the implicit assumptions are invalid here (thin lens stop at the lens; $D_{XP} = D_{EP}$, etc.)