Name Solutions

Closed book and notes. Use back sides if needed. The time limit is 2 hours.
Do not use any pre-stored information or programs in your calculator.
Show your work. Note any assumptions you make in solving the problems.

Thanks for all your efforts this semester. Have a great holiday!!

1) (10 points) An optical system has third-order astigmatism W_{222} and field curvature W_{220} only. The design goal is to produce a flat medial focus surface. What is the required relative amount of these two coefficients? There is no defocus.

$$W = W_{222} H^2 y_p^2 + W_{220} H^2 \rho^2$$

$$\varepsilon_y = -2 \frac{R}{r_c} W_{222} H^2 y_p - 2 \frac{R}{r_c} W_{220} H^2 y_p$$

$$\varepsilon_x = -2 \frac{R}{r_c} W_{220} H^2 x_p$$

Medial Focus: $\varepsilon_x = -\varepsilon_y$ for any ρ

$$W_{222} + W_{220} = -W_{220}$$

$$W_{222} = -2 W_{220}$$

$$W_{220} = -\frac{1}{2} W_{222}$$

2) (15 points) One of the ninth-order aberration terms is W_{553}.

a. Give the expression for the wavefront aberration.

$$W = W_{553} H^5 \rho^5 \cos^3\theta = W_{553} H^5 \rho^2 y_p^3$$

b. Derive the general expressions for the transverse ray aberrations ε_x and ε_y for this aberration. Give the results in terms of x_p and y_p.

$$W = W_{553} H^5 (x_p^2 y_p^3 + y_p^5)$$

$$\varepsilon_y = -\frac{R}{r_c} \frac{\partial W}{\partial y_p}$$

$$\varepsilon_y = -\frac{R}{r_c} W_{553} H^5 (3 x_p^2 y_p^2 + 5 y_p^4)$$

$$\varepsilon_x = -\frac{R}{r_c} \frac{\partial W}{\partial x_p}$$

$$\varepsilon_x = -\frac{R}{r_c} W_{553} H^5 (2 x_p y_p^3)$$

c. Now provide the equations for the tangential and sagittal ray fans for this aberration.

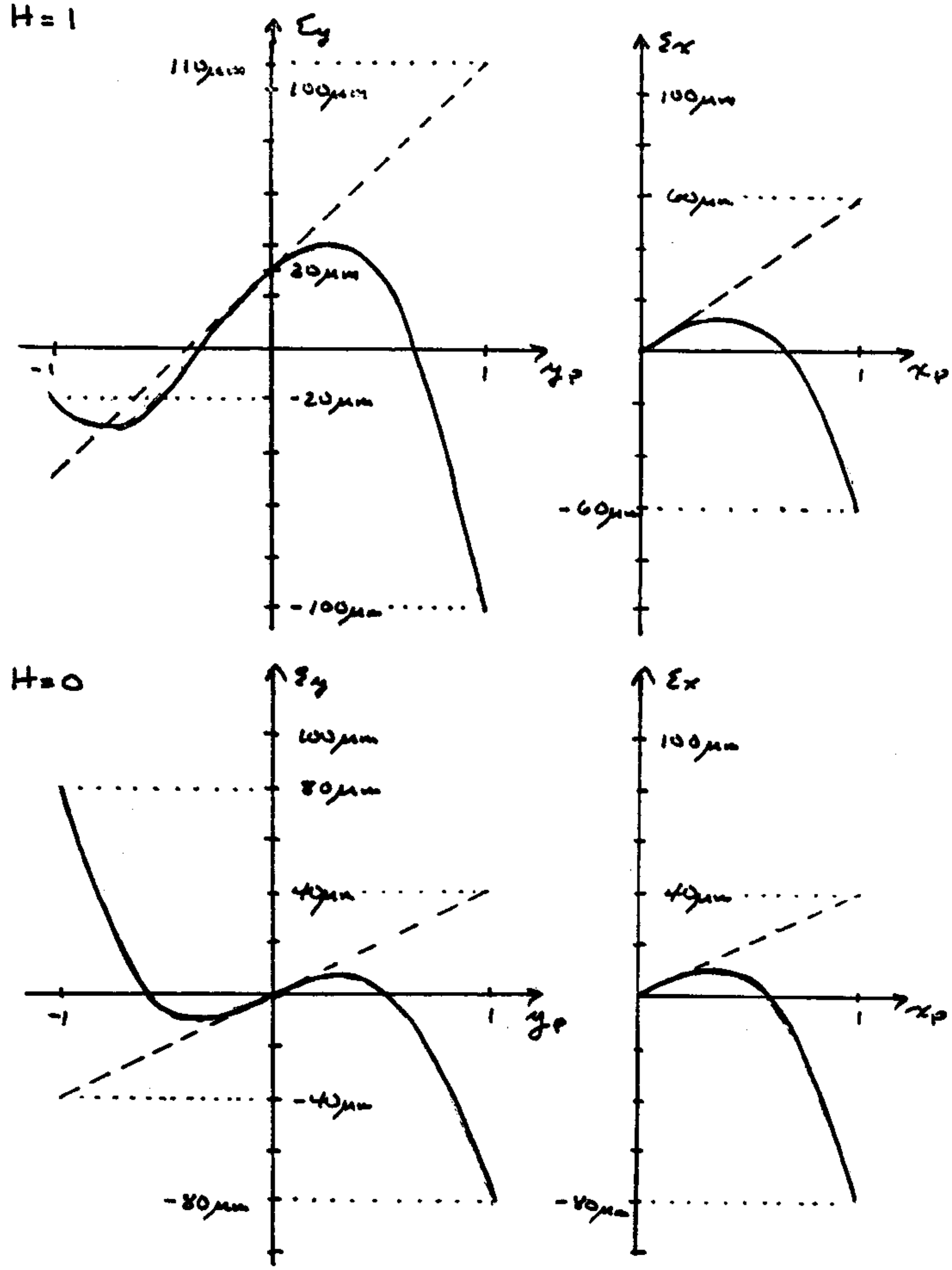
Tangential : $x_p = 0$

$$\varepsilon_y = -5 \frac{R}{r_c} W_{553} H^5 y_p^4$$

Sagittal : $y_p = 0$

$$\varepsilon_x = 0$$

3) (20 points) You are given the following set of rayfans. The ray fans are plotted at full field and on axis for an f/5 optical system. Only first and third order aberrations are present. The dashed lines indicate the slope of the fans at the origin. Calculate the wavefront aberration coefficients W_{xyz} for each aberration present.



$R/r_L \approx 2\, f/\# = 10$

Tilt: H=1 y-fan offset (or Distortion)

$\varepsilon_y(y_p=0;\ H=1) = -R/r_L\, W_{111} H = 30\mu m$

$W_{111} = -3\mu m$

Defocus: x-fan slope at origin (H=0)

$\varepsilon_x^0(x_p=1) = -2\, R/r_L\, \Delta W_{20} = 40\mu m$

$\Delta W_{20} = -2\mu m$

SA: x-fan (H=0)

$\varepsilon_x(x_p=1) = -2\, R/r_L\, \Delta W_{20} - 4\, R/r_L\, W_{040} = -80\mu m$

$W_{040} = 3\mu m$

FC: x-fan slope at the origin (H=1)

$\varepsilon_x^0(x_p=1) = -2\, R/r_L\, \Delta W_{20} - 2\, R/r_L\, W_{220} = 60\mu m$

$W_{220} = -1\mu m$

Astig: y-fan slope at the origin (H=1)

$\varepsilon_y^0(y_p=1) = -2\, R/r_L\, \Delta W_{20} - 2\, R/r_L\, W_{220}$

$- 2\, R/r_L\, W_{222} - R/r_L\, W_{111} = 110\mu m$

$W_{222} = -1\mu m$

Coma: all terms

$\varepsilon_y(y_p=1) = -2\, R/r_L\, \Delta W_{20} - 4\, R/r_L\, W_{040} - R/r_L\, W_{111}$

$- 2\, R/r_L\, W_{220} - 2\, R/r_L\, W_{222} - 3\, R/r_L\, W_{131} = -100\mu m$

$W_{131} = 3\mu m$

$\Delta W_{20} = -2\mu m$	$W_{131} = 3\mu m$
$W_{111} = -3\mu m$	$W_{220} = -1\mu m$
$W_{040} = 3\mu m$	$W_{222} = -1\mu m$

4) (10 points) The following is a list of wavefront aberration coefficients. Which are valid or invalid coefficients for the wavefront expansion of a rotationally symmetric optical system?

$$(H^2)^i (\rho^2)^j (H\rho\cos\theta)^k$$

W_{911} Valid $(H^2)^4 (H\rho\cos\theta)$

W_{683} Not Valid

W_{666} Valid $(H\rho\cos\theta)^6$

W_{321} Not Valid

$W_{25,6,24}$ Not Valid

(For all you fans of old hits by Chicago)

5) (10 points) Design a 250 mm focal-length thin-lens achromatic doublet using the following two glasses:

Glass 1:	SK N 18	639554
Glass 2:	SF8	689312

$n_{d1} = 1.639 \quad \nu_1 = 55.4$

$n_{d2} = 1.689 \quad \nu_2 = 31.2$

$$\phi = 1/f = .004/mm$$

$$\phi_1/\phi = \frac{\nu_1}{\nu_1 - \nu_2} \qquad \phi_2/\phi = \frac{-\nu_2}{\nu_1 - \nu_2}$$

$$\phi_1/\phi = \frac{55.4}{24.2} \qquad \phi_2/\phi = \frac{-31.2}{24.2}$$

$$\phi_1 = .00916/mm \qquad \phi_2 = -.00516/mm$$

Note $\phi_1 + \phi_2 = .004/mm$

$$f_1 = 109.2 \text{ mm} \qquad f_2 = -193.9 \text{ mm}$$

6) (15 points) For these two situations, determine the size and location of the entrance and exit pupils. A thin lens with a focal length 50 mm is used.

a) A 10 mm diameter stop is located 25 mm to the right of the lens.

Stop is XP : $z = 25$ mm $D = 10$

EP : Image of stop through lens
(light R → L $n = -1$)

$\frac{-1}{z'} = \frac{-1}{z} + \frac{1}{f}$ $z' = 50$ mm

$m = z'/z = 2$ $D = 20$ mm

(to the right of the lens)

b) A 10 mm diameter stop is located 25 mm to the left of the lens.

Stop is EP : $z = -25$ mm $D = 10$

XP : Image of stop through lens
(light L → R $n = +1$)

$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$ $z' = -50$

$m = z'/z = 2$ $D = 20$
(to the left of the lens)

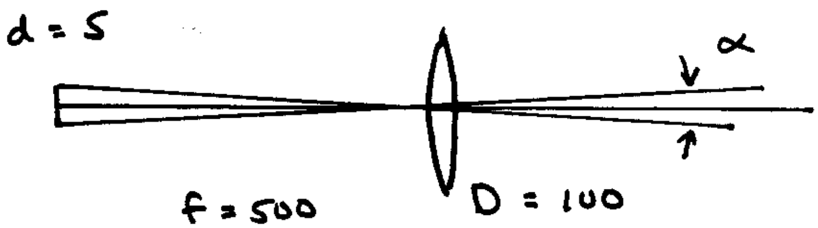
Of course, a + b are just flipped.

7) (20 points) A 5 mm diameter extended source is used to project a spot of light onto a distant wall. A 500 mm focal length lens with a diameter of 100 mm is to be used.

a) The source is placed at the front focal point of the lens to "collimate" it. What is the projected spot size on a wall 50 m away? On a wall 100 m away?

Note: compute the maximum extent of the projected spot. Do not determine the irradiance profile of the spot.

Must include the source divergence



$\alpha = \frac{5}{500} = .01$

$D' = D + \alpha z'$

$D'(50\,m) = 600\,mm$

$D'(100\,m) = 1100\,mm$

or treat it as a vignetting problem

$u' = 0$ $\qquad \bar{u}' = \frac{2.5}{500} = .005$

$y' = 50$ $\qquad \bar{y}' = \bar{u}' z'$

$a' = |y| + |\bar{y}|$

$a' = 50 + .005 z'$

$D' = 2a'$

$D'(50\,m) = 600\,mm$

$D'(100\,m) = 1100\,mm$

b) In an attempt to improve the performance, the source is focused on the wall at 50 m. What is the spot size on this wall, and what is the resulting spot size when the projector is now pointed at the more distant wall at 100 m?

Image at 50 m:

$$m = z'/z \approx z'/f = 50\,m/500\,mm = 100$$

$$D' = 100\,D$$

$$D'(50\,m) = \underline{500\,mm}$$

Spot at 100 m:

Construct marginal + chief rays

$u' = \frac{-50\,mm}{50\,m} = -.001$ $\bar{u}' = \frac{250\,mm}{50\,m} = .005$

$y' = 50\,mm - u'z'$ $\bar{y}' = \bar{u}'z'$

Check: at 50 m $y' = 0$; $\bar{y}' = 250\,mm$

at 100 m

$y' = -50\,mm$ $\bar{y}' = 500\,mm$

$$a' = |y| + |\bar{y}| = 550\,mm$$

$$D'(100\,m) = \underline{1100\,mm}$$

Without the approximation $z = f$:

$z = -505\,mm$ $m\,(at\ 50\,m) = 99$

$D'(50\,m) = 495\,mm$ $D'(100\,m) = 1090\,mm$