Name: Solutions

Closed book and notes. Use back sides if needed. The time limit is 2 hours.
Do not use any pre-stored information or programs in your calculator.
Show your work. Note any assumptions you make in solving the problems.

Thanks for all your efforts this semester. Have a great holiday!!

1) (15 points) An f/10 optical system has the following first and third order aberrations:

$W_{020} = 2\ \mu m$
$W_{040} = -2\ \mu m$
$W_{131} = 1\ \mu m$
$W_{220} = 3\ \mu m$
$W_{222} = -3\ \mu m$
$W_{311} = -2\ \mu m$

a) Write the expression of the wavefront error.

$$W = W_{020}\rho^2 + W_{040}\rho^4 + W_{131}H\rho^3\cos\theta$$
$$+ W_{220}H^2\rho^2 + W_{222}H^2\rho^2\cos^2\theta + W_{311}H^3\rho\cos\theta$$

b) Provide the expressions for the tangential and sagittal ray fans. $R/r_c \sim 2f/\# = 20$

Tangential $(x_p = 0)$:

$$\varepsilon_y = -2\,R/r_c\,W_{020}\,y_p - 4\,R/r_c\,W_{040}\,y_p^3 - 3\,R/r_c\,W_{131}\,H y_p^2$$
$$-2\,R/r_c\,W_{220}\,H^2 y_p - 2\,R/r_c\,W_{222}\,H^2 y_p - R/r_c\,W_{311}\,H^3$$

$$\varepsilon_y = -20\left[4y_p - 8y_p^2 + 3Hy_p^2 - 2H^3\right]\mu m$$

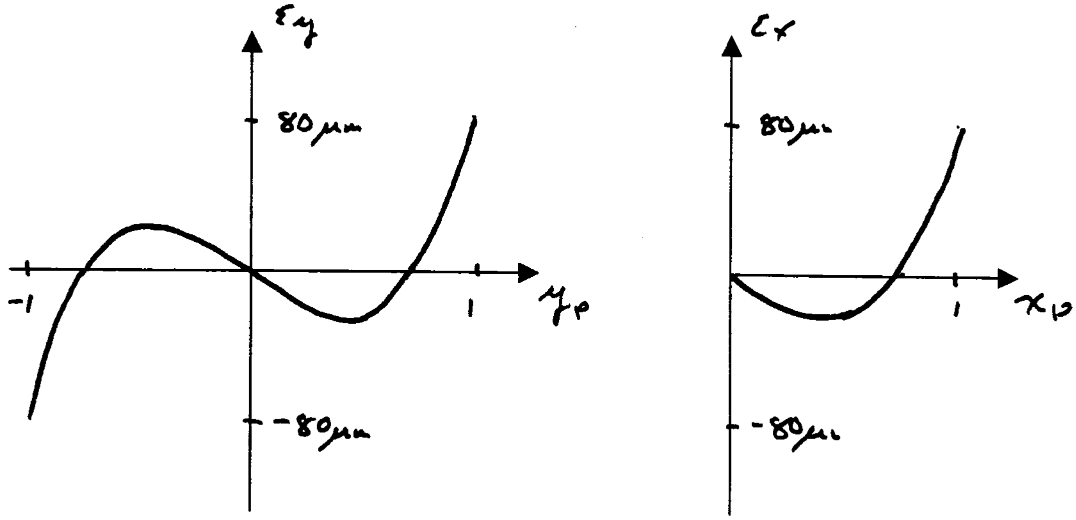
Sagittal $(y_p = 0)$

$$\varepsilon_x = -2\,R/r_c\,W_{020}\,x_p - 4\,R/r_c\,W_{040}\,x_p^3 - 2\,R/r_c\,W_{220}\,H^2 x_p$$

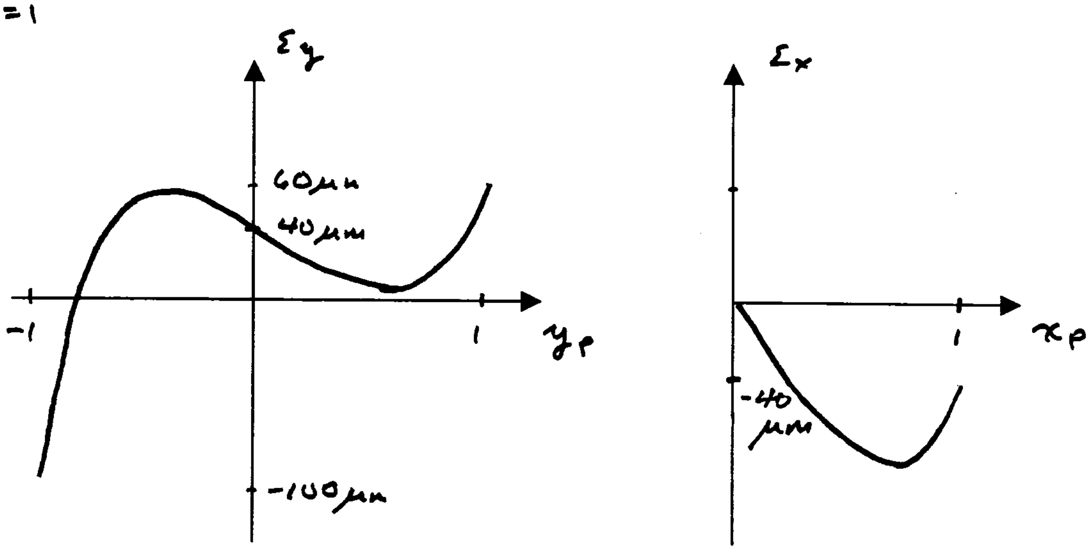
$$\varepsilon_x = -20\left[4x_p - 8x_p^3 + 6H^2x_p\right]\mu m$$

c) Plot the tangential and sagittal ray fans for H = 0 and H = 1. Be sure to label the axes and provide the scale of the plots.

H = 0



H = 1



2) (15 points) One of the ninth-order aberration terms is W_{272}.

Note: W_{272} is not a valid coefficient of a rotational system. However, the problem can be worked by usual methods.

a. Give the expression for the wavefront aberration.

$$W = W_{272} H^2 \rho^7 \cos^2\theta$$

$$W = W_{272} H^2 (x_p^2 + y_p^2)^{5/2} y_p^2$$

b. Derive the general expressions for the transverse ray aberrations ε_x and ε_y for this aberration. Give the results in terms of x_p and y_p.

$$\varepsilon_y = -\frac{R}{r_c}\frac{dW}{dy_p} = -\frac{R}{r_c} W_{272} H^2 \left[\frac{5}{2}(x_p^2+y_p^2)^{3/2} 2y_p^3 + 2y_p(x_p^2+y_p^2)^{5/2}\right]$$

$$\varepsilon_y = -\frac{R}{r_c} W_{272} H^2 \left[5y_p^3(x_p^2+y_p^2)^{3/2} + 2y_p(x_p^2+y_p^2)^{5/2}\right]$$

$$\varepsilon_x = -\frac{R}{r_c}\frac{dW}{dx_p} = -\frac{R}{r_c} W_{272} H^2 \left[\frac{5}{2}(x_p^2+y_p^2)^{3/2} 2x_p y_p^2\right]$$

$$\varepsilon_x = -\frac{R}{r_c} W_{272} H^2 \left[5x_p y_p^2 (x_p^2+y_p^2)^{3/2}\right]$$

c. Now provide the equations for the tangential and sagittal ray fans for this aberration.

Tangential ($x_p = 0$):

$$\varepsilon_y = -\frac{R}{r_c} W_{272} H^2 \left[5y_p^6 + 2y_p^6\right]$$

$$\varepsilon_y = -7\frac{R}{r_c} W_{272} H^2 y_p^6$$

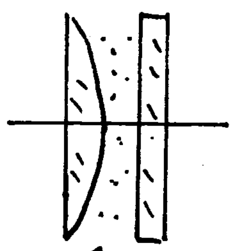
Sagittal ($y_p = 0$):

$$\varepsilon_x = 0$$

3) (15 points) The goal of this problem is to design a variable focal length lens using a plano-convex lens, a plane parallel plate, and a fluid that has a variable index of refraction. The fluid is placed between the lens and the plane parallel plate to produce a sandwich of thin lenses.

Plano-convex lens:	focal length = 100 mm	n = 1.517
Plane parallel plate:	thickness = 2 mm	n = 1.517

a) Sketch the arrangement of elements and give the equation for the system focal length as a function of the index of the fluid. Assume all of the elements are thin lenses in contact.



- The plane parallel plate has no effect.
- The fluid must be against the curved lens surface.

Lens alone: $\phi = \frac{1}{f} = (C_2 - C_1)(n_2 - n_1)$

$f = 100$ $\quad C_1 = 0$ $\quad n_2 = 1$ $\quad n_1 = 1.517$

$C_2 = -.0193/mm$

Only the lens-fluid interface has power

$$\phi = (n_F - 1.517)\, C_2 = (1.517 - n_F)\, .0193/mm$$

$$f = 1/\phi = \frac{51.7\ mm}{(1.517 - n_F)}$$

b) If the index of this fluid can vary from 1.3 to 1.7, what range of focal lengths can be obtained with the system? Note that this is a fictitious fluid!

$n_F = 1.3$ $\qquad f = 238\ mm$

$n_F = 1.7$ $\qquad f = -282\ mm$

When $n_F = 1.517$ the system has zero power.

4) (10 points) Design a 500 mm focal-length thin-lens achromatic doublet using the following two glasses:

Glass 1:	PSK 3	552635	$n_{d_1} = 1.552$	$\nu_1 = 63.5$
Glass 2:	BaSF 2	664358	$n_{d_2} = 1.664$	$\nu_2 = 35.8$

$$\phi = 1/f = .002/mm$$

$$\phi_1/\phi = \frac{\nu_1}{\nu_1 - \nu_2} \qquad \phi_2/\phi = \frac{-\nu_2}{\nu_1 - \nu_2}$$

$$\nu_1 - \nu_2 = 27.7$$

$$\phi_1/\phi = \frac{63.5}{27.7} \qquad \phi_2/\phi = \frac{-35.8}{27.7}$$

$$\phi_1 = .00458/mm \qquad \phi_2 = -.00258/mm$$

Note $\phi_1 + \phi_2 = \phi = .002/mm$

$$f_1 = 218.1\ mm \qquad f_2 = -386.9\ mm$$

5) (15 points) A 5X Keplerian telescope has a 200 mm focal length objective.

a) Determine the focal length of the eye lens and the overall length of the telescope.

$m = -\frac{1}{MP} = -\frac{1}{5}$ $f_2 = f_1/5 = 40\ mm$

$m = -f_2/f_1$ $L = f_1 + f_2 = 240\ mm$

b) If the stop of the telescope is at the objective, what is the eye relief?

XP is the image of the stop by the eye lens.

$z = -L$ $\frac{1}{z'} = \frac{1}{z} + \frac{1}{f_2}$ $z' = 48\ mm$

Eye Relief $= 48\ mm$

c) The objective lens has a diameter of 40 mm. What is the required eye lens diameter for the telescope to have an unvignetted field of view of +/- 2 degrees?

Need y_2 and $\bar{y}_2$ at lens 2. $\phi_1 = .005/mm$

Chief: $\bar{u} = \tan(2°) = .0349$

$\bar{y}_1 = 0$

$\bar{y}_2 = \bar{u}L = 8.38\ mm$

Marginal: $y_1 = 20$ or $y_2 = m y_1$

$u' = -y_1\phi_1 = -0.1$ $y_2 = \frac{20}{-5} = -4\ mm$

$y_2 = y_1 + u'L = 20 - 24$

$y_2 = -4\ mm$

Vignetting $a_2 > |y| + |\bar{y}|$

$a_2 > 12.38\ mm$

$D_2 > 24.76\ mm$

6) (15 points) Convert the 5X Keplerian telescope from the previous problem (a 200 mm focal length objective) into a doubly telecentric system.

a) Determine the stop location and a pair of conjugate planes.

The stop is at the common focal point.

One set: F_1 of L1 ($z = -200$) and F_2' of L2 ($z' = 40$)

Alternate: L1 and the XP

z relative to L1

z' relative to L2

b) Two objects are located 400 mm and 100 mm to the left of the objective lens. Where are the respective image planes (located relative to the second lens element)?

$\bar{m} = m^2 = 1/25$

Use $\bar{m}$ with the distances measured relative to a known set of conjugates.

Using F_1 and F_2':

<u>Object at 400 mm:</u>

$\Delta z = -200$ mm $\qquad \Delta z' = \bar{m}\,\Delta z = -8$ mm

or 8 mm inside of F_2'

$z' = 32$ mm to the right of L2

<u>Object at 100 mm:</u>

$\Delta z = 100$ mm $\qquad \Delta z' = \bar{m}\,\Delta z = 4$ mm

or 4 mm outside of F_2'

$z' = 44$ mm to the right of L2

Note that for the 2 objects: $\Delta z = 300$ mm

$\Delta z' = 12$ mm $= 300/25$

7) (15 points) An optical system has astigmatism ($W_{222} = 2\ \mu m$). The maximum image height is 10 mm and the system has a numerical aperture of NA = 0.1. What is the radius of curvature of the tangential image surface?

$NA = 0.1 \quad \Rightarrow \quad f/5$

At tangential focus, $\varepsilon_y = 0$

$$\varepsilon_y = -2\frac{R}{r_p} W_{222} H^2 y_p - 2\frac{R}{r_p}\Delta W_{20} y_p = 0$$

$$\Delta W_{20} = -W_{222} H^2$$

$$\delta z = 8(f/\#)^2 \Delta W_{20} = -8(f/\#)^2 W_{222} H^2$$

Surface sag: $\quad sag = \delta z = \dfrac{h^2}{2R}$

at $H = 1 \quad h = 10\ mm$

$$\frac{h^2}{2R} = -8(f/\#)^2 W_{222} H^2$$

$H = 1$
$h = 10\ mm$
$f/\# = 5$
$W_{222} = 2\ \mu m$

$\underline{R = -125\ mm}$

Inward bending surface.