Name Solutions

Closed book; closed notes. Equation sheets are attached and can be removed.
Use the back sides if required. The time limit is 2 hours.
Calculators are permitted, but do not use any pre-stored information or programs.
Note any assumptions you make in solving the problems.
Show your work. Present it in a neat and logical fashion.

Distance Students: Please return the original exam only; do not FAX an additional copy.

1) (10 points) Name and briefly describe the three classifications of illumination systems.

Diffuse – no source imaging; all angles of illumination.

Specular – source imaged into the pupil of the projection lens.

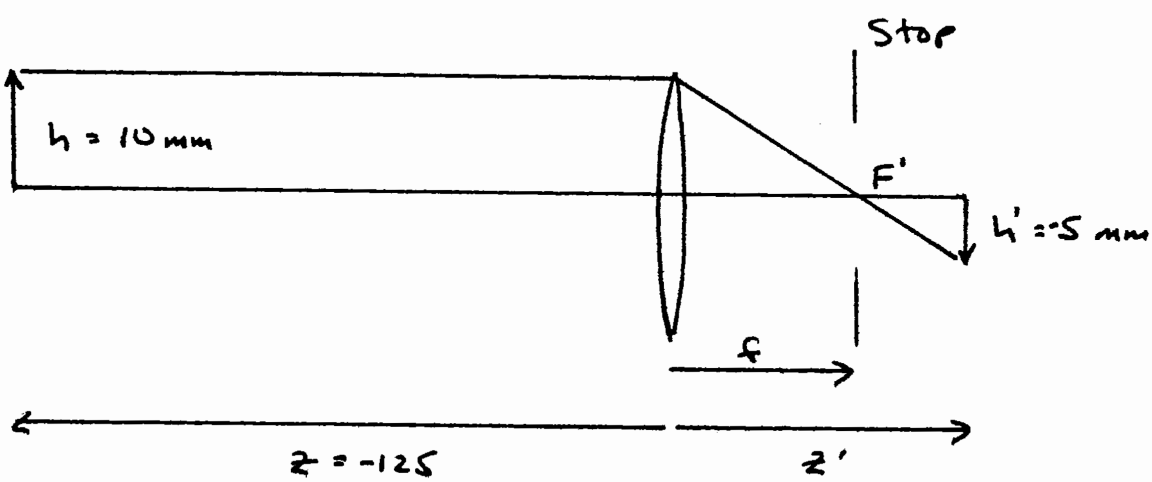
Critical – source imaged onto the object.

2) (20 points) A camera objective is telecentric in object space and designed to image an object of size ±10 mm onto a detector of size ±5 mm. The working distance (from the object to the objective) is 125 mm. The system must be unvignetted over this field of view and have an image-space working F-number of f/4.

Model the objective as a single thin lens in air.

Determine the system layout including the position and diameter of the thin lens and the position and diameter of the system stop.

The stop is at the rear focal point of the objective:



$m = \frac{h'}{h} = \frac{z'}{z} = -\frac{1}{2}$ $\qquad \underline{z' = 62.5 \text{ mm}}$

$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$ $\qquad \underline{f = 41.67 \text{ mm}}$

$f/\#_w = \frac{1}{2NA} \approx \frac{1}{2|u'|} = 4$

$u' = -\frac{r_{STOP}}{z'-f}$

$f/\#_w = 4 = \frac{z'-f}{D_{STOP}}$ $\qquad \underline{D_{STOP} = 5.21 \text{ mm}}$

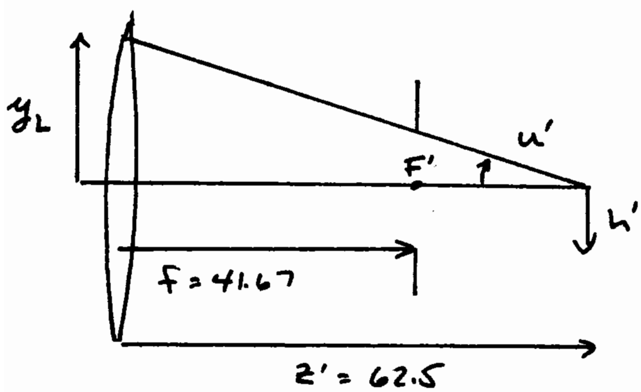
Note: In this telecentric system, the EP is at ∞ and is of ∞ diameter.

The relationship:
$f/\#_w = (1-m)\, f/\#$
is not valid. The approximations used – thin lens; stop at the lens – are not valid.

The defining relationship for $f/\#_w$ must be used:

$f/\#_w = \frac{1}{2NA} \approx \frac{1}{2|u'|}$

To determine the required lens diameter, extend the marginal ray back to the lens:



$$u' = -\frac{D_{STOP}/2}{z' - f} = -.125$$

or

$$u' = -1/2\, f/\#_W = -.125$$

$$y_L = -z' u'$$

$$y_L = 7.81 \text{ mm}$$

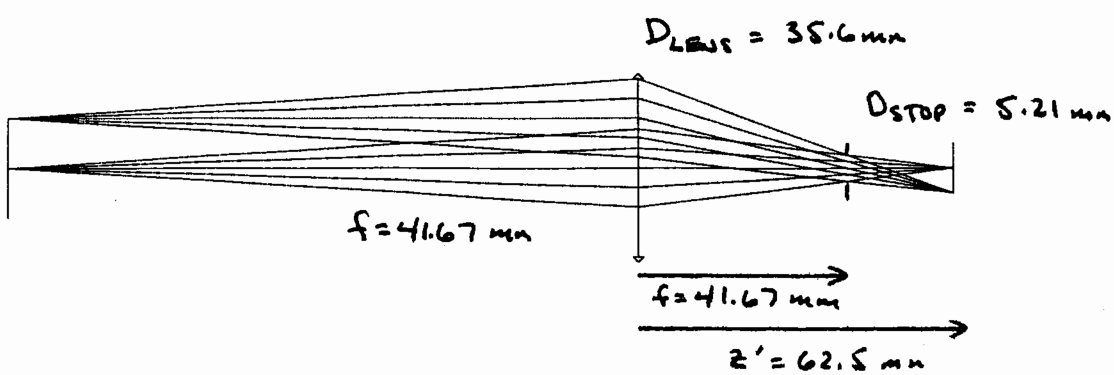
Since the system is telecentric in object space

$$\bar{y}_L = h = 10 \text{ mm}$$

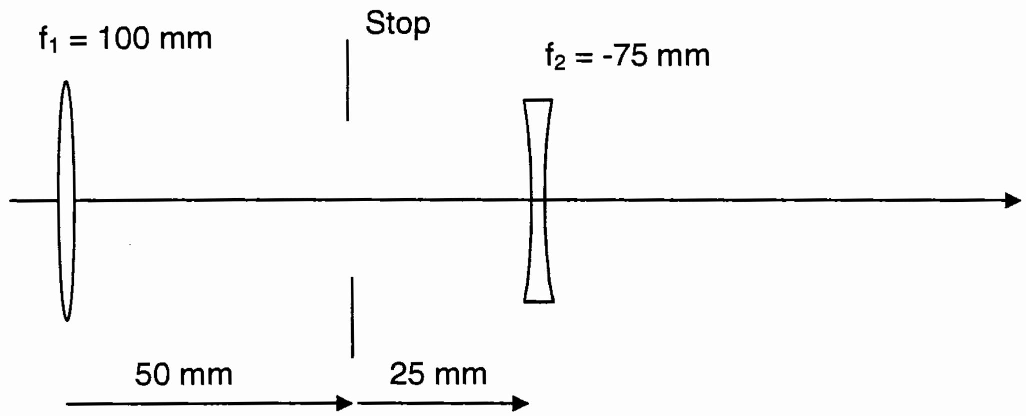
For no vignetting

$$a_L = |y_L| + |\bar{y}_L| = 17.8 \text{ mm}$$

$$\underline{D_{LENS} = 35.6 \text{ mm}}$$



3) (20 points) The following thin-lens telephoto lens places the system stop between the two elements.



The system F-number is f/5. Use *Gaussian methods* to determine the Entrance Pupil location and diameter, the Exit Pupil location and diameter, and the Stop diameter.

$\phi_1 = .01/mm$ $\phi_2 = -.01333/mm$

System Focal Length:

$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t$ $t = 75mm$

$\phi = .006667/mm$ $\underline{f = 150\ mm}$

Entrance Pupil Diameter:

$f/\# = 5 = \frac{f}{D_{EP}}$ $\underline{D_{EP} = 30\ mm}$

Entrance Pupil Location:

Image stop through f_1 (Light from R → L)

$z_S = 50$ $n = n' = -1$

$\frac{n'}{z'_{EP}} = \frac{n}{z_S} + \frac{1}{f_1}$ $\frac{-1}{z'_{EP}} = \frac{-1}{50} + \frac{1}{100}$

$\underline{z'_{EP} = 100\ mm}$ to the right of f_1

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Stop Size:

$$m_{EP} = \frac{z'_{EP}/n'}{z_S/n} = \frac{100}{50} = 2 \qquad m_{EP} = \frac{D_{EP}}{D_{STOP}}$$

$$\underline{D_{STOP} = D_{EP}/m_{EP} = 15\ mm}$$

Exit Pupil:

Image stop through f_2

$z_S = -25 \qquad n = n' = 1$

$$\frac{1}{z'_{XP}} = \frac{1}{z_S} + \frac{1}{f_2} \qquad \frac{1}{z'_{XP}} = \frac{1}{-25} + \frac{1}{-75}$$

$\underline{z'_{XP} = -18.75\ mm}$ to the left of f_2

$$m_{XP} = \frac{z'_{XP}}{z_S} = \frac{-18.75}{-25} = .75 \qquad m_{XP} = \frac{D_{XP}}{D_{STOP}}$$

$$\underline{D_{XP} = m_{XP} D_{STOP} = 11.25\ mm}$$

Summary:

$D_{STOP} = 15\ mm$

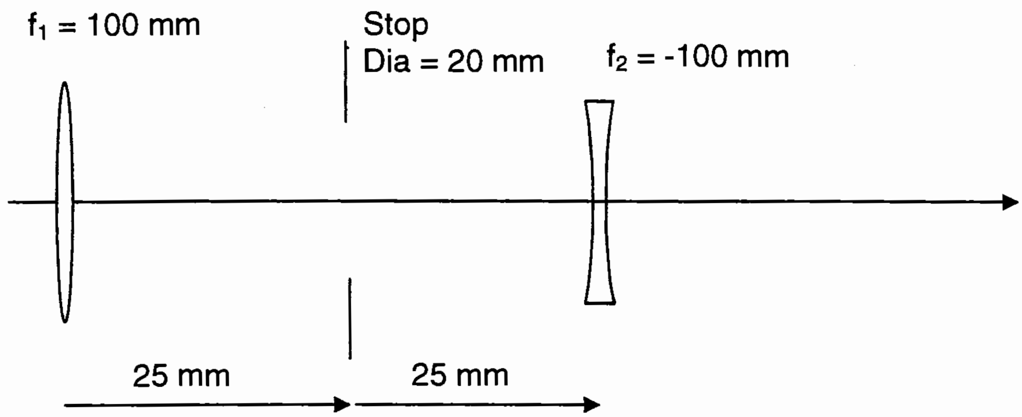
$D_{EP} = 30\ mm$

$z'_{EP} = 100\ mm$ (to the right of f_1)

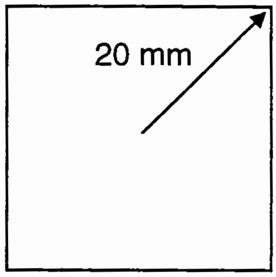
$D_{XP} = 11.25\ mm$

$z'_{XP} = -18.75\ mm$ (to the left of f_2)

4) (20 points) A different thin-lens telephoto lens also places the system stop between the two elements. The object is at infinity.

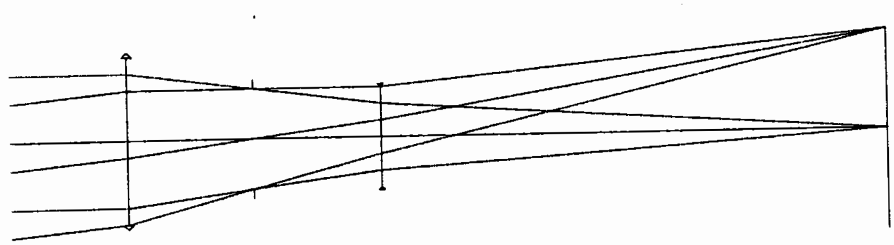


This lens is used with a sensor that has a corner-to-corner dimension of 40 mm. What are the required element diameters for this system to be unvignetted over this field of view?



Use paraxial raytrace methods.

Ray Layout:



raytrace form on next page…

Stop Radius = 10 mm
Image Height = 20 mm

Surface	0	1 (f_1)	2 (Stop)	3 (f_2)	4 (Image)	5	6
f		100	–	−100	–		
$-\phi$		−.01	–	.01			
t	∞	25	25	100			
Potential Marginal Ray							
y	1	1	.75	.50	0		
u	0	−.01	−.01	−.005			
Marginal Ray							
y	13.33	13.33	10	6.67	0		
u	0	−.1333	−.1333	−.06667			
Potential Chief Ray							
$\bar{y}$		−2.5	0	2.5	15		
$\bar{u}$	.075	.1	.1	.125			
$\bar{y}$		−3.333	0	3.333	20		
$\bar{u}$	.10	.1333	.1333	.1667			

Procedure:
1) Trace a potential marginal ray
2) Scale to stop radius (Scale factor = 10/.75 = 13.33)
3) Trace a potential chief ray
4) Scale to full image height (scale factor = 20/15 = 1.333)

At f_1:

$y_1 = 13.33$ mm

$\bar{y} = -3.33$

At f_2:

$y_2 = 6.67$ mm

$\bar{y}_2 = 3.33$

Unvignetted: $a \geq |y| + |\bar{y}|$

$a_1 \geq 16.67$ mm

$D_1 \geq 33.33$ mm

$a_2 \geq 10.0$ mm

$D_2 \geq 20.0$ mm

5) (15 points) A doubly telecentric system has a magnification of m = -3 and a length of 200 mm. The length is defined to be from the first lens element to the last lens element.

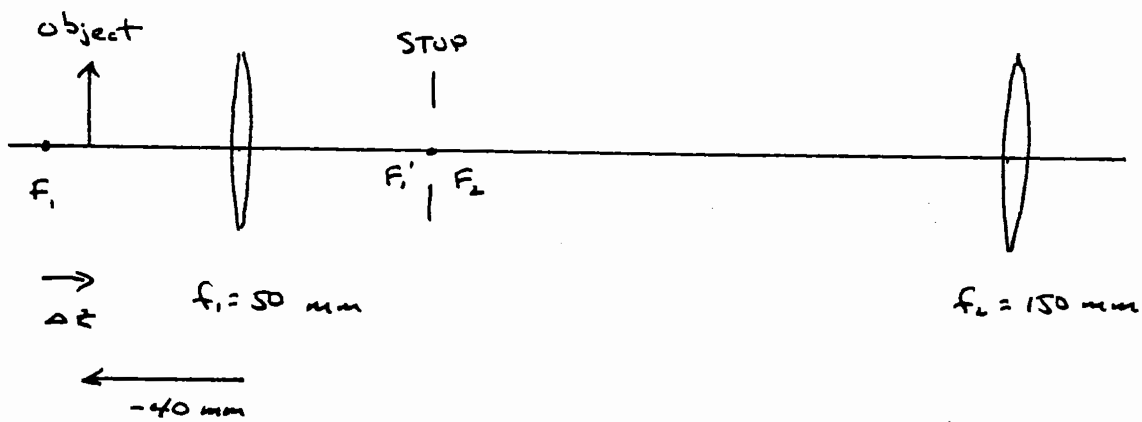
a) Show the system layout with focal lengths and spacings.

Afocal ; Stop at the common focal point.

$m = -f_2/f_1 = -3$ $L = f_1 + f_2 = 200$ mm

$f_2 = 3f_1$ $4f_1 = 200$ mm

$f_1 = 50$ mm $f_2 = 150$ mm



b) An object is located 40 mm to the left of the first element. Where is the image relative to the final element?

The reference conjugate locations are F_1 and F_2'

$\Delta z = 10$ mm (from F_1)

$\bar{m} = m^2 = 9$

$\Delta z' = \bar{m}\, \Delta z = 90$ mm (from F_2')

Image Location = 90 mm + f_2 = 240 mm (from lens 2)

6) (15 points) A thin lens has a power ϕ in air. The glass has an index of refraction n_d with and Abbe number of ν_1. The lens is immersed into a fluid with the same index of refraction n_d. The fluid however has a different Abbe number ν_2.

When the lens is in the fluid, what is the longitudinal chromatic aberration of the power?

$$\delta\phi = \phi_F - \phi_C$$

$$\phi = (n_d - 1)(C_1 - C_2) = (n_d - 1)\Delta C$$

Glass: n_{F1} n_{C1} n_d

$$\nu_1 = \frac{n_d - 1}{n_{F1} - n_{C1}}$$

Fluid: n_{F2} n_{C2} n_d

$$\nu_2 = \frac{n_d - 1}{n_{F2} - n_{C2}}$$

$$\phi_F = (n_{F1} - n_{F2})C_1 + (n_{F2} - n_{F1})C_2$$

$$\phi_F = (n_{F1} - n_{F2})(C_1 - C_2)$$

$$\phi_F = (n_{F1} - n_{F2})\Delta C$$

$$\phi_C = (n_{C1} - n_{C2})\Delta C$$

$$\delta\phi = \phi_F - \phi_C = (n_{F1} - n_{F2})\Delta C - (n_{C1} - n_{C2})\Delta C$$

$$\delta\phi = \left[(n_{F1} - n_{C1})\Delta C - (n_{F2} - n_{C2})\Delta C\right]\frac{n_d - 1}{n_d - 1}$$

$$\delta\phi = \frac{n_{F1} - n_{C1}}{n_d - 1}(n_d - 1)\Delta C - \frac{n_{F2} - n_{C2}}{n_d - 1}(n_d - 1)\Delta C$$

$$\delta\phi = \frac{\phi}{\nu_1} - \frac{\phi}{\nu_2}$$

$$\delta\phi = \frac{\nu_2 - \nu_1}{\nu_1 \nu_2}\phi$$