

Name Solutions

Closed book; closed notes. Equation sheets are attached and can be removed.
 Use the back sides if required. The time limit is 2 hours.
 Calculators are permitted, but do not use any pre-stored information or programs.
 Note any assumptions you make in solving the problems.
 Show your work. Present it in a neat and logical fashion.

Distance Students: Please return the original exam only; do not FAX an additional copy.

1) (20 points) Design a slide projector using a projection-condenser (specular) illumination system for the following conditions:

Slide:	25 mm x 25 mm
Screen:	2 m x 2 m
Projection Lens to Screen Distance:	4 m
Filament:	8 mm diameter (circular)
Condenser Lens:	f/0.8

To simplify the problem, assume that the condenser lens is a thin lens in contact with the slide. No source mirror is to be used in this system.

Provide the element locations, focal lengths and diameters. Sketch your system.

$$\text{Slide Diagonal} = D_s = 35.4 \text{ mm}$$

$$\text{Filament Diameter} = D_f = 8 \text{ mm}$$

$$\text{Projection Magnification} = m_p = h_i/h_o = - \frac{2 \text{ m}}{25 \text{ mm}} = -80$$

Projection Distances:

$$m_p = z'_p / z_p$$

$$z'_p = 4 \text{ m} = 4000 \text{ mm}$$

$$z_p = -50 \text{ mm}$$

Projection Focal Length:

$$\frac{1}{z'_p} = \frac{1}{z_p} + \frac{1}{f_p}$$

$$f_p = 49.4 \text{ mm}$$

Condenser: f/0.8

$$\text{Diameter} = D_c = D_s = 35.4 \text{ mm}$$

$$f/\# = f_c/D_c = 0.8$$

$$f_c = 28.32 \text{ mm}$$

Blank page follows...

Filament Imaging: $z_c' = -z_p = 50 \text{ mm}$

$$\frac{1}{z_c'} = \frac{1}{z_c} + \frac{1}{f_c} \quad z_c = -65.3 \text{ mm}$$

$$m_c = z_c'/z_c = -.77$$

Filament Image

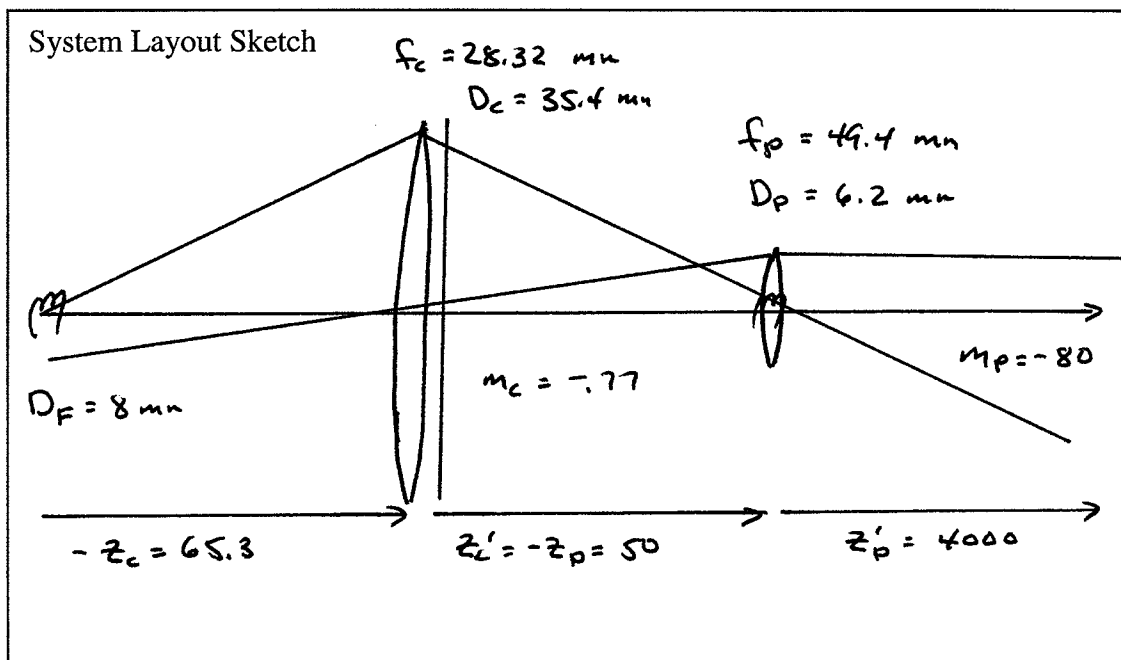
$$D_F = 8 \text{ mm}$$

$$D_F' = |m_c| D_F = |m_c| (8 \text{ mm}) = 6.2 \text{ mm}$$

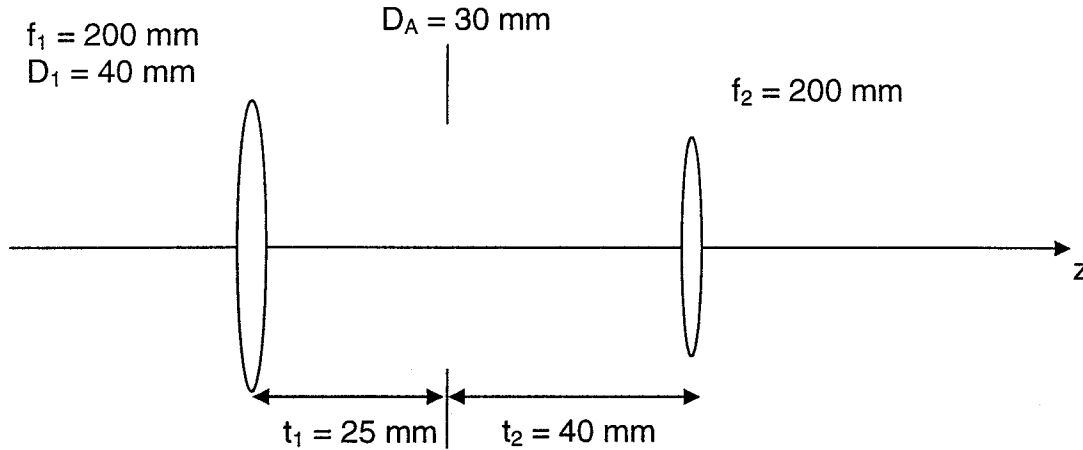
Projection Lens Diameter:

$$D_P = D_F' = 6.2 \text{ mm}$$

$$f/\#_P = f_P/D_P = 49.4 \text{ mm}/6.2 \text{ mm} \quad f/8.0$$



2) (20 points) The following imaging system consists of two thin lenses and an aperture. The spacings and the focal lengths of the lenses are specified. The diameter of the first lens and the aperture are also given.



- a) For an object at infinity, use a paraxial raytrace to determine if the first lens or the aperture serves as the System Aperture Stop.
- b) Determine the required diameter of the second lens so that the system is unvignetted for an object FOV of $\pm 5^\circ$.

a) Trace a potential Marginal Ray and determine the ray height at the two potential stops:

$$\tilde{y}_1 = 10 \quad a_1 = 20 \quad a_1 / \tilde{y}_1 = 2.0$$

$$\hat{y}_A = 8.75 \quad a_A = 15 \quad a_A / \hat{y}_A = 1.714$$

The ray is fractionally closer to the aperture.

The aperture is the system stop.

Scale the potential Marginal ray by 1.714 to obtain the Marginal Ray

Not required:

$$\text{BFD} = 80.6 \text{ mm}$$

$$\phi = .008375 / \text{mm}$$

$$f = 119.4 \text{ mm}$$

Raytrace sheet follows...

Surface	0	Lens 1 1	Aper 2	Lens 2 3	F' 4	5	6
f		200	-	200			
$-\phi$		-0.005	-	-0.005			
t		∞	25	40	?		
Potential Marginal Ray					80.6		
\tilde{y}		10*	10	8.75	6.75	0	
\tilde{u}		0	-0.05	-0.05	-0.08375		
Marginal Ray - Scale by 1.714							
y		17.14	17.14	15.0	11.57	0	
u		0	-0.0857	-0.0857	-0.143		
Potential Chief Ray							
\tilde{y}			-2.5	0	.40	1.045	
\tilde{u}			.00875	0.01*	0.01*	.008	
Chief Ray - Scale by 10.0							
y			-2.5	0	4.0	10.45	
u			.0875	0.1	0.1	0.08	
y							
u							

b) Trace a potential Chief Ray starting at the Stop.

The object space angle of this ray $\tilde{u} = 0.00875$

FOV requirement = $\pm 5^\circ$ $\bar{u} = \tan(5^\circ) = .0875$

Scale the ray by $\bar{u}/\tilde{u} = 10.0$ to get the chief ray

No vignetting at Lens 2:

$$a_2 \geq |y_1| + |y_2|$$

$$a_2 \geq 15.57 \text{ mm}$$

$$D_2 \geq \underline{31.14 \text{ mm}}$$

$$y_2 = 11.57 \text{ mm}$$

$$\bar{y}_2 = 4.00 \text{ mm}$$

Note also that Lens 1
is larger than required
 $D_1 \geq 39.28 \text{ mm}$

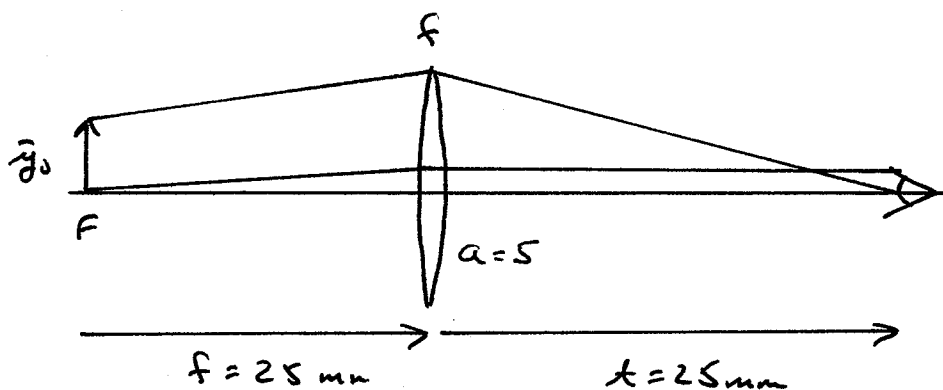
Not required: the image height is 10.45 mm

3) (20 points) A magnifier that is marked 10X is used to examine an object. The magnifier lens has a diameter of 10 mm, and the magnifier is used with a relaxed eye. This implies that the eye is focused at infinity, and that the virtual image produced by the magnifier is also at infinity.

a) The magnifier is first used as an eye loupe. The separation between the magnifier lens and the eye is 25 mm. What is the diameter of the half-vignetted field of view (object size in mm) seen through the magnifier? Assume that the eye has a pupil diameter of 4 mm.

$$MP = 10X = \frac{250 \text{ mm}}{f} \quad f = 25 \text{ mm}$$

- The object must be at the front focal point of the magnifier to produce an image at infinity
- The marginal ray is collimated between the lens and the eye.
- The system stop is at the eye since the eye pupil is smaller than the lens diameter
- For half-vignetted: $a = |y|$
and only the chief ray matters.



Continues...

	Obj	Mag	Stop	Eye
Surface	0	1	2	3
f		25		
$-\phi$		-.04		
t	25	25		
Chief Ray \bar{y}	5	5	0	
\bar{u}	0	-0.200		
y				
u				

$\bar{y} = 5$ at the Magnifier Lens - Half vignetting

$\bar{y}_0 = 5$

FOV = ± 5 mm = 10 mm

b) The magnifier is now used as a magnifying glass by increasing the separation between the magnifier lens and the eye to 250 mm. What is the diameter of the half-vignetted field of view (object size in mm) seen through the magnifier? Once again, assume that the eye has a pupil diameter of 4 mm.

All of the conditions are the same except $t = 250$ mm:

	Obj	Mag	Stop	Eye
Surface	0	1	2	3
f		25		
$-\phi$		-.04		
t	25	250		
Chief Ray \bar{y}	0.5	5	0	
\bar{u}	.18	-0.02		
y				
u				

$\bar{y} = 5$ at the magnifier lens - Half vignetting

Continues...

$$\bar{y}_0 = 0.5 \text{ mm}$$

$$\text{FOV} = \pm 0.5 \text{ mm} = 1 \text{ mm}$$

The FOV is greatly reduced from the situation of the eye loupe.

c) What do these results imply about the way a magnifier is best used?

- This is a common problem and easy to observe.
- When a magnifier (as a magnifying glass) is pulled away from the object to place it at F , the FOV becomes very small. A small portion of the object fill the entire aperture.
- As a result a magnifying glass must be used with the object well inside F . A finite conjugate virtual image results, a usable FOV results, but the eye must now accommodate (not relaxed)
- This situation also limits the power of a magnifying glass
- A magnifier works better when placed close to the eye.

4) (20 points) A biconvex thick lens in air has the following prescription and is used with a distant object:

$$\begin{aligned} R_1 &= 250 \text{ mm} \\ R_2 &= -500 \text{ mm} \\ t &= 50 \text{ mm} \end{aligned}$$

$$\begin{aligned} n_F &= 1.828 \\ n_d &= 1.805 \\ n_C &= 1.796 \end{aligned}$$

Use Gaussian reduction to determine the longitudinal chromatic focus shift of the lens.

$$\phi_i = (n' - n) / R_i$$

$$\phi = \frac{1}{f} = \phi_1 + \phi_2 - \phi_2 \tau$$

$$\tau = t/n$$

$$d' = -\frac{\phi_1}{\phi} \tau$$

$$BFD = f + d'$$

	<u>F</u>	<u>d</u>	<u>C</u>
ϕ_1	.00312	.00322	.003184
ϕ_2	.001656	.00161	.001592
τ	27.352	27.701	27.840
ϕ	.004818	.004686	.004635
f	207.56	213.38	215.75
d'	-18.80	-19.03	-19.12
BFD	188.76 mm	194.35 mm	196.63 mm

$$\text{Focus Shift} = BFD_C - BFD_F = \underline{7.87 \text{ mm}}$$

The longitudinal chromatic aberration does not include the principal plane shifts of this thick lens.

5) (10 points) A doubly telecentric system is constructed out of two thin lenses. The spacing between the lenses is 250 mm, and the magnitude of the magnification $|m|$ is $1/4$.

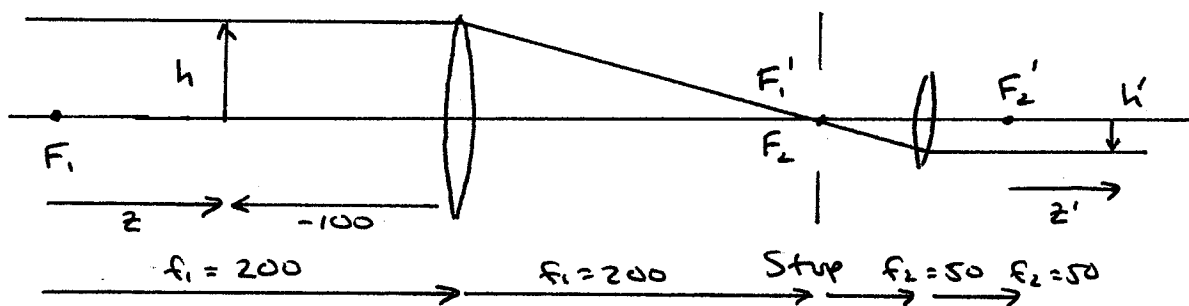
a) Sketch the layout of the system. Show the required spacings and focal lengths.

A focal system (2 positive lenses) with the stop at the common focal point.

$$m = -1/4 = -f_2/f_1$$

$$t = f_1 + f_2 = 250 \text{ mm}$$

$$f_1 = 4f_2 = 200 \text{ mm} \quad f_2 = 50 \text{ mm}$$



b) A 12 mm high object is located 100 mm to the left of the first lens of this system. Determine the location and size of the image. Do not use a raytrace for this problem.

$$m = -1/4$$

$$h = 12 \text{ mm}$$

$$\bar{m} = m^2 = 1/16$$

$$h' = mh = -3 \text{ mm}$$

Use the longitudinal magnification with F_1 and F_2' as the reference points.

$$z = 100 \text{ mm}$$

$$z' = \bar{m}z = 6.25 \text{ mm}$$

The image is located $f_2 + z'$ from the second lens

$$s' = 56.25 \text{ mm}$$

6) (10 points) Provide relationships for each of the following. The relationships should be in terms of focal lengths, object/image distances, etc.

Magnification of a focal imaging system:

$$m = h'/h = \frac{z'/n'}{z/n} \quad (\text{Gaussian Distances})$$

Magnification of an afocal imaging system:

$$m = -\frac{f_2}{f_1}$$

Magnifying power of a telescope:

$$MP = \frac{1}{m} = -\frac{f_1}{f_2}$$

Magnifying power of a magnifier:

$$MP = \frac{250\text{mm}}{f}$$

Visual magnification of a microscope:

$$m_V = m_{\text{obj}} MP_{\text{EYE}} = \frac{z_0'}{z_0} \frac{250}{f_{\text{EYE}}}$$

(z_0, z_0' Gaussian Distances)