Name Solutions

Closed book; closed notes. Equation sheets are attached and can be removed.
Spare raytrace forms are also attached.
Use the back sides if required. The time limit is 2 hours.
Calculators are permitted, but do not use any pre-stored information or programs.
If a method of solution is specified in the problem, it must be used.
Note any assumptions you make in solving the problems.
Show your work. Present it in a neat and logical fashion.

Distance Students: Please return the original exam only; do not FAX an additional copy.

1) (10 points) A 100 mm focal length thin lens (in air) in made out of glass KzFSN5. The glass code for this glass is 654396. What is the longitudinal chromatic aberration of this lens?

$654396 \longrightarrow n_d = 1.654$

$\nu = 39.6$

$$\frac{\delta f}{f} = \frac{1}{\nu}$$

$$\delta f = \frac{f}{\nu} = \frac{100\text{mm}}{39.6}$$

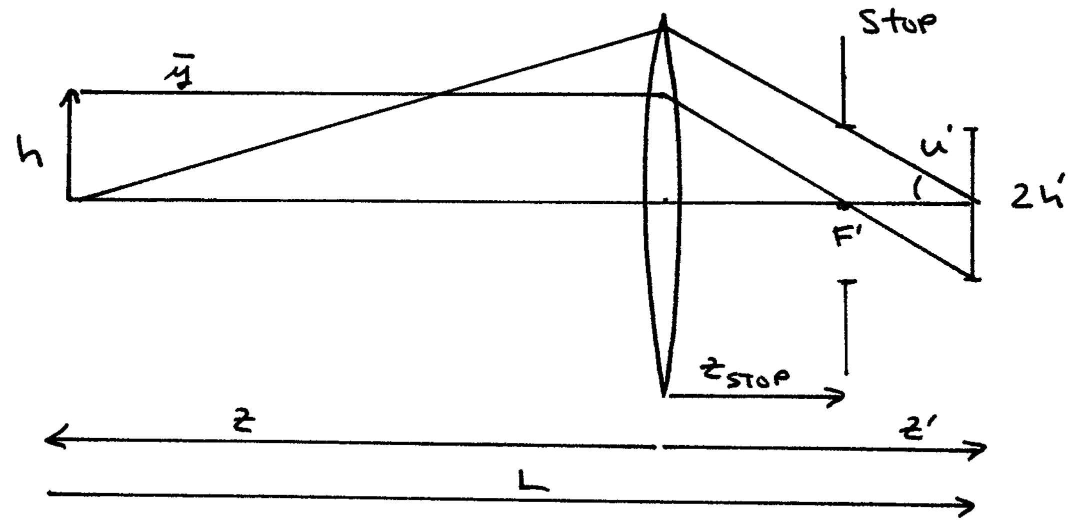
$$\delta f = 2.53\text{mm}$$

Longitudinal Chromatic Aberration = 2.53 mm

2) (25 points) Design an object-space telecentric imaging system consisting of a thin lens and a stop. The system has an object-to-image distance of 450 mm, and it images a 20 mm diameter object onto a 10 mm square detector. The diameter of the image fills the width of the detector. The system operates at a working f-number of 4 ($f/\#_W = 4$ or NA = 0.125) and is unvignetted for this object.

Provide the focal length and diameter of the lens, the stop diameter, and the required spacings.

Stop at F' of the Lens.



$$L = z' - z = 450 \text{ mm}$$

$$m = -\frac{h'}{h} = -\frac{2h'}{2h} = -\frac{10 \text{ mm}}{20 \text{ mm}} = -\frac{1}{2}$$

$$m = z'/z \qquad z = -2z'$$

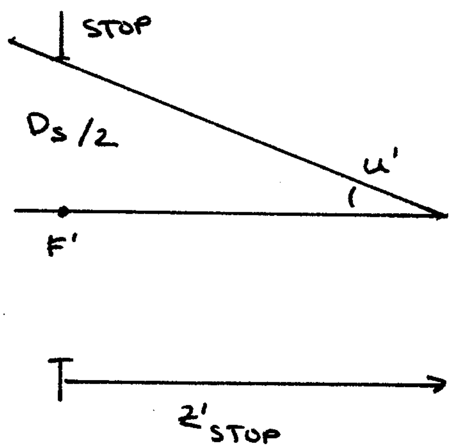
$$z = -300 \text{ mm} \qquad z' = 150 \text{ mm}$$

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \qquad\qquad z_{STOP} = f = 100\text{mm}$$

$$\underline{f = 100\text{mm}}$$

Continues...

D_S = Stop Diameter



$z'_{STOP} = z_{STOP} - z' = 50\,mm$

$$u' = -\frac{D_s/2}{z'_{STOP}}$$

$NA = 0.125 = |u'|$

$D_S = 12.5\ mm$

or

$$f/\#_w = 4 = \frac{D_s}{z'_{STOP}} = \frac{D_s}{50\,mm}$$

$D_S = 12.5\ mm$

At the lens:

$\bar{y}$ = object radius $= h = 10\ mm$

$|y| = |z'| \cdot |u'|$ or $y = 3\,(D_s/2)$

$|y| = 18.75\ mm$

$a_L = |y| + |\bar{y}|$ for no vignetting

$a_L = 28.75\ mm$

$D_L = 57.5\ mm$

Focal Length = 100 mm

Lens Diameter = 57.5 mm

Stop Diameter = 12.5 mm

Stop Location: 100 mm to the Right of the Lens

Image Location: 150 mm to the Right of the Lens

3) (30 points) The telephoto ratio is defined as the ratio of the overall length of a lens system (as measured from the first element to the image plane) to the focal length of the system:

$$\text{Telephoto Ratio} = \frac{\text{System Length}}{\text{System Focal Length}}$$

The object is assumed to be distant. In other words, the telephoto ratio describes how much shorter an optical system is when compared to an equivalent single thin lens system.

An f/5.6 lens system is constructed with two thin lenses in air.

The system has a telephoto ratio of 0.75.
The system has a focal length of 200 mm.
The system provides a back focal distance of 70 mm.
The system stop is located halfway between the two thin lenses.
The object is at infinity.
The lens is unvignetted when used with a detector that has a diameter of 30 mm.

Determine the focal lengths and diameters of the two thin lenses, the stop diameter, and the element spacings. Also determine the angular Field of View in Object Space.

$f = 200$ mm

$T.R. = \dfrac{L}{f} = 0.75$

$L = 150$ mm

$L = t + BFD$

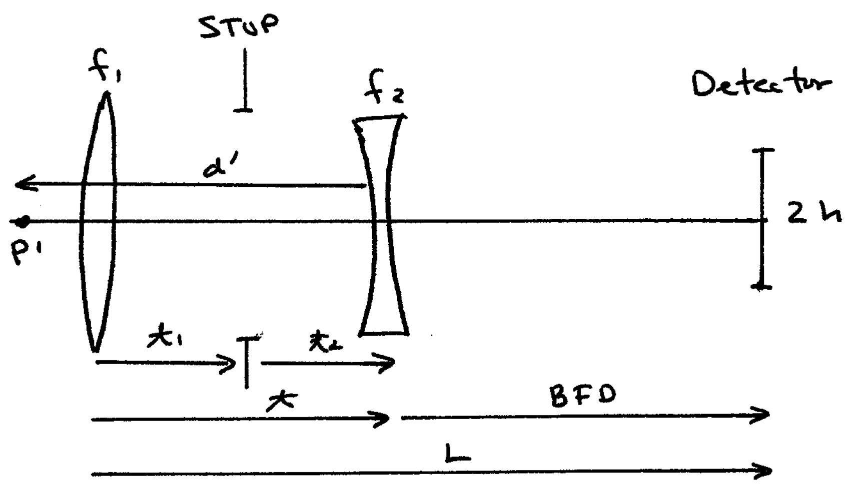
$BFD = 70$ mm

$t = \underline{80 \text{ mm}}$

$t = t_1 + t_2$

$t_1 = t_2$

$t_1 = t_2 = \underline{40 \text{ mm}}$



$BFD = f + d'$

$d' = -130$ mm

Continues...

$$d' = -\frac{\phi_1}{\phi} t \qquad \phi = \frac{1}{f} = .005/mm$$

$$-130mm = -\frac{\phi_1}{.005/mm} \cdot 80mm$$

$\phi_1 = .008125/mm$

$f_1 = 123.1mm$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t$$

$.005/mm = .008125/mm + \phi_2 (.008125/mm) 80mm$

$-.003125/mm = \phi_2 (1 - .65)$

$\phi_2 = -.008928/mm$

$f_2 = -112.0mm$

$f/5.6 \rightarrow f/D_{EP} = 5.6$

$D_{EP} = 35.71mm$

$r_{EP} = 17.857mm \leftarrow$ Marginal Ray Height

- Trace Marginal Ray and Potential Chief Ray
- Scale Chief Ray to image radius of 15mm

		L1	STOP	L2	Image		
Surface	0	1	2	3	4	5	6
f		123.1	—	−112.0			
−φ		−.008125	—	.008928			
t	∞	40	40	70			
Marginal Ray							
y	17.857	17.857	12.053	6.250	0		
u	0	−.1451	−.1451	−.0893			
Potential Chief Ray							
$\tilde{\bar{y}}$		−4	0	4	13.50		
$\tilde{\bar{u}}$	.0675	.1	.1	.1351			
$\bar{y}$		−4.44	0	4.44	15		
$\bar{u}$	.0750	.1111	.111	.1501			

Scale chief ray by $\frac{15}{13.50} = 1.111$

Continues…

STOP Radius = y_{STOP} = 12.053 mm

D_{STOP} = 24.1 mm

For Unvignetted:

$a = |y| + |\bar{y}|$

L1: $y_1 = 17.857$ mm

$\bar{y}_1 = -4.44$ mm

$a_1 = 22.3$ mm

$D_1 = 44.6$ mm

L2: $y_2 = 6.250$ mm

$\bar{y}_2 = 4.44$ mm

$a_2 = 10.7$ mm

$D_2 = 21.4$ mm

Object Space FOV:

HFOV = $\tan^{-1}(\bar{u}) = \tan^{-1}(.0750)$

HFOV = +/- 4.29°

Lens 1: Focal Length = 123.1 mm

Diameter = 44.6 mm

Lens 2: Focal Length = -112.0 mm

Diameter = 21.4 mm

Separation between Lens 1 and Lens 2 = 80 mm

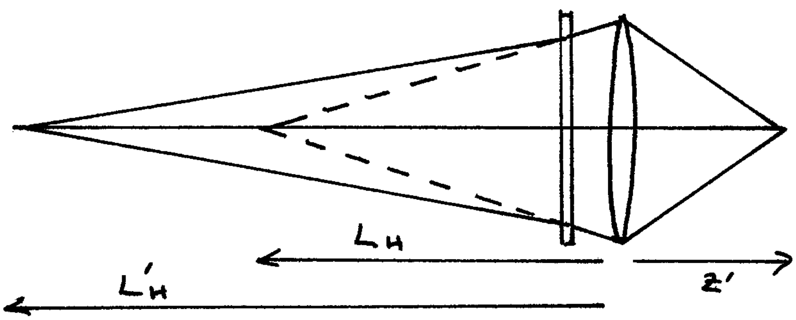
Stop Diameter = 24.1 mm

Angular FOV = +/- 4.29 degrees

4) (15 points) As is common, a fixed focus camera has a hyperfocal distance of 8.0 feet, providing in focus images for objects from 4 feet to infinity. The camera is used in air.

To use this same camera underwater, a water-proof box is provided with a thin, flat window. The camera lens is in close proximity to the window. No adjustments to the camera are made.

When the camera is used underwater, what is the hyperfocal distance and range of in-focus objects? The index of refraction of water is 1.33.



An object in the water will appear closer than it really is.

Air: $\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$ At Hyperfocal: $\frac{1}{z'} = \frac{1}{L_H} + \frac{1}{f}$

$L_H = -8$ feet $\qquad \frac{1}{L_H} = \frac{1}{z'} - \frac{1}{f} = \frac{1}{-8 \text{ feet}}$

Water: z' and f do not change:

$\frac{1}{z'} = \frac{n}{L_H'} + \frac{1}{f} \qquad \frac{n}{L_H'} = \frac{1}{z'} - \frac{1}{f} = \frac{1}{-8 \text{ feet}}$

$L_H' = nL_H = -10.6$ feet

$L_N' = L_H'/2 = -5.3$ feet

$L_F' = L_F = \infty$

or Use Reduced Thicknesses:

$L_H'/n = L_H \qquad L_H' = -10.6$ feet

Hyperfocal Distance = 10.6 feet

Range of In-Focus Objects: 5.3 feet to ∞ feet

5) (20 points) An earth-resource satellite will contain an refractive imaging system that images a 100 kilometer wide swath of the earth onto a 100 mm wide detector array. The satellite orbits at a height of 700 kilometers.

The lens system is to be tested on earth and in air. How much should the detector array be shifted (refocused) prior to launch to compensate for the fact that the optical system will operate in vacuum? Which direction is the detector shifted?

Model the optical system as a thin lens with an index of refraction of 1.5000 relative to air. The index of refraction of air is 1.0003.

In Air:

$$z = -700{,}000\ m$$

$$m = \frac{h'}{h} = -\frac{100mm}{100{,}000m} = -\frac{0.1m}{10^5 m}$$

$$m = -10^{-6}$$

$$z'_A \approx f'_R = n_A f_A = \frac{n_A}{\phi_A}$$

$$m = \frac{z'/n_A}{z/n_A} = -10^{-6}$$

$$z'_A = mz = 0.7m = n_A f_A$$

$$z'_A = 700mm$$

$$\phi_A = (n - n_A)(C_1 - C_2) = (n - n_A)\Delta C$$

$$\Delta C = \frac{\phi_A}{(n - n_A)}$$

$$(n - n_A) = .5000$$

$$\Delta C = \frac{\phi_A}{.5000}$$

In Vacuum:

Since $n_V < n_A$, the index difference in vacuum is larger than in air:

$$(n - n_V) = .5003$$

$$\phi_V = (n - n_V)\Delta C$$

$$\Delta C = \frac{\phi_V}{(n - n_V)}$$

$$\Delta C = \frac{\phi_V}{.5003}$$

Continues…

But ΔC does not change

$$\frac{\phi_A}{.5000} = \frac{\phi_V}{.5003} \qquad \phi_V = \frac{.5003}{.5000}\phi_A$$

$$f_V = \frac{.5000}{.5003} f_A$$

Image location in vacuum:

$$z'_V \approx f'_R = n_V f_V = f_V$$

Image Shift

$$\Delta z = z'_V - z'_A = f_V - z'_A \qquad z'_A = n_A f_A = 700\ mm$$

$$\Delta z = f_V - n_A f_A$$

$$\Delta z = \frac{.5000}{.5003} f_A - n_A f_A$$

$$\Delta z = z'_A \left[\frac{.5000}{.5003} \frac{1}{n_A} - 1 \right] \qquad n_A = 1.0003$$

$$\Delta z = -.629\ mm$$

Since Δn is larger in vacuum, the focal length in vacuum is shorter and the detector must be shifted towards the lens.

Detector shift = .629 mm

Shift Direction: Towards the Lens