Name **SOLUTIONS**

Closed book; closed notes. The time limit is 2 hours.
Equation sheets are attached and can be removed.
Spare raytrace forms are also attached.
Use the back sides if required.
Do not use any pre-stored information or programs in your calculator.
Assume thin lenses in air if not specified.
If a method of solution is specified in the problem, it must be used.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy.

1) (10 points) The following list identifies six optical glasses by their six-digit glass code.

Identify which of these glasses is a crown (K) or a flint (F) by circling the appropriate letter.

Crown Glass: Abbe Number > 50-55 *Flint Glass: Abbe Number < 50-55*

552635	K	F	$v = 63.5$
664360	K	F	$v = 36.0$
589613	K	F	$v = 61.3$
548458	K	F	$v = 45.8$
517642	K	F	$v = 64.2$
606437	K	F	$v = 43.7$

Which of these two glasses should be to produce a first-order achromatic doublet with the least excess power?

The least excess power results when the difference in the Abbe numbers of the two glasses is greatest.

Crown Glass: *517642* Flint Glass: *664360*

2) (20 points) During the semester, we have discussed a variety of optical systems, components and concepts. For each of the topics that is listed, provide a brief description and discuss some of the features or properties of this item. (for example, from a design perspective, what does it mean for the performance of the system or the use of the system?). Try to be practical, I am looking for concepts not equations.

Limit your answers to the space provided. Legibility does count – I need to be able to read it!

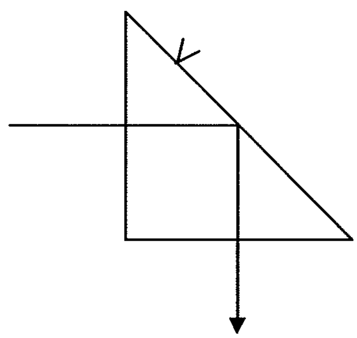
a) <u>Field Lenses</u>

- Placed at or near an intermediate image plane.
- Bend the bundles of light corresponding to off-axis object points back into the optical system to increase the field of view or to reduce vignetting.
- Do not change the system magnification or magnifying power.
- Shift the exit pupil location to reduce the eye relief in visual instruments.
- Since there are located in image planes, a field lens must be of high optical quality and free of scratches and dirt.

b) <u>Reverse Telephoto Lens</u>

- Compound imaging lens or objective consisting of a negative lens or group followed by a positive lens or group.
- The rear principal plane of the system is behind the system. The back focal distance is greater than the focal length.
- Useful in applications to provide additional working distance. Commonly used for systems with short focal lengths.

c) <u>Right angle Roof Prism (Amici)</u>



- Provides a 90 degree beam deviation with no parity change (2 reflections).
- The extra reflection comes from a roof on the hypotenuse of the prism.
- The use of the roof does not increase the path length through the prism.

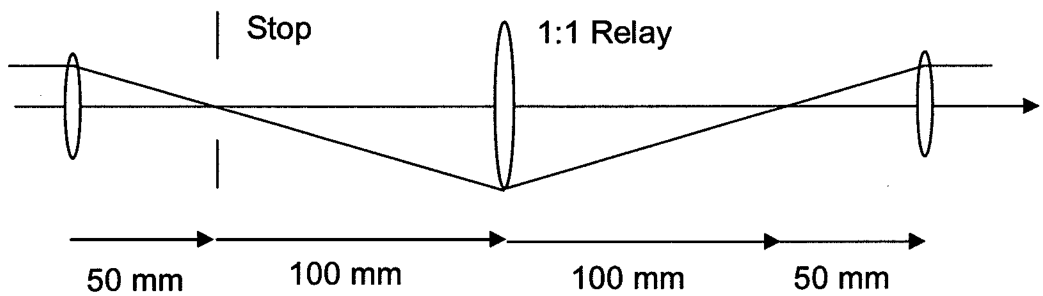
d) <u>Specular Illumination System</u>

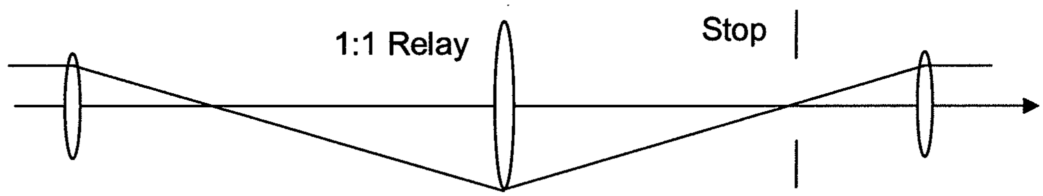
- A condenser lens is used to image the source into the entrance pupil of the projection lens.
- Each point on the source illuminates the entire object, and all of this light is directed into the projection lens.
- Good uniformity.
- Good efficiency.
- Examples are projection-condenser illumination and Koehler illumination.
- The most common form of illumination system.

3) (15 points) Using only 50 mm focal length thin lenses, provide the layout of a double-telecentric system with a lateral magnification of +1.0. You may use up to four of these thin lenses in your design. Provide a sketch of the system clearly indicating the spacings of the lenses and the location of the system stop.

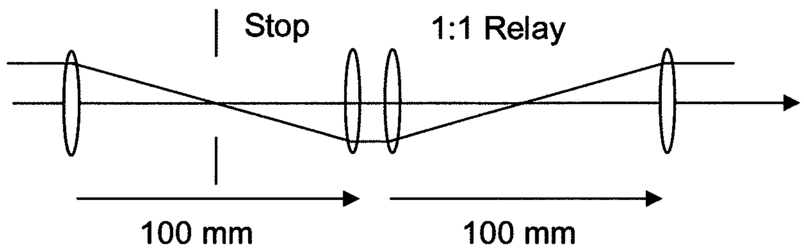
Note: The system magnification must be POSITIVE.
The lens diameters are not required.
There are several possible designs – you need to provide only one.

A double telecentric system must be afocal. An afocal system comprised of two 50 mm lenses separated by 100 mm will have a magnification of -1.0. A 1:1 relay inserted in the system will introduce an additional magnification of -1.0, resulting in a net system magnification of +1.0 using three lenses. The 1:1 relay will be in the 2f-2f configuration, and it adds a length of 200 mm to the system. To make the system telecentric, the stop can be located either at the rear focal point of the first lens or the front focal point of the third lens.





In this configuration, the relay lens must have twice the diameter of the first and third lenses. A shorter configuration with the same size lenses can be constructed by using two of the 50 mm lenses for the relay (for a total of 4 lenses). Once again, the stop can be at either location. Note that the separation between the two relay lenses does not effect the system magnification, stop location or the telecentricity.



4) (20 points) In a 3X Galilean telescope, the separation between the objective lens and the eye lens is 100 mm. The objective lens diameter is 30 mm and the eye lens diameter is 12 mm. The telescope is to be used with an eye that has a 4 mm diameter entrance pupil. The separation between the eye lens and the eye pupil is 15 mm. The object is at infinity.

What is the unvignetted object space Field of View of this system in degrees?
Which element limits the unvignetted Field of View?

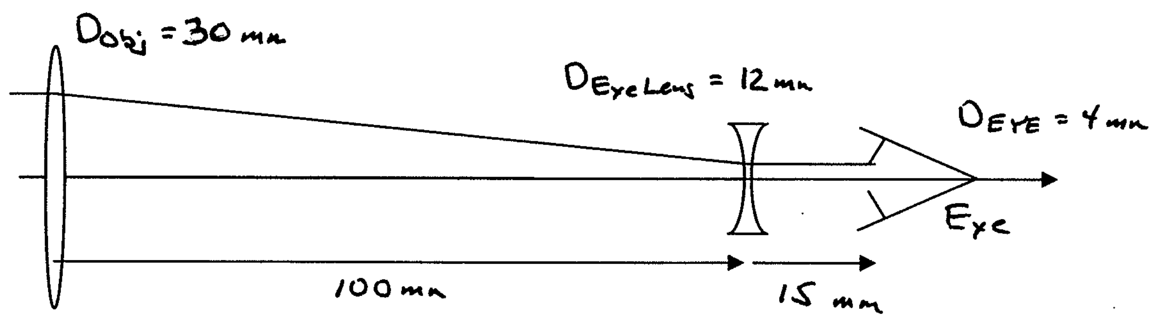
First Design the telescope:

$$MP = 3 = -\frac{f_{obj}}{f_{eye\ lens}} \qquad t = f_{obj} + f_{eyelens} = 100\ mm$$

$$-2 f_{eyelens} = 100\ mm$$

$$f_{obj} = -3 f_{eyelens}$$

$$\underline{f_{eye\ lens} = -50\ mm} \qquad \underline{f_{obj} = 150\ mm}$$



The next step is to determine the system stop (including the eye). Trace a potential marginal ray and scale it to the aperture that has the smallest ratio of aperture size to potential marginal ray height. See raytrace sheet. The eye is the system stop.

This determination can also be done by observation and using the collimated rays for an object at infinity:

Is the eye lens the stop? No $D_{EYE} < D_{Eye\ Lens}$

Is the objective the stop? No $D_{EYE} < D_{Obj}/MP$

Continues...

Since the eye is the stop, the Field of View can be limited by either the Objective Lens or the Eye Lens of the telescope.

Trace a potential chief ray starting at the eye. Evaluate the vignetting condition at each lens using a scaled version of this potential chief ray. The lens which provides the smallest Field of View will limit the Field of View and determine the unvignetted Field of View.

Potential Chief Ray: $\tilde{\bar{y}}_{EYE} = 0$ $\tilde{\bar{u}}_{EYE} = 0.1$ (arbitrary)

Unvignetted Limit: $a = |y| + |\bar{y}| = |y| + \text{Constant}\,|\tilde{\bar{y}}|$

At Objective: $y_{obj} = 6.0\,mm$ $\tilde{\bar{y}}_{obj} = -14.5\,mm$ $a_{obj} = 15\,mm$

$$a_{obj} = 15\,mm = |y_{obj}| + A\,|\tilde{\bar{y}}_{obj}| = 6.0\,mm + A(14.5\,mm)$$

$$\text{Scaling Factor} = A = \underline{0.621}$$

At Eye Lens: $y_{eyelens} = 2.0\,mm$ $\tilde{\bar{y}}_{eyelens} = -1.5\,mm$ $a_{eyelens} = 6.0\,mm$

$$a_{eyelens} = 6.0\,mm = |y_{eyelens}| + B\,|\tilde{\bar{y}}_{eyelens}| = 2.0\,mm + B(1.5\,mm)$$

$$\text{Scaling Factor} = B = \underline{2.66}$$

The smaller scaling factor is for the objective.

The objective lens limits the unvignetted FOV. Scale the potential chief ray by the scaling factor $A = 0.621$

Resulting object space chief ray: $\bar{u} = 0.0207$

$$\theta_{1/2} = \tan^{-1}(\bar{u}) = \underline{1.18\ deg}$$

Unvignetted Field of View = $\pm$ 1.18 degrees in object space

FOV Limited by Objective

Ray Trace Form Follows...

			Obj		Eye Lens		Eye						
Surface	0		1		2		3		4		5		6
f			150		-50								
$-\phi$			-.00667		.02								
t		—		100		15							
Potential Marginal Ray													
$\tilde{y}$	1		1		.333		.333						
$\tilde{u}$		0		-.00667		0							
Marginal Ray - Scale by 2/.333 *													
y	6		6		2		2						
u		0		-.04		0							
Potential Chief Ray													
$\tilde{\bar{y}}$			-14.5		-1.5		0						
$\tilde{\bar{u}}$		.0333		.13		.1							
Chief Ray - Scale by 0.621													
$\bar{y}$			-9.0		-0.931		0						
$\bar{u}$		0.0207		0.0807		0.0621							
y													
u													
y													
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* Marginal Ray Scaling:

The smallest ratio is at the eye - the eye is the system stop - use the ratio at the eye to scale the marginal ray.

Objective: $\dfrac{a_{obj}}{\tilde{y}_{obj}} = \dfrac{25}{1} = 25$

Eye Lens: $\dfrac{a_{eyelens}}{\tilde{y}_{eyelens}} = \dfrac{6}{.333} = 18$

Eye: $\dfrac{a_{eye}}{\tilde{y}_{eye}} = \dfrac{2}{.333} = 6$

5) (15 points) A swimming pool has a sloped bottom (i.e. deeper on one end than the other). You notice that at noon on a sunny day, an interesting light pattern has formed on the bottom of the pool. The pattern is due to imaging of the sun by the waves on the pool surface. Especially sharp line images of the sun occur at a depth of 2 m, and the spacing of the lines is about 300 mm.

Using reasonable, simple assumptions, what can you say about the waves on the surface of the water? In particular, what are the approximate amplitude and period of the waves? State your assumptions.

$n_{water} = 1.33$

Single Refracting Surface

Linear Waves:

Period = Line Spacing = 300 mm

The depth for the sharp lines gives the focal length

$$f_R' = 2000 \text{ mm}$$

$$f = f_E = f_R'/n = 1500 \text{ mm}$$

$$\phi = 1/f = .00067/\text{mm}$$

Wave Curvature:

$$\phi = (n-1)/R \qquad R = 0.5 \text{ m} = 500 \text{ mm}$$

The form of the wave:

$$h(y) = A \cos\left(2\pi y/300\text{mm}\right)$$

The amplitude of the wave must now be determined
– there are several methods

Continues…

1) Differentiate the height of the wave twice to determine its curvature:

$$h'(y) = -A\left(\frac{2\pi}{300\,mm}\right)\sin(2\pi y/300\,mm)$$

$$h''(y) = -A\left(\frac{2\pi}{300\,mm}\right)^2\cos(2\pi y/300\,mm)$$

$$A\left(\frac{2\pi}{300\,mm}\right)^2 = C = \frac{1}{R} = \frac{1}{500\,mm}$$

$A = 4.5\ mm$ P-V Amplitude = $\underline{9.0\ mm}$

2) Expand the cosine and compare to the sag of a surface

$$\cos x \approx 1 - \frac{x^2}{2}$$

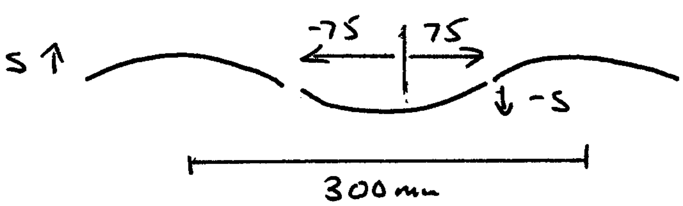
$$h(y) = A\cos\left(\frac{2\pi y}{300\,mm}\right) \approx A\left(1 - \underbrace{\frac{1}{2}\left(\frac{2\pi}{300\,mm}\right)^2 y^2}_{Sag}\right)$$

$$Sag = \frac{y^2}{2R} = A\left(\frac{2\pi}{300\,mm}\right)^2\frac{y^2}{2} \qquad R = 500\ mm$$

$A = 4.5\ mm$ P-V Amplitude = $\underline{9.0\ mm}$

3) Assume each peak or trough of the wave is a convex or concave cylindrical section, and evaluate the sag using the sag equation

$$S = Sag = \frac{y^2}{2R} = \frac{(75\,mm)^2}{2(500\,mm)}$$

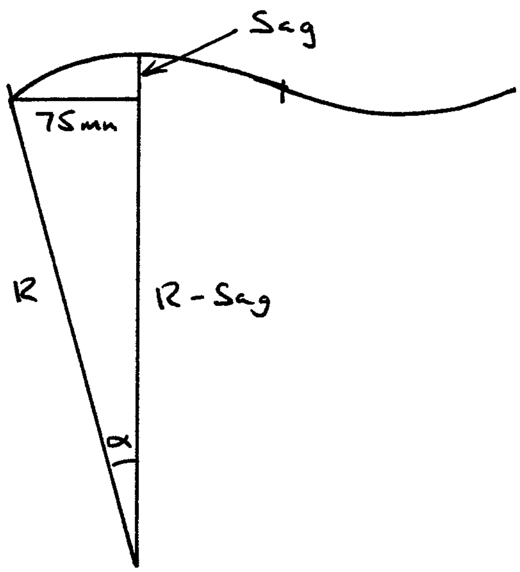


$S = 5.6\ mm$

P-V Amplitude $\approx 11\ mm$

(This method slightly over-estimates the amplitude)

Wave period = $\underline{\ 300\ }$ mm Wave peak-to-valley amplitude ≈ $\underline{\ 9-11\ }$ mm



4) $\alpha \approx \frac{75\,mm}{R} = \frac{75\,mm}{500\,mm}$

$\alpha \approx 0.15$

$\cos\alpha = \frac{R - Sag}{R} = 0.15$

$Sag = 5.6\,mm$

P-V Amplitude $\approx 11\,mm$

5) $(75\,mm)^2 + (R - Sag)^2 = R^2$

$(75\,mm)^2 + \cancel{R^2} - 2R\,Sag + Sag^2 = \cancel{R^2}$

$Sag^2 << 2R\,Sag$ or $(75\,mm)^2$

$Sag = \frac{75\,mm^2}{2R}$

$Sag = 5.6\,mm$

P-V Amplitude $\approx 11\,mm$

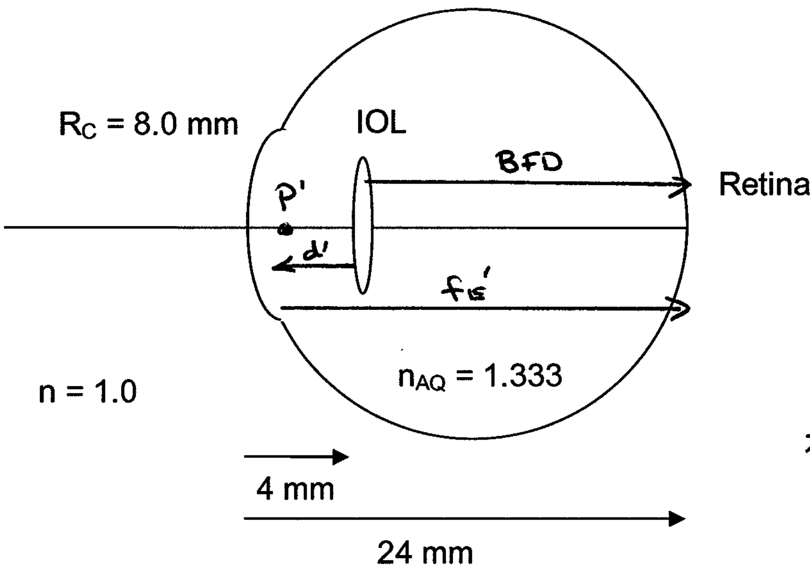
(This actually re derives the sag equation).

Wave period = 300 mm Wave peak-to-valley amplitude ≈ 9 - 11 mm

6) (20 points) In cataract surgery, the natural lens of the eye (now white and opaque) is removed and replaced by an artificial intraocular lens (IOL). The goal of this lens is to work with the cornea to focus images of distant objects on the retina. The cornea can be assumed to be a single refracting surface ($R_C = 8.0$ mm), and the IOL is assumed to be a thin lens. The IOL is usually made of plastic and it is immersed in the aqueous of the eye ($n_{AQ} = 1.333$). The IOL is located 4 mm behind the cornea, and the retina is 24 mm behind the cornea.

Both parts of this problem are to be done using Gaussian methods. No credit will be given for raytrace analysis.

ϕ_E = power of the total system



$t = 4$mm

a) What is the power and focal length of the IOL required to image distant objects onto the retina? This power and focal length are for the thin lens immersed in the aqueous.

$$BFD = 24\text{mm} - 4\text{mm} = 20\text{mm}$$

$$BFD = f_E' + d' = 20\text{ mm}$$

Cornea: $\phi_C = (n_{AQ} - 1)/R_C = 0.046/\text{mm}$

Principal Plane: $\dfrac{d'}{n_{AQ}} = -\dfrac{\phi_C}{\phi_E}\dfrac{t}{n_{AQ}}$

Continues...

Note that both t and d' are in the same index

$$d' = -\frac{\phi_C}{\phi_E} t$$

$$f_E' = n_{AQ} f_E = \frac{n_{AQ}}{\phi_E}$$

$$BFD = f_E' + d' = \frac{n_{AQ}}{\phi_E} - \frac{\phi_C}{\phi_E} t = 20\text{mm}$$

$$n_{AQ} = 1.333$$
$$\phi_C = 0.0416/\text{mm}$$
$$t = 4.0 \text{ mm}$$

$$\frac{1}{\phi_E}\left(n_{AQ} - \phi_C t\right) = 20\text{mm}$$

$$\phi_E = 0.0583/\text{mm}$$

$$f_E = 17.1 \text{ mm}$$

Eye Power:

$$\phi_E = \phi_C + \phi_{IOL} - \phi_C \phi_{IOL} \frac{t}{n_{AQ}} = 0.0583/\text{mm}$$

$$0.0416/\text{mm} + \phi_{IOL}\left(1 - 0.0416/\text{mm}\left(\frac{4\text{mm}}{1.333}\right)\right) = 0.0583/\text{mm}$$

$$\phi_{IOL} = \underline{0.0191/\text{mm}}$$

$$f_{IOL} = \underline{52.4 \text{ mm}}$$

ϕ_{IOL} = 0.0191 /mm f_{IOL} = 52.4 mm

Continues…

b) If the plastic IOL is removed from the aqueous, what are its power and focal length in air? The index of the IOL is 1.5.

$n_{IOL} = 1.5$

For a thin Lens:

$$\phi = (n' - n)\Delta C$$

ΔC does not change with a change of medium

In Aqueous:

$$n' = n_{IOL} \qquad n = n_{AQ}$$

$$\phi_{AQ} = \phi_{IOL} = 0.0191/mm = (n_{IOL} - n_{AQ})\Delta C$$

$$0.0191/mm = (1.5 - 1.333)\Delta C$$

$$\Delta C = 0.114/mm$$

In Air: $n' = n_{IOL} \qquad n = 1.0$

$$\phi_{Air} = (n_{IOL} - 1.0)\Delta C = (1.5 - 1.0)\, 0.114/mm$$

$$\phi_{Air} = 0.057/mm$$

$$f_{Air} = 17.5\ mm$$

ϕ_{Air} = <u>0.057</u> /mm f_{Air} = <u>17.5</u> mm