Name Solutions

Closed book; closed notes. The time limit is 2 hours.
Equation sheets are attached and can be removed.
Spare raytrace forms are also attached.
Use the back sides if required.
Do not use any pre-stored information or programs in your calculator.
Assume thin lenses in air if not specified.
If a method of solution is specified in the problem, it must be used.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy.

1) (10 points) Use two thin lenses in air to design a telephoto objective with a focal length of 400 mm and a back focal distance of 200 mm. The separation of the two lenses must be 75 mm.

$f = 400\ mm \qquad \phi = 0.0025/mm$

$t = 75\ mm$

$BFD = 200\ mm$

$$BFD = f_{12}' + d' = f + d'$$

$$d' = -200\ mm$$

$$d' = -\frac{\phi_1}{\phi} t = -\frac{\phi_1}{.0025} 75 = -200$$

$$\phi_1 = .006667/mm$$

$$f_1 = 150\ mm$$

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t$$

$$.0025 = .006667 + \phi_2 (1 - .5000)$$

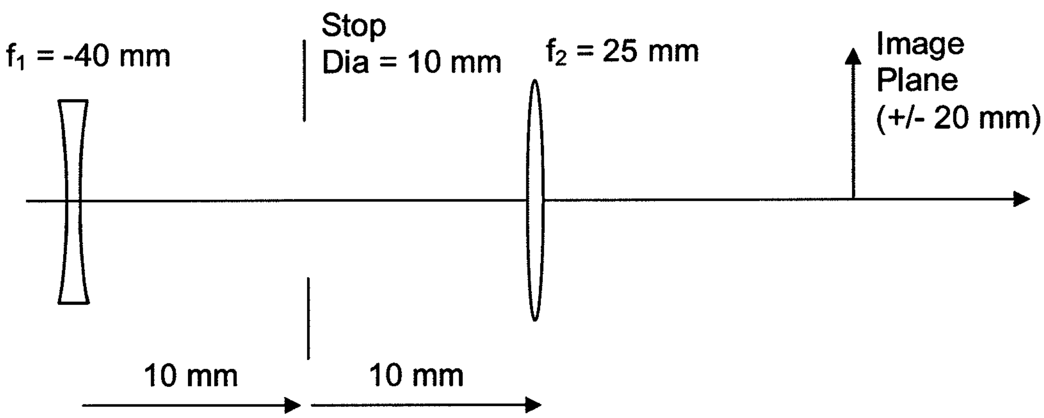
$$\phi_2 = -.008333\ /mm$$

$$f_2 = -120\ mm$$

f1 = 150 mm f2 = -120 mm

2) (25 points) The following diagram shows the design of a reverse telephoto objective that is comprised of two thin lenses in air. The system stop is located between the two lenses.

The diameter of the stop is 10 mm.
The object is at infinity.
The maximum image size is +/- 20 mm.



Determine the following:
- Entrance pupil and exit pupil locations and sizes.
- System focal length.
- Angular field of view (in object space).
- Required diameters for the two lenses for the system to be unvignetted over the specified maximum image size.

NOTE: This problem is to be worked using raytrace methods only. Gaussian imaging methods may not be used for any portion of this problem. Be sure to clearly label your rays on the raytrace form.

Your answers must be entered below. Be sure to provide details on the pages that follow to indicate your method of solution (how did you get your answer: which ray was used, analysis of ray data, etc.)

Entrance Pupil: 8.0 mm to the R of the first lens. D_{EP} = 8.0 mm

Exit Pupil: 16.667 mm to the L of the second lens. D_{XP} = 16.66 mm

System Focal Length = 28.57 mm

FOV = +/- 35.0 deg

Lens 1 Diameter = 19.2 mm Lens 2 Diameter = 23.2 mm

	Obj		EP		L1		Stop		L2		XP		F' Image
Surface	0		1		2		3		4		5		6
f			–		−40		–		25		–		
−φ			–		.025		–		−.04		–		
t				−8.0		10		10		−16.667		59.52	

Potential Chief Ray

	0		1		2		3		4		5		6
$\tilde{\bar{y}}$			0		−1.0		0		1.0		0		
$\tilde{\bar{u}}$		.125		.125		.1*		.1*		.06			

Potential Marginal Ray

59.52

	0		1		2		3		4		5		6
$\tilde{y}$	1*		1		1		1.25		1.5		2.0833		0
$\tilde{u}$		0		0		.025		.025		−.035		−.035	

Potential Chief Ray - Extended

	0		1		2		3		4		5		6
$\tilde{\bar{y}}$			0		−1.0		0		1.0		0		3.571
$\tilde{\bar{u}}$		.125		.125		.1		.1		.06		.06	

Marginal Ray - Scale Factor = 4.0

	0		1		2		3		4		5		6
y	4.0		4.0		4.0		5.0		6.0		8.333		0
u		0		0		0.1		0.1		−0.14		−0.14	

Chief Ray - Scale Factor = 5.60

	0		1		2		3		4		5		6
$\bar{y}$			0		−5.60		0		5.60		0		20
$\bar{u}$		0.700		0.700		0.560		0.560		0.336		0.336	

y													
u													

y													
u													

Continues…

* arbitrary

Provide Method of Solution:

EP/XP Location: Trace a potential chief ray starting at the center of the stop. Pupils are located where this ray crosses the axis in object / image space.

$L1 \rightarrow EP = 8.0$ mm (Right of L1)

$L2 \rightarrow XP = -16.667$ mm (Left of L2)

Focal Length: Trace a potential marginal ray parallel to the axis in object space ($y = 1$). The rear focal point is located where this ray crosses the axis.

$XP \rightarrow F' = 59.52$ mm

$BFD = (L2 \rightarrow XP) + (XP \rightarrow F') = -16.667 + 59.52$

$BFD = 42.85$ mm

$\phi = -\frac{u'}{y_1}$ $u' = -.035$

$\phi = .035/\text{mm}$

$f = 1/\phi = 28.57$ mm

Extend the potential chief ray to F'

Pupil Sizes: Scale the marginal ray to the desired stop radius $r_{STOP} = D_{STOP}/2 = 5.0$ mm

Scale Factor $= 5.0\text{ mm}/1.25\text{ mm} = 4.00$

$r_{EP} = 4.0$ mm $r_{XP} = 8.333$ mm

$D_{EP} = 8.0$ mm $D_{XP} = 16.666$ mm

Continues...

Provide Method of Solution:

FOV: Scale the potential chief ray to an image height of 20 mm (from the potential ray value at 3.571 mm)

$$\text{Scale Factor} = \frac{20\text{ mm}}{3.571\text{ mm}} = 5.60$$

Object space chief ray:

$$\bar{u}_0 = 0.700$$

$$HFOV = \tan^{-1}(\bar{u}_0) = \tan^{-1}(0.700)$$

$$HFOV = 35.0°$$

$$FOV = \underline{70°} \quad \text{or} \quad +/- \ 35.0°$$

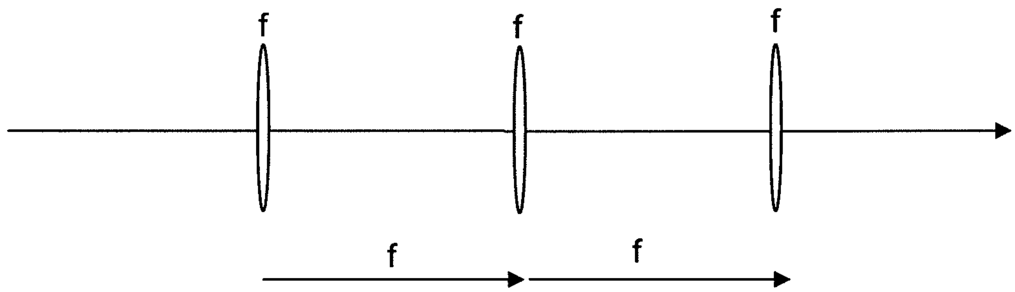
Lens Diameters: For Unvignetted

$$a \geq |y| + |\bar{y}|$$

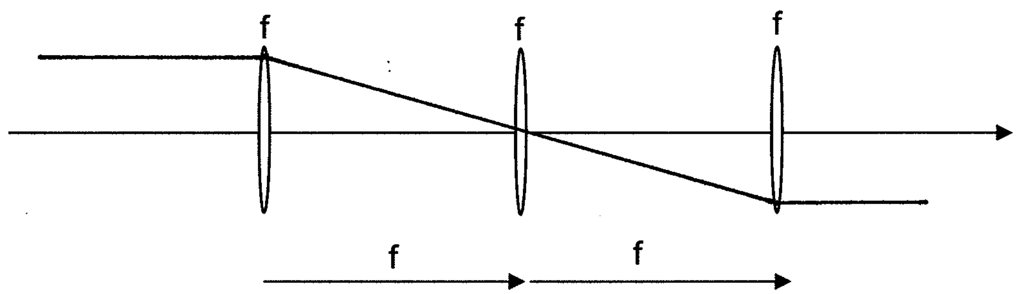
L1: $y_1 = 4.0$ mm, $\bar{y}_1 = -5.6$ mm $\qquad a_1 \geq 9.6$ mm $\qquad D_1 \geq \underline{19.2\text{ mm}}$

L2: $y_2 = 6.0$ mm, $\bar{y}_2 = 5.60$ mm $\qquad a_2 \geq 11.6$ mm $\qquad D_2 \geq \underline{23.2\text{ mm}}$

3) (20 points) Consider the following optical system comprised of three identical thin lenses of focal length f that are each separated by this same distance f:



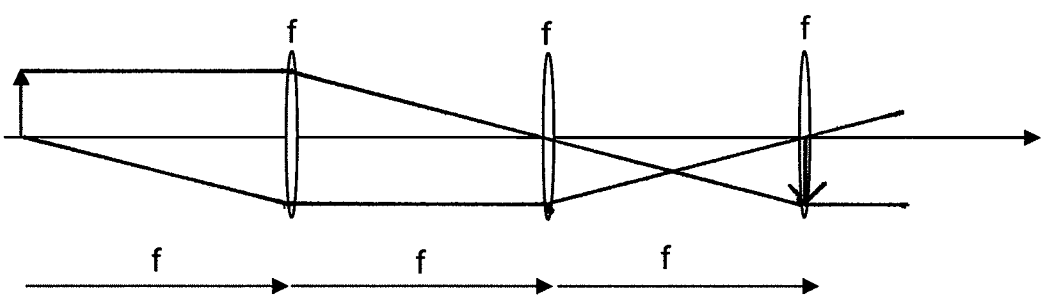
a) Determine the focal length of this system by sketching rays. No calculations are required or permitted.



Sketch a ray parallel to the axis

– the system is afocal.

b) An object is located at the front focal point of the first lens element. Determine the image location and size by sketching rays. No calculations are required or permitted.



Trace rays from the top and bottom of the object.

The image is located at the second lens with a magnification of –1 (inverted)

c) Here equations are permitted. Explain the result of parts (a) and (b) by combining the second and third lenses into a reduced component that is used in conjunction with the first lens. Hint: consider the system to be analogous to a telescope with an objective lens and a compound eyepiece.

For L2 + L3:

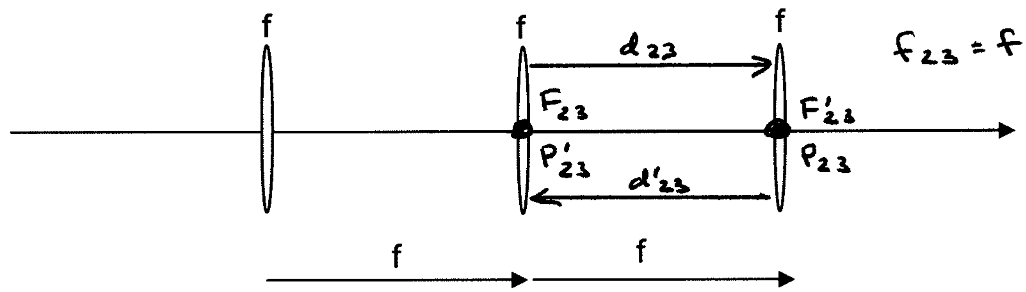
$\phi_{23} = \phi_2 + \phi_3 - \phi_2\phi_3 t_2$

$\phi_{23} = \frac{1}{f} + \frac{1}{f} - \frac{f}{f^2} = \frac{1}{f}$

$f_{23} = f$

$d_{23} = \frac{\phi_3}{\phi_{23}} t_2 = \frac{f}{f} f = f$

$d'_{23} = -\frac{\phi_2}{\phi_{23}} t_2 = -\frac{f}{f} f = -f$



The front focal point (F_{23}) of the L2-L3 combination is at the rear focal point of L1 - the system is afocal

$m = -f_{23}/f_1 = -1$

In an afocal system, it is well known that an object at F_1 images to F'_{23}. It does here as well as F'_{23} is located at the second element.

d) Where should the stop be located for the system to be telecentric in object space? Is there anything else you can then say about the telecentricity of the system?

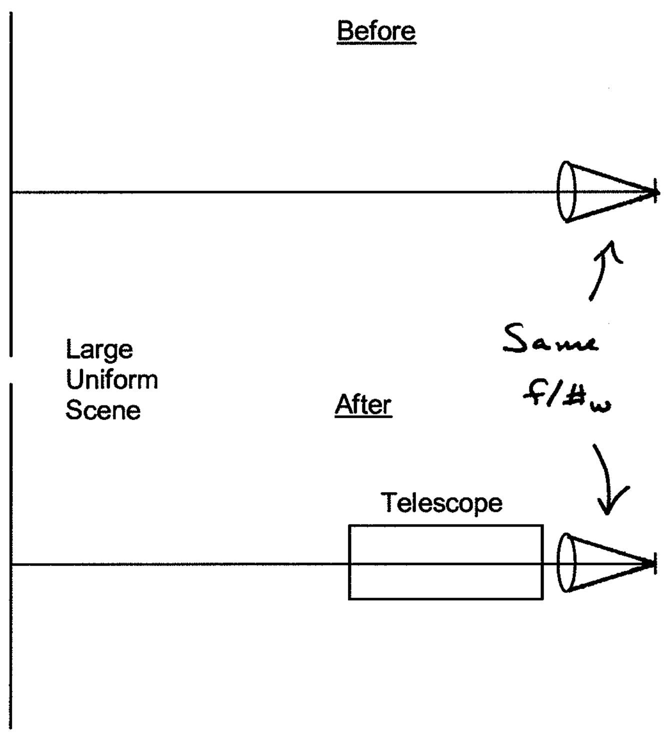
The stop would be located at the second lens element.

The system is afocal and is doubly telecentric.

4) (10 points) An imaging lens is used to image a large extended uniform object. The object greatly overfills the field of view defined by the detector used with the lens. Under these conditions, a certain irradiance E′ is incident on the detector.

A 5X telescope is now placed in front of the imaging lens. The telescope (aperture sizes, position, etc.) is designed so that the system stop of the combination of the telescope and the imaging lens remains at the imaging lens.

How does the irradiance on the detector change? You must explain how you obtained your answer.



The system stop (and XP) for both systems is the same. The image forming cones of light are identical.

- Same $f/\#_w$

The camera equation gives the image plane irradiance:

$$E' = \frac{\pi L_o}{4(f/\#_w)^2}$$

The scene radiance L_o is constant.

Therefore, the image plane irradiance <u>does not</u> change!

For a given dA on the object, the larger telescope aperture does collect 25X more light. However, the image of this dA is 25X larger on the detector. The net result is that the irradiance does not change. This result requires that the telescope XP is larger than the EP of the imaging lens.

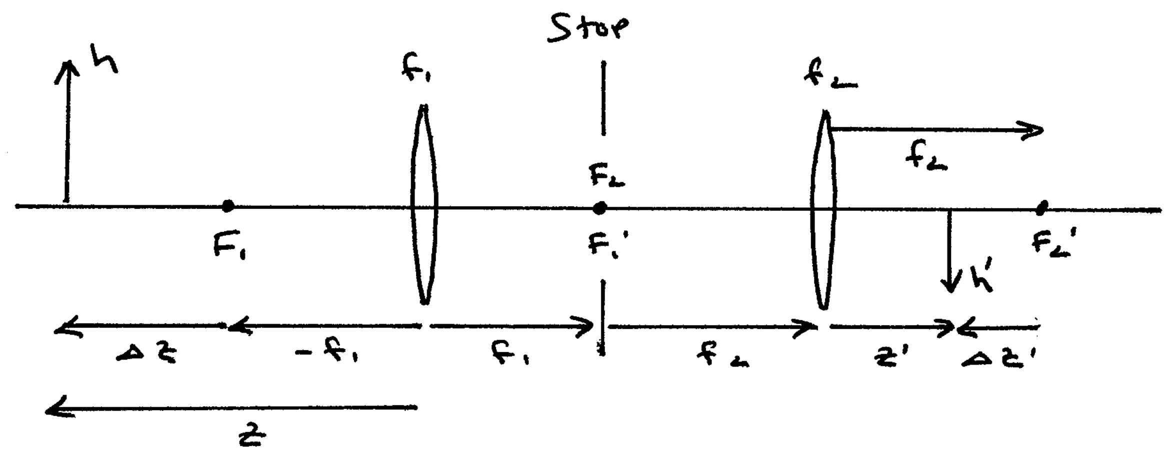
5) (20 points) An object is located 75 mm to the left of the first element of a doubly-telecentric system. The conjugate image is located 10 mm to the right of the second element of the system. The image size is half the object size.

The system uses two thin lenses in air. Determine the system layout by providing the focal lengths of the two lenses, the required spacings and the stop position.

A doubly telecentric system must be afocal.

$$m = -\frac{f_2}{f_1} \qquad \bar{m} = \frac{\Delta z'}{\Delta z} = m^2$$

Conjugate locations can be determined by using the longitudinal magnification ($\bar{m}$) and distances measured from a pair of known conjugates. The front focal point of the first Lens (F_1) and the rear focal point of the second Lens (F_2') are convenient.



For doubly telecentric, the stop must be located at the common focal point.

Continues...

$z = \Delta z - f_1$ $\qquad$ $z' = \Delta z' + f_2$

For the afocal system: $m = -\frac{1}{2}$ (inverted)

$z = -75$ mm

$z' = 10$ mm

$m = -f_2/f_1$

① $f_2 = -m f_1$

② $\Delta z = z + f_1$

$\frac{\Delta z'}{\Delta z} = m^2$

$\Delta z' = m^2 \Delta z$

$z' = \Delta z' + f_2$

$z' = m^2 \Delta z + f_2$

Using ① and ②

$z' = m^2 (z + f_1) - m f_1$

$10 = \frac{1}{4}(f_1 - 75) + \frac{1}{2} f_1$

$f_1 = 38.333$ mm

$f_2 = 19.166$ mm

$t = 47.50$ mm

f_1 = 38.333 mm $\qquad$ f_2 = 19.166 mm $\qquad$ t = 57.50 mm

Stop Location: Common Focal Point

6) (15 points) An 8X Keplerian telescope has a 240 mm focal length objective.

a) Determine the focal length of the eye lens and the overall length of the telescope.

$MP = -f_1/f_2 = -8 \quad f_1 = 240\text{ mm}$

$f_2 = f_1/8$

$L = f_1 + f_2 = 270\text{ mm}$

$f_2 = 30\text{ mm}$

f_2 = 30 mm L = 270 mm

b) If the stop of the telescope is at the objective, what is the eye relief?

XP is image of the stop by the eye lens.

$z = -L = -270\text{ mm} \qquad \frac{1}{z'} = \frac{1}{ER} = \frac{1}{z} + \frac{1}{f_2}$

$z' = ER = 33.75\text{ mm}$

ER = 33.75 mm

c) The objective lens has a diameter of 40 mm. What is the required eye lens diameter for the telescope to have an unvignetted field of view of +/- 2 degrees?

Need y_2 and $\bar{y}_2$ at the eye lens.

$\phi_1 = .0041666/\text{mm}$

Chief: $\bar{u} = \tan(2°) = .0349$

$\bar{y}_1 = 0$

$\bar{y}_2 = \bar{u} L = 9.423\text{ mm}$

Marginal: $y_1 = 20$

$u' = -y_1 \phi_1 = -.08333$

$y_2 = y_1 + u' L = 20 - 22.5$

$y_2 = -2.5\text{ mm}$

or $y_2 = m\, y_1$

$m = \frac{1}{MP} = -\frac{1}{8}$

$y_2 = -\frac{20}{8} = -2.5\text{ mm}$

Unvignetted: $a_2 \geq |y| + |\bar{y}|$

$a_2 \geq 11.9\text{ mm}$

$D_2 \geq 23.8\text{ mm}$

D_2 = 23.8 mm

Extra Credit (5 points)

A direct vision prism uses two opposing thin prisms to provide dispersion without deviation of the d light. For a desired dispersion Δ, the deviation δ is zero.

The properties of the two glasses are n_{d1}, ν_1 and n_{d2}, ν_2

Provide the equations for the two required prism angles α_1 and α_2 in terms of these glass values and the net dispersion Δ.

$$\delta = \delta_1 + \delta_2 \qquad \delta_1 = -\delta_2 \qquad \delta_1 = -(n_{d_1}-1)\alpha_1$$

$$\delta_2 = -(n_{d_2}-1)\alpha_2$$

$$\Delta = \Delta_1 + \Delta_2 = \frac{\delta_1}{\nu_1} + \frac{\delta_2}{\nu_2}$$

$$\Delta = -\left(\frac{\nu_1-\nu_2}{\nu_1\nu_2}\right)\delta_1 \qquad \underline{\text{or}} \qquad \Delta = \left(\frac{\nu_1-\nu_2}{\nu_1\nu_2}\right)\delta_2$$

$$\Delta = \left(\frac{\nu_1-\nu_2}{\nu_1\nu_2}\right)(n_{d_1}-1)\alpha_1 \qquad \Delta = -\left(\frac{\nu_1-\nu_2}{\nu_1\nu_2}\right)(n_{d_2}-1)\alpha_2$$

$$\frac{\alpha_1}{\Delta} = \left(\frac{\nu_1\nu_2}{\nu_1-\nu_2}\right)\left(\frac{1}{n_{d_1}-1}\right) \qquad \frac{\alpha_2}{\Delta} = -\left(\frac{\nu_1\nu_2}{\nu_1-\nu_2}\right)\left(\frac{1}{n_{d_2}-1}\right)$$