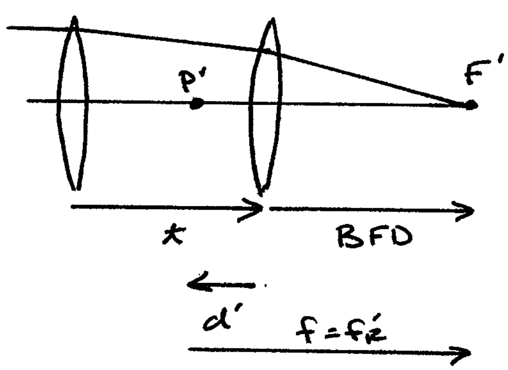
Name Solutions

Closed book; closed notes. Time limit: 2 hours.
An equation sheet is attached and can be removed. A spare raytrace sheet is also attached.
Use the back sides if required.
Assume thin lenses in air if not specified.
If a method of solution is specified in the problem, that method must be used.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.
Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy.

1) (10 points) Use two thin lenses in air to design a Petzval objective with a focal length of 100 mm and a back focal distance of 80 mm. The separation of the two lenses must be 40 mm. **Use Gaussian methods.**



$f'_R = f = 100\ mm$ $\phi = .01/mm$

$t = 40\ mm$

$BFD = 80\ mm = d' + f'_R$

$d' = -20\ mm$

$d' = -\frac{\phi_1}{\phi} t = -20\ mm$

$\phi_1 = .005/mm$ $f_1 = 200\ mm$

$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t = .01/mm$

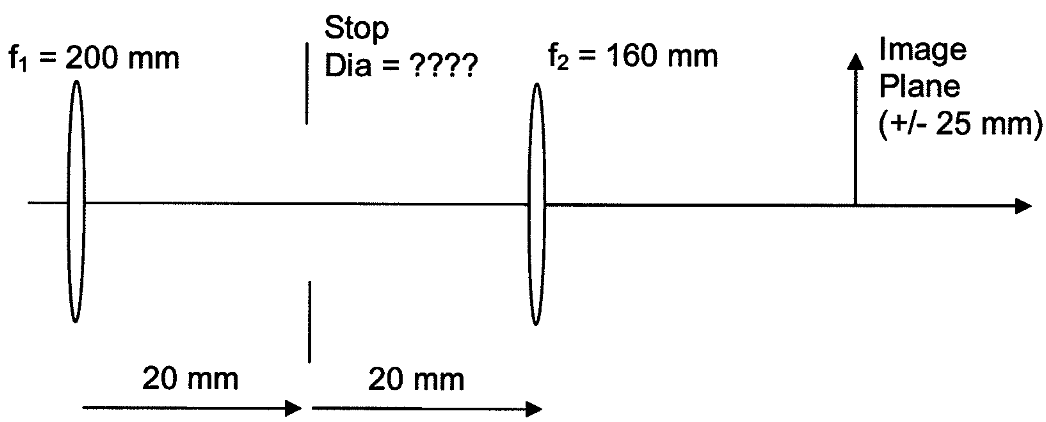
$.005/mm + \phi_2 (1 - .005/mm \cdot 40\ mm) = .01/mm$

$\phi_2 = .00625/mm$ $f_2 = 160\ mm$

f1 = 200 mm f2 = 160 mm

2) (30 points) The following diagram shows the design of an objective that is comprised of two thin lenses in air. The system stop is located between the two lenses.

The system operates at f/4.
The object is at infinity.
The maximum image size is +/- 25 mm.



Determine the following:
- Entrance pupil and exit pupil locations and sizes.
- System focal length and back focal distance.
- Stop diameter.
- Angular field of view (in object space).
- Required diameters for the two lenses for the system to be unvignetted over the specified maximum image size.

NOTE: This problem is to be worked using raytrace methods only. Gaussian imaging methods may not be used for any portion of this problem. The field of view must be determined from the chief ray.

Be sure to clearly label your rays on the raytrace form. Your answers must be entered below. Be sure to provide details on the pages that follow to indicate your method of solution (how did you get your answer: which ray was used, analysis of ray data, etc.).

Entrance Pupil: 22.22 mm to the R of the first lens. D_{EP} = 25.0 mm

Exit Pupil: 22.86 mm to the L of the second lens. D_{XP} = 25.72 mm

System Focal Length = 100 mm Back Focal Distance = 80 mm

Stop Diameter = 22.5 mm FOV = +/- 14.0 deg in object space

Lens 1 Diameter = 36.1 mm Lens 2 Diameter = 31.1 mm

	Obj		EP		L1		Stop		L2		XP		F' Image
Surface	0		1		2		3		4		5		6
f			–		200		—		160		—		
$-\phi$					-.005		-		-.00625		-		
t				-22.222		20		20		-22.857		102.86	

Potential Chief Ray

$\tilde{\bar{y}}$			0		-2.0		0		2.0		0		
$\tilde{\bar{u}}$				.09		.1 *		.1 *		.0875			

Potential Marginal Ray 102.86

y	1 *		1		1		0.9		0.8		1.0286		0
u		0		0		-.005		-.005		-.01		-.01	

Potential Chief Ray - Extended

y			0		-2.0		0		2.0		0		9.00
u				.09		.1		.1		.0875		.0875	

Marginal Ray - Scale Factor = 12.5

y	12.5		12.5		12.5		11.25		10.0		12.86		0
u		0		0		-.0625		-.0625		-.125		-.125	

Chief Ray - Scale Factor = 2.778

$\bar{y}$			0		-5.56		0		5.56		0		25.0
$\bar{u}$				.250		.2778		.2778		.2431		.2431	

y													
u													

y													
u													

Continues…

* arbitrary

Provide Method of Solution:

EP/XP Locations: Trace a potential chief ray starting at the center of the stop. The pupils are located where this ray crosses the axis in object/image space.

L1 → EP = 22.22 mm (R of L1)
L2 → XP = -22.86 mm (L of L2)

Focal Length / BFD: Trace a potential chief ray parallel to the axis in object space ($y = 1$). The rear focal point is located where this ray crosses the axis.

XP → F' = 102.86 mm

BFD = (L2 → XP) + (XP → F') = -22.86 mm + 102.86 mm

BFD = 80.0 mm

$\phi = -\frac{u'}{y_1}$ $u' = -.01$ $y_1 = 1.0$

$\phi = .01/\text{mm}$ $f = 100\text{ mm}$

Extend the potential chief ray to the image plane F'.

Pupil Sizes: The system operates at f/4 $f = 100\text{mm}$

$f/\# = 4 = f/D_{EP}$ $D_{EP} = 25\text{ mm}$

$r_{EP} = 12.5\text{ mm}$

Scale the marginal ray

Scale Factor = 12.5

$r_{STOP} = 11.25\text{ mm}$ $r_{XP} = 12.86\text{ mm}$

$D_{STOP} = 22.5\text{ mm}$ $D_{XP} = 25.72\text{ mm}$

Provide Method of Solution:

FOV: Scale the potential chief ray to an image height of 25mm (from the potential value of 9.00 mm)

Scale Factor = $25.0/9.0 = 2.778$

Object Space Chief Ray:

$\bar{u}_0 = .250$

$HFOV = \tan^{-1}(\bar{u}_0) = 14.0°$

$FOV = 28.0°$ or $\pm 14.0°$

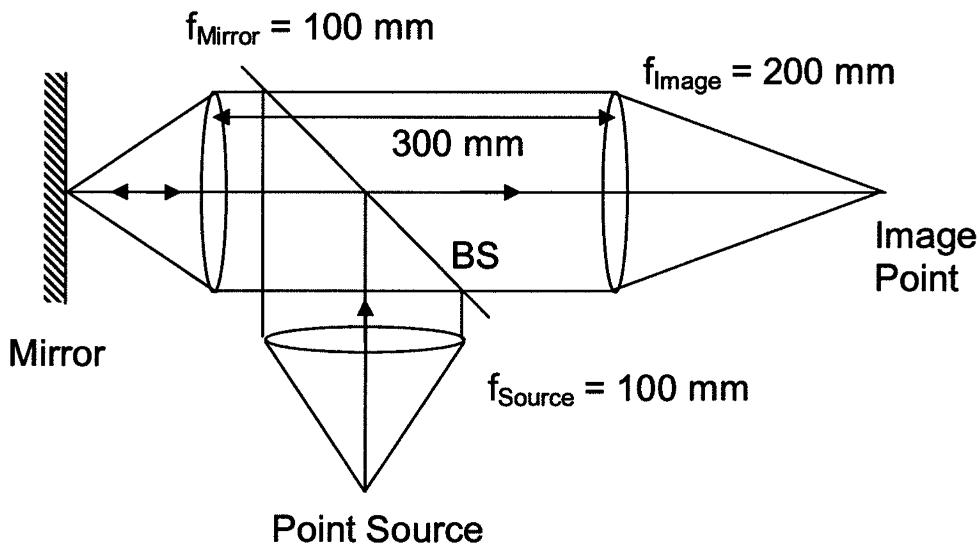
Lens Diameters: For Unvignetted

$a \geq |y| + |\bar{y}|$

L_1 $y_1 = 12.5$ mm
$\bar{y}_1 = -5.56$ mm
$a_1 \geq 18.06$ mm
$D_1 \geq 36.1$ mm

L_2 $y_2 = 10.0$ mm
$\bar{y}_2 = 5.56$ mm
$a_2 \geq 15.56$ mm
$D_2 \geq 31.1$ mm

3) (30 points) Consider the following optical system:



A point source is imaged onto a mirror by two lenses ("Source Lens" and "Mirror Lens") and a beam splitter. The reflected light is the imaged by two lenses ("Mirror Lens" and "Image Lens") to a final image point as shown. The mirror is located at the focal point of the "Mirror Lens".

The separation between the "Mirror Lens" and the "Image Lens" is 300 mm forming an afocal system. The point source is located at the front focal point of the "Source Lens", and the combination of the "Source Lens" and the "Mirror Lens" also forms an afocal system.

The method of solution is not specified.

In this configuration, the image point is located at the rear focal point of the Image Lens.

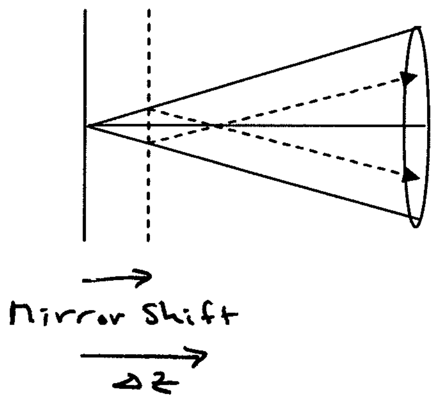
Source/Mirror Lens Combination:

Afocal $m = -1$ $\bar{m} = 1$

Mirror/Image Lens Combination:

Afocal $m = -2$ $\bar{m} = 4$

a) The mirror is translated 5 mm towards the "Mirror Lens" (to the right). How much does the image point move?



The point source seen by the mirror lens will move towards the mirror lens by twice the mirror shift.

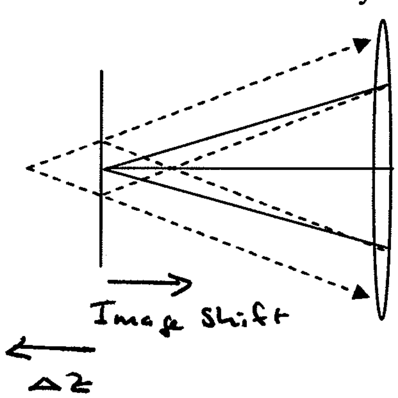
$\Delta z = 10\ mm$ $\bar{m} = 4$

$\Delta z' = \bar{m}\,\Delta z = 40\ mm$

to the right or away from the image lens

Image Shift = 40 Direction Right

b) Returning to the original configuration, the point source is translated down or away from the "Source Lens" by 2 mm. How much does the image point move?



Because $\bar{m}$ of the source/mirror lens combination is 1.0, the source image produced by the mirror lens will be 2.0 mm in front of the mirror.

The reflected source image seen by the mirror lens is 2 mm behind the mirror

$\Delta z = -2\ mm$

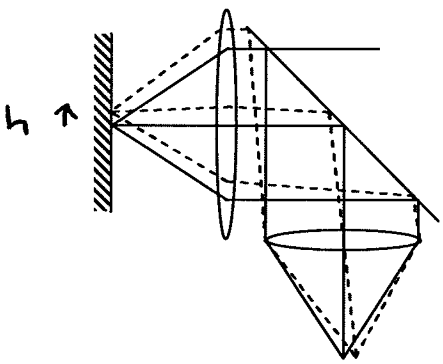
$\Delta z' = \bar{m}\,\Delta z$

$\bar{m} = 4$

$\Delta z' = -8\ mm$ to the left

Image Shift = - 8 Direction Left

c) Returning to the original configuration, the point source is translated to the right by 2 mm (the distance to the "Source Lens" does not change). How much does the image point move?



The magnification of the source/mirror lens combination is -1. The point source imaged onto the mirror will move up by 2 mm

$$h = 2.0 \text{ mm}$$

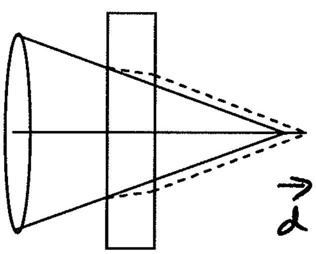
For the re-imaging

$$m = -2$$

$$h' = -4 \text{ mm}$$

Image Shift = -4 mm Direction Down

d) Returning to the original configuration, a block of glass 5 mm thick (n = 1.5) is inserted after (to the right of) the "Image Lens". How much does the image point move?



The image shift associated with a glass plate:

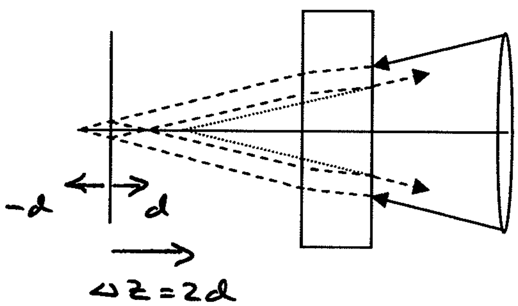
$$d = t\left(\frac{n-1}{n}\right) = 5 \text{ mm}\left(\frac{1.5-1.0}{1.5}\right)$$

$$d = 1.67 \text{ mm}$$

The final image point will move to the right by this distance.

Image Shift = 1.67 mm Direction Right

e) Returning to the original configuration, a block of glass 5 mm thick (n = 1.5) is between the "Mirror Lens" and the mirror. How much does the image point move?



The point image produced by the mirror lens will be "d" behind the mirror. Upon reflection the image will be "d" to the right of the mirror. An additional shift of "d" occurs when the light goes through the glass plate a second time

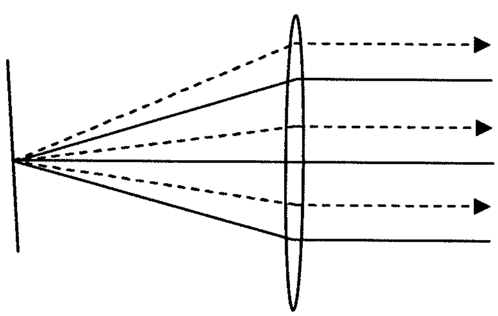
$\Delta z = 2d = 3.33$ mm

$\Delta z' = \bar{m}\,\Delta z = 13.33$ mm $\qquad \bar{m} = 4$

to the right

Image Shift = 13.33 mm Direction Right

f) Returning to the original configuration, the mirror is tilted by 2 degrees about its intersection point with the optical axis. How much does the image point move?



The mirror tilt rotates the ray bundle about the point source imaged onto the mirror.

There is no change in the final image location.

Image Shift = No Shift Direction ——

g) The original system has been changed so that the separation between the "Mirror Lens" and the "Image Lens" is now 200 mm. Repeat part a) for this configuration: The mirror is translated 5 mm towards the "Mirror Lens" (to the right). How much does the image point move?

The re-imaging system is no longer afocal, and exact imaging must be used.

Gaussian reduction could be applied to the Mirror / Image Lens combination, but it is probably easier just to sequentially image the shifted source through the two lenses.

$\Delta z = 10$ mm

$z_1 = -90$ mm $\qquad \frac{1}{z_1'} = \frac{1}{z_1} + \frac{1}{f_M} \qquad f_M = 100$ mm

$z_1' = -900$ mm

$z_2 = -t - z_1' \qquad t = 200$ mm

$= -1100$ mm

$\frac{1}{z_2'} = \frac{1}{z_2} + \frac{1}{f_I} \qquad f_I = 200$ mm

$z_2' = 244.44$ mm

$\Delta z' = 244.44 - f_{Image} = 44.44$ mm to the right

Image Shift = 44.44 mm Direction Right

4) (15 points) Design an object-space telecentric system consisting of a thin lens and a stop. The focal length of the lens is 100 mm, and the system operates at 1:1 conjugates. The system covers a field of view of ±10 mm, and it operates at a working f-number of 4 ($f/\#_W = 4$ or NA = 0.125). The system is unvignetted over this field of view.

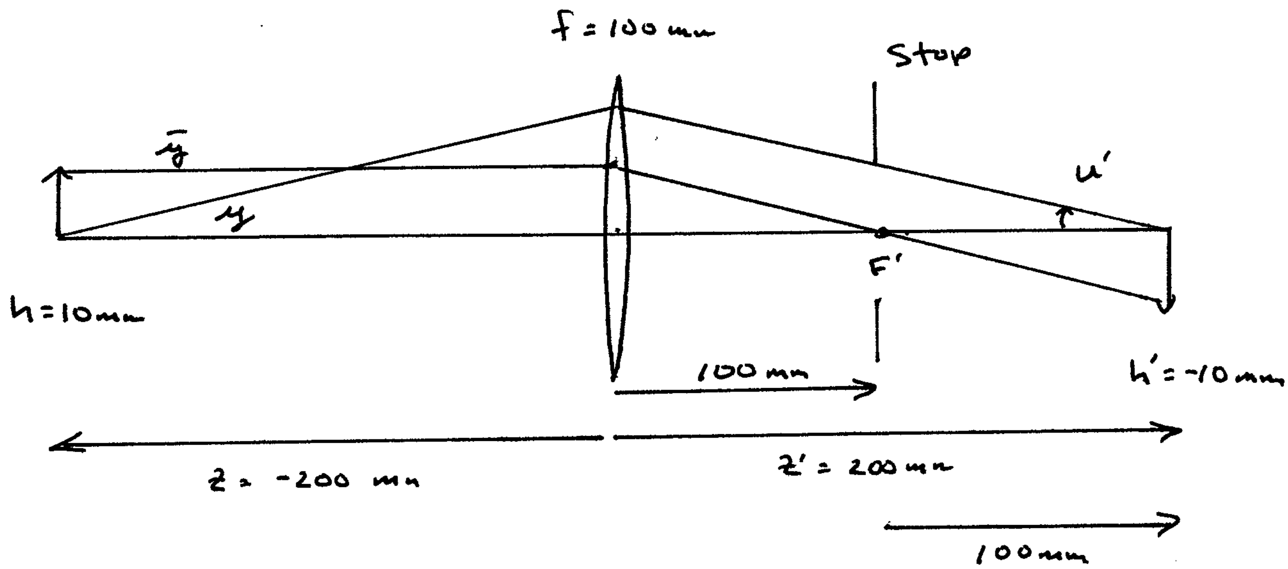
Sketch the system and provide the required spacings, the diameter of the lens and the stop diameter.

The method of solution is not specified.

At 1:1 conjugates, $z = -2f$ $z' = 2f$

$z = -200$ mm $z' = 200$ mm

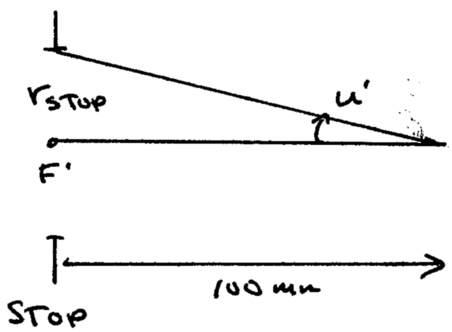
For object space telecentricity, the stop must be at the rear focal point of the lens.



The chief ray is parallel to the axis in object space.

Continues...

$f/\#_w = 4$



$$u' = -\frac{r_{STOP}}{100\,mm}$$

$NA = 0.125 = |u'|$

$r_{STOP} = 12.5\ mm = D_{STOP}/2$

or

$$f/\#_w = 4 = \frac{100\,mm}{D_{STOP}}$$

$D_{STOP} = 25\ mm$

At the lens:

$y_L = 2\, r_{STOP} = 25\ mm$

$\bar{y}_L =$ object height $= h = 10\ mm$

For no vignetting

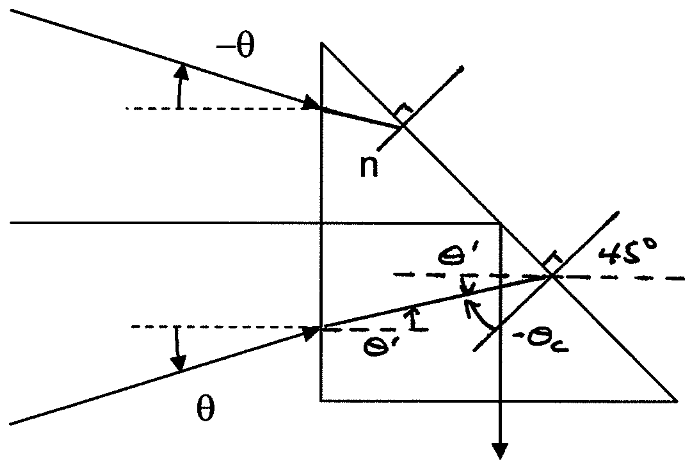
$a_L \geq |y_L| + |\bar{y}_L| = 35\ mm$

$D_L \geq 70\ mm$

$D_{Stop} =$ 25 mm $D_{Lens} =$ 70 mm

Stop Location: 100 mm to the Right of the lens

5) (15 points) A right angle prism of index n is used to deviate a converging beam by 90 degrees. The fastest f/# or NA that the prism will support is limited by the loss of Total Internal Reflection at the hypotenuse of the prism. The prism is used in air.



a) Derive the expression for the fastest NA that is supported by the prism. At this limit, the entire converging beam undergoes TIR at the hypotenuse.

The bottom ray is incident on the hypotenuse at an angle that is closer to perpendicular than that of the top ray – the bottom ray will lose TIR first.

Critical Angle: $\theta_c = \sin^{-1}\left(\frac{1}{n}\right)$

From Diagram: $45° = \theta' + \theta_c$

$\theta' = 45° - \theta_c$

$\theta' = 45° - \sin^{-1}\left(\frac{1}{n}\right)$

Continues...

$$\sin\theta = n\sin\theta'$$

$$\sin\theta = n\sin\left\{45^\circ - \sin^{-1}\left(\frac{1}{n}\right)\right\}$$

In air: $NA = \sin\theta$

$$NA = n\sin\left\{45^\circ - \sin^{-1}\left(\frac{1}{n}\right)\right\}$$

b) Evaluate this expression for BaK4 glass (glass code = 569560). What is the f/# corresponding to this NA?

BaK4 $n_d = 1.569$ $\nu = 56.0$ (not needed)

Limiting $NA = 0.148$

$$f/\# \approx \frac{1}{2NA} = 3.38$$

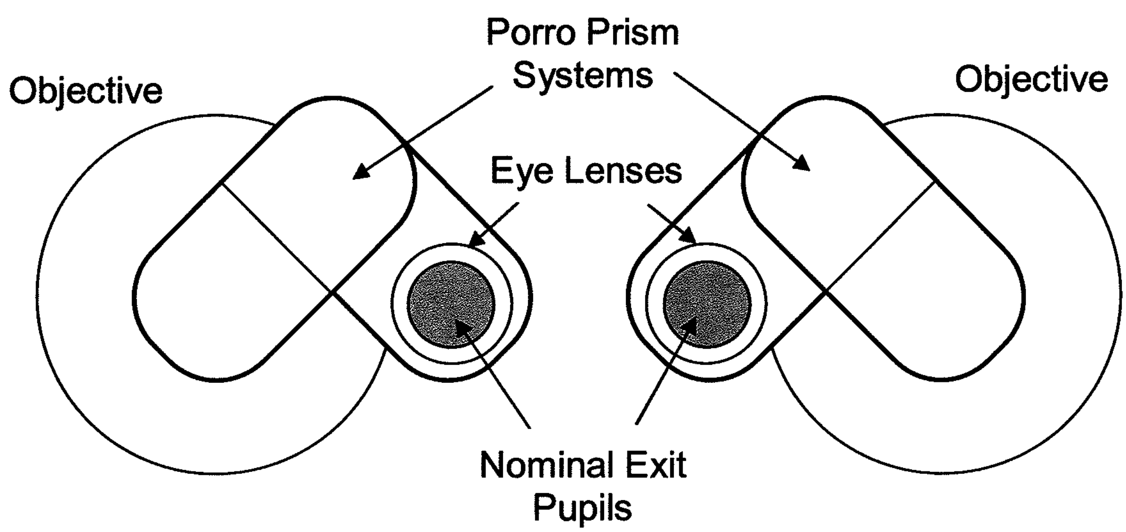
NA = 0.148 f/#: 3.38

Extra Credit (5 points)

The Porro Prism erecting system in a pair of binoculars can be thought of as four right angle prisms. If the objective lens used in the binoculars is faster than the TIR limit found in the preceding problem, what is the effect on the exit pupils?

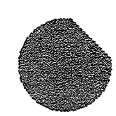
Sketch the appearance of the exit pupils in this situation.

The orientation of the prisms is shown in this drawing of the binoculars as seen from the back. The prisms are oriented at ±45 degrees.



Sketches of Exit Pupils Showing the Effects of the Loss of TIR:

- At each reflection, the loss of TIR will clip an edge of the pupil.
- The prism orientation will determine which side is clipped.



one reflection

- Each of the four reflections in a Porro-Prism System will clip a different side of the pupil. The final XP becomes "squared-off."