**Porro Prism Binoculars - Solutions**

Sections A and B

| | |
|---|---|
| Prism Size | $D_P$ = 26.56 mm |
| Half-Vignetted Field of View | $FOV_{HALF}$ = +/- 4.38 degrees |
| Eye Lens Focal Length | $f_E$ = 25.0 mm |
| Field Lens Focal Length | $f_F$ = 47.62 mm |
| Objective Lens Diameter | $D_O$ = 40.0 mm |
| Eye Lens Diameter | $D_E$ = 23.4 mm |
| Field Lens Diameter | $D_F$ = 30.6 mm |
| Objective Lens to Prism Entrance Face | t = 103.40 mm |
| Overall Mechanical System Length | L = 154.96 mm |

Section C

| | |
|---|---|
| Prism Size | $D_P$ = 43.13 mm* |
| Half-Vignetted Field of View | $FOV_{HALF}$ = +/- 7.83 degrees |
| Eye Lens Focal Length | $f_E$ = 25.0 mm |
| Field Lens Focal Length | $f_F$ = 47.62 mm |
| Objective Lens Diameter | $D_O$ = 40.0 mm |
| Eye Lens Diameter | $D_E$ = 38.0 mm |
| Field Lens Diameter | $D_F$ = 55.0 mm |
| Objective Lens to Prism Entrance Face | t = 43.13 mm* |
| Overall Mechanical System Length | L = 111.26 mm* |

* Additional solutions exist

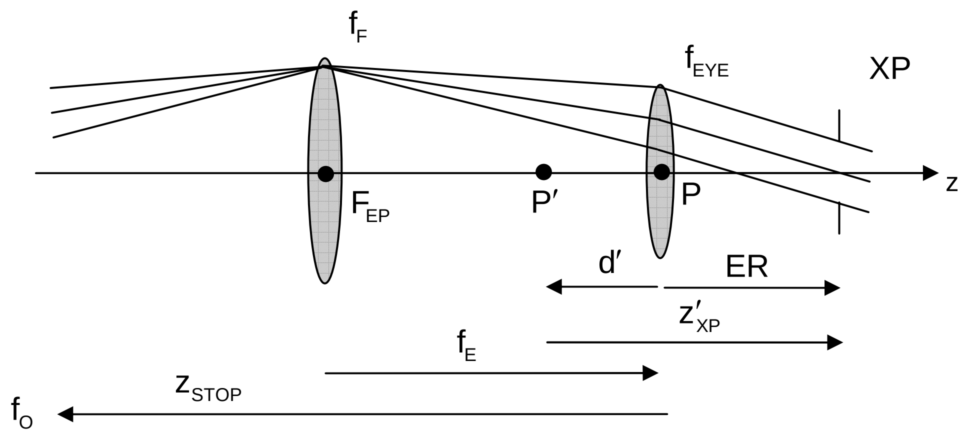
The fist step in the solution is to solve for the design of the base Keplerian telescope that is common to all parts of the problem.

MP = -8 Objective: $f_O$ = 200 mm
Stop at the Objective $D_O$ = 40 mm

$$MP = -\frac{f_O}{f_E} = -8$$

Eye Lens: $f_E$ = 25 mm

The field lens is at the front focal point of the eye lens:




$$z_{STOP} = -(f_O + f_E) = -225\text{mm}$$

$$\frac{1}{z'_{XP}} = \frac{1}{z_{STOP}} + \frac{1}{f_E} \qquad z'_{XP} = 28.125\text{mm}$$

$$ER = 15\text{mm} = z'_{XP} + d' \qquad d' = -13.125$$

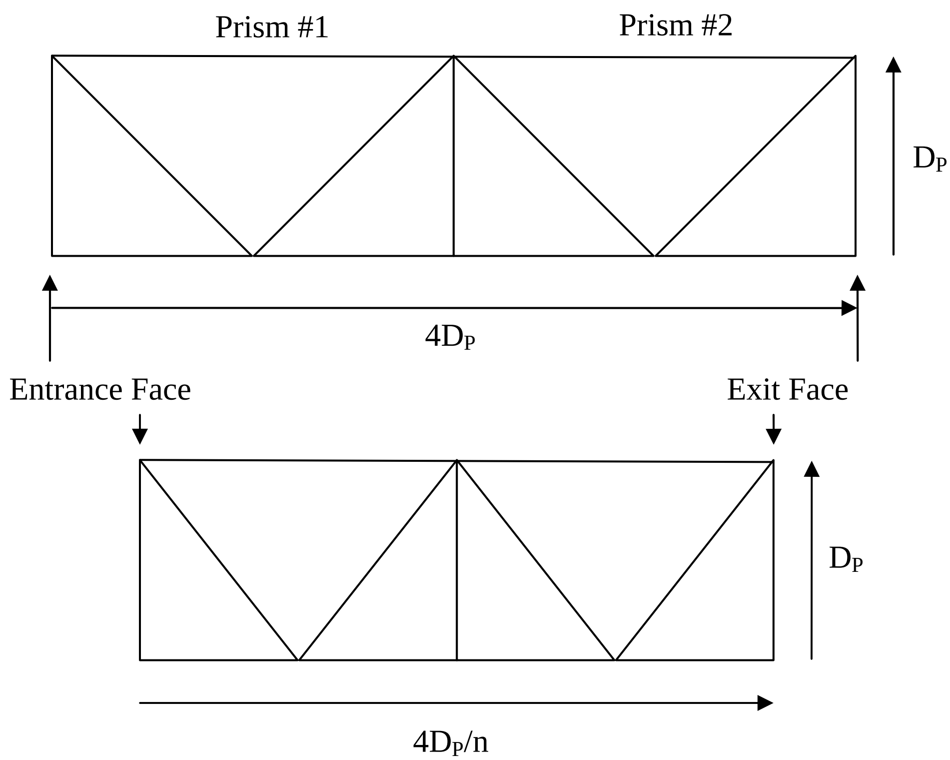
$$d' = -13.125 = -\frac{\phi_F}{\phi_{EP}} t \qquad \phi_{EP} = \phi_E \qquad t = f_E$$

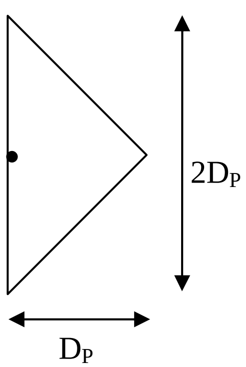
$$d' = -13.125 = -\frac{f_E^2}{f_F}$$

Field Lens: $f_F$ = 47.62 mm

Now examine the Porro prism system:

The tunnel diagram and the reduced tunnel diagram are




Because of the dihedral line locations of the two component prisms in the Porro prism system, the length of the mechanical volume of the Poro prism system is $2D_P$. The entrance and exit faces of the Porro system are co-planar.

To prevent mechanical mounting issues (and to satisfy the resulting requirement that no lens elements be within the mechanical volume of the Porro system), the minimum spacing between a lens element and either the entrance or exit face of the Porro system is $D_P$.

<u>Part A</u> – Determine the minim prism size for an unvignetted FOV of +/- 2 degrees.

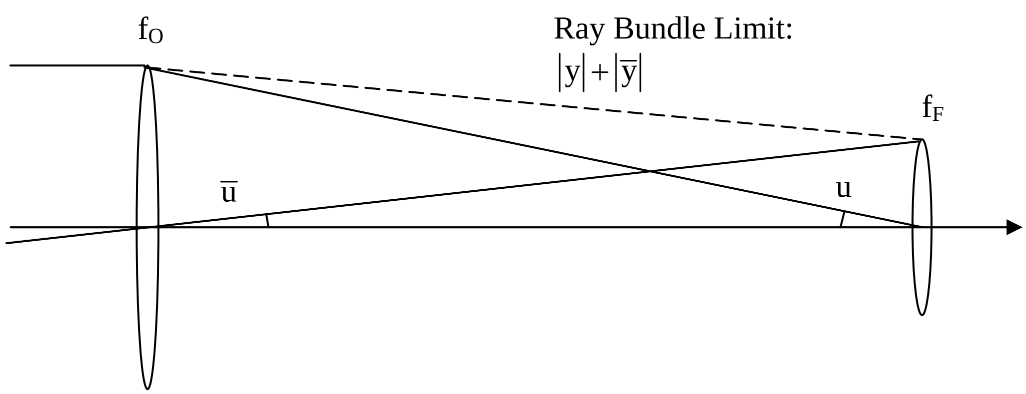
$$\mathrm{HFOV} = 2\,\mathrm{deg}$$

$$\text{Chief Ray:} \quad \bar{u} = \tan(\mathrm{HFOV}) = 0.035$$

Look at the marginal and chief rays to determine the ray bundle lime for no vignetting:

$$\text{Ray Bundle Limit} = |y| + |\bar{y}|$$

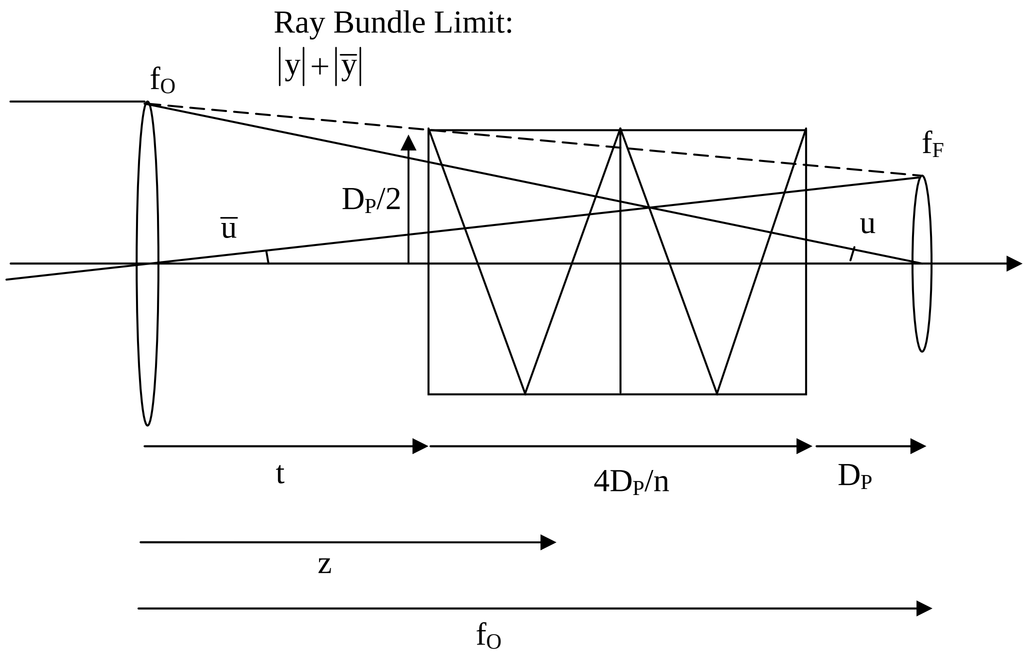
This can be evaluated at and axial position between the objective lens and field lens to determine the required prism aperture size.




The Porro prism system should be placed as close to the field lens as possible to minimize its size. The exit face of the prism system will be $D_P$ from the field lens.

The prism must be sized as to not vignette. The unvignetted FOV is limited by the aperture size at the entrance face of the Porro system.

Using the reduced tunnel diagram:




Using the distance z as the distance from the objective lens

$$\bar{y} = \bar{u}z$$

$$u = -y_O \phi_O$$
$$u = -\left(D_O / 2\right) / f_O$$

$$y = \left(D_O / 2\right) - \left(D_O / 2\right) z / f_O$$

For no vignetting, the required minimum prism aperture radius at a specific z is then

$$a = D_P / 2 = |y| + |\bar{y}| = \bar{u}z + \left(D_O / 2\right) - \left(D_O / 2\right) z / f_O$$

$$D_P / 2 = 20\text{mm} - 0.065z$$

This is the ray bundle limit.

For a given $D_P$, the front face of the prism system is located at

$$z_P = t = f_O - 4D_P / n - D_P \qquad n = 1.517$$

$$z_P = t = 200\text{mm} - 3.637 D_P$$

Using this position in the equation for the required apertures size allows for the determination of the minimum prism size:

$$D_P / 2 = 20\text{mm} - 0.065\left(200\text{mm} - 3.637 D_P\right)$$

$$D_P = 26.56\text{mm}$$

And the lengths of the tunnel diagrams are

$$4D_P = 106.24\text{mm}$$

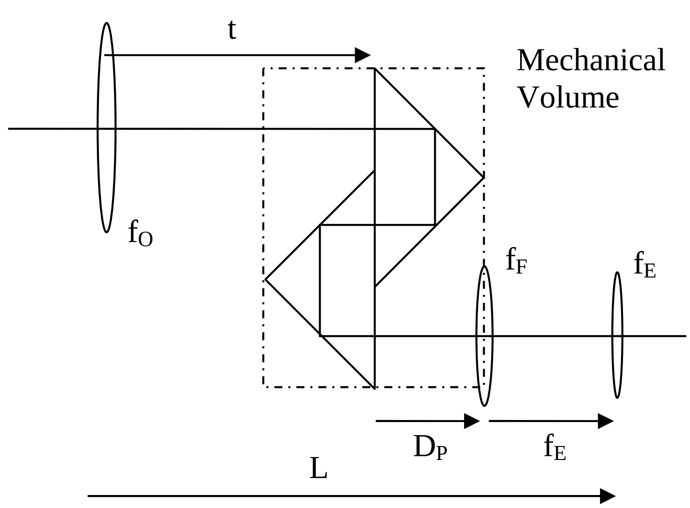
$$4D_P / n = 70.03\text{mm}$$

The distance form the objective to the entrance face is then

$$t = f_O - 4D_P / n - D_P = 103.41\text{mm}$$

And because the entrance and exit faces of the Porro prism system are coplanar, the total length of the binoculars is

$$L = t + D_P + f_E = 154.96\text{mm}$$




The second prism is actually rotated 90 deg out of the plane of the paper.

<u>Part B</u> – Determine the resulting half-vignetted FOV

The half-vignetted FOV is determined by the maximum chief ray angle that is passed through the system by the Porro prism system. A quick examination of the diagram in part A (with the chief ray and the reduced tunnel diagram) shows that the exit face of the Porro system will limit the half-vignetted FOV.

At this location and condition, the chief ray height will equal the prism size:

$$\bar{y} = D_P / 2 = \bar{u}z \qquad z = f_O - D_P$$

$$26.56\text{mm} / 2 = \bar{u}\left(200\text{mm} - 26.56\text{mm}\right)$$

$$\bar{u} = 0.07657$$

$$\text{HFOV} = \tan^{-1}\left(\bar{u}\right) = 4.38\,\text{deg}$$

$$\text{FOV} = +/- 4.38\,\text{deg}$$

The required field lens and eye lens diameters are determined by tracing this chief ray through the system (raytrace attached):

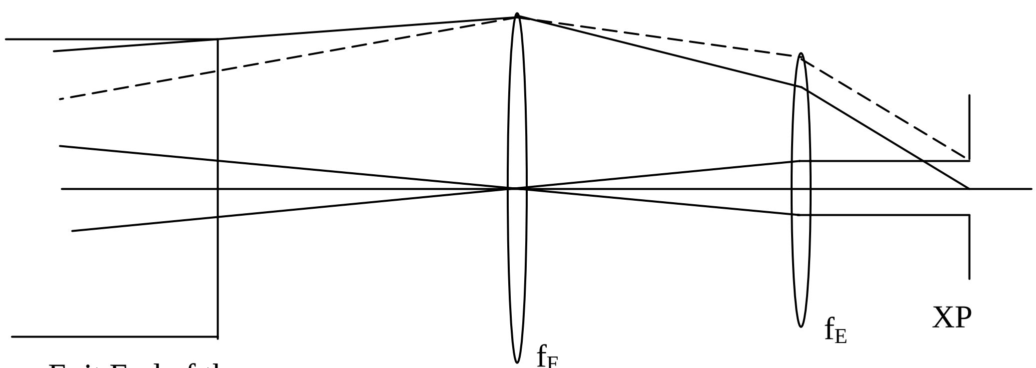
Field Lens:

$$\bar{y}_F = 15.314\text{mm} \qquad y_F = 0$$

$$a_F = \left|\bar{y}\right| = 15.314\text{mm} \qquad D_F = 30.6\text{mm}$$

Eye Lens: $\bar{y}_E = 9.188\text{mm} \qquad y_E = -2.5\text{mm}$

The simple solution for the diameter of the eye lens needed to support this half vignetted FOV is to use twice the chief ray height at the eye lens ($D_E$ = 18.4 mm), but this diameter will actually totally vignette the FOV. A closer examination of the off axis ray bundle in the vicinity of the field and eye lenses is needed.




The dashed ray is the ray from the bottom of the objective lens. Note that to the left of the field lens this ray is below the chief ray. To the right of the field lens, this ray is now above the chief ray. If the eye lens radius was chosen to be equal to the chief ray height, this ray would be blocked and no light from that FOV would make it through the XP and the system.

The eye lens diameter must be chosen according to the conditions for no vignetting.

Eye Lens:

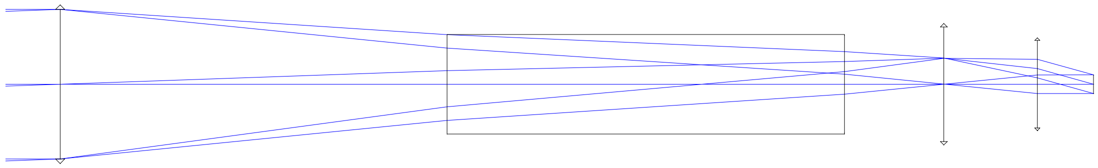
$$\bar{y}_E = 9.188\text{mm} \qquad y_E = -2.5\text{mm}$$

$$a_E = |y_E| + |\bar{y}_E| = 11.69\text{mm} \qquad D_E = 23.4\text{mm}$$

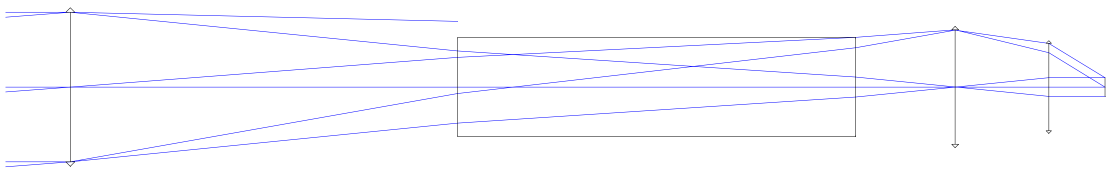
The eye lens diameter must also support the 2 degree unvignetted FOV. For this 2 degree chief ray, the chief ray height at the eye lens is 4.20 mm (see raytrace), and the required lens diameter would be 2(4.20 mm + 2.5 mm) = 13.4 mm. This value is less than what is already specified by the larger half-vignetted FOV. Because the eye lens diameter was actually specified as that needed for no vignetting, this last check is actually unnecessary.

This design was analyzed with commercial lens design software to see the actual raypaths and the vigneting limits.

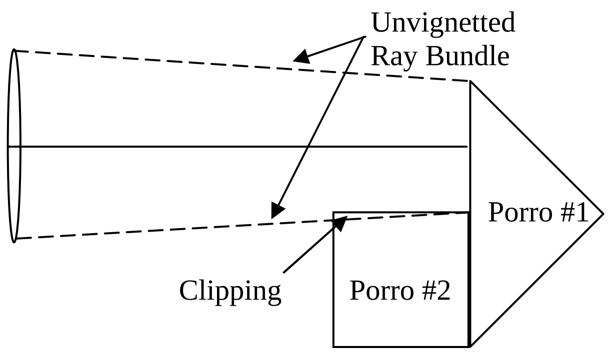
2 degree unvignetted FOV – Note that the ray bundle is limited by the entrance face of the Porro prism system.



4.38 degree half-vignetted FOV – the chief ray is limited by the exit face of the Porro prism system. The amount of vignetting that occurs at the entrance face of the prism system is relatively small (much less than 50%).



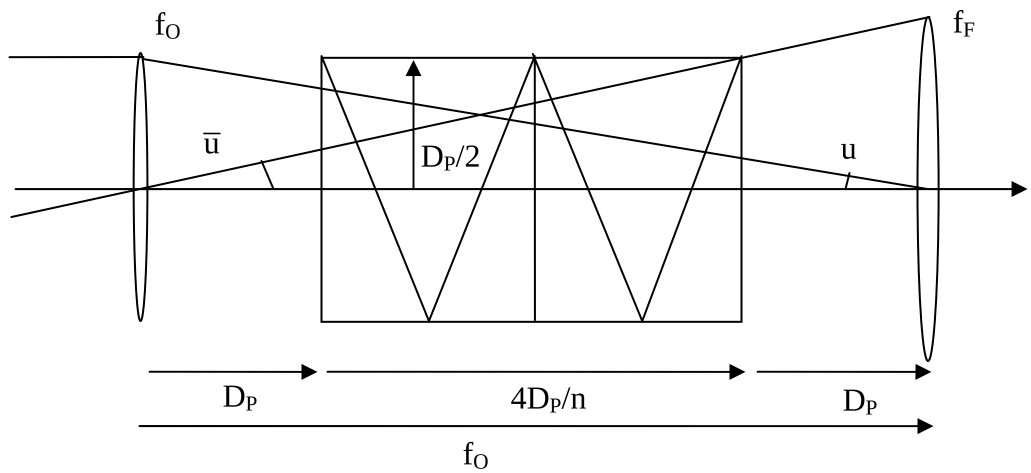
The system designed here is fairly representative of what is found in commercial binoculars. The unvignetted FOV is relatively small and is limited by the size of the entrance face of the prism. Larger FOVs are partially vignetted, but the vignetting grows slowly because the entrance face is optically about halfway between the objective lens and the field lens. Eventually the exit face of the prism begins to vignette the off-axis FOV, and this vignetting proceeds quickly with FOV as the exit face is close to the intermediate image plane. In practice, a field stop is often used to limit the FOV to a value somewhat less than the half-vignetted FOV supported by the prism. A uniform brightness across the FOV is desired. There is not a large difference between the half-vignetted FOV and the FOV of the field stop.




One small detail that has been ignored in this solution is the physical relationship between the two Porro prisms in the Porro system. Remember that the second prism is physically in front of the first prism. The side of this second prism extends straight forward at the edge of the prism aperture $D_P$ and can slightly clip one side of the beam from one side of the unvignetted FOV. The effect is small and could be prevented by using larger prisms or reducing the aperture used for a given size prism. The chief ray defining the half-vignetted FOV is similarly clipped by the first prism beyond the exit face of the second prism.

Part C – Redesign the system for the largest possible half-vignetted FOV.

The larges FOV will be obtained by making the prism as large as possible subject to the mechanical constraints of the mechanical volume of the prism. The objective lens-entrance face separation as well as the exit face-field lens separation will be equal to the prism size $D_P$.




$$f_O = 200mm = 2D_P + 4D_P / n \qquad n = 1.517$$

$$D_P = 43.13mm$$

Because the entrance and exit faces are coplanar, the actual system length is

$$L = 2D_P + f_E = 111.26mm$$

The reduced tunnel diagram length is

$$4D_P / n = 113.72mm$$

The half vignetted FOV can be determined from the chief ray:

$$\bar{u} = \frac{D_P / 2}{f_O - D_P} = \frac{21.56mm}{156.87mm} = 0.1374$$

$$HFOV = \tan^{-1}\left(\bar{u}\right) = 7.83\deg$$

$$FOV = +/- 7.83\deg$$

As in Part B, the lens diameters are determined by raytrace analysis:

Field Lens:

$$\bar{y}_F = 27.48\text{mm} \qquad y_F = 0$$

$$a_F = |\bar{y}| = 27.48\text{mm} \qquad D_F = 55.0\text{mm}$$

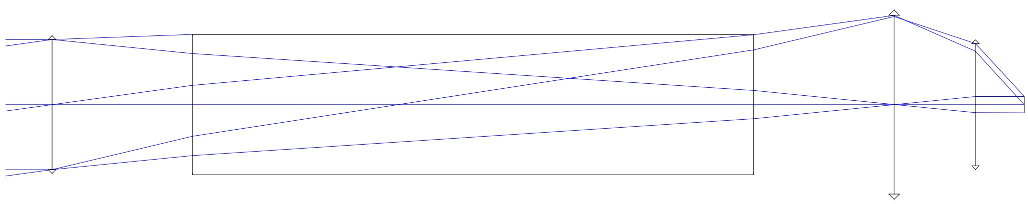
Eye Lens:

$$\bar{y}_E = 16.49\text{mm} \qquad y_E = -2.5\text{mm}$$

$$a_E = |y_E| + |\bar{y}_E| = 18.99\text{mm} \qquad D_E = 38.0\text{mm}$$

Both of these lenses are too fast for this to be a practical system. In addition the very large prisms would be excessively heavy.

An optical design software representation of this system:

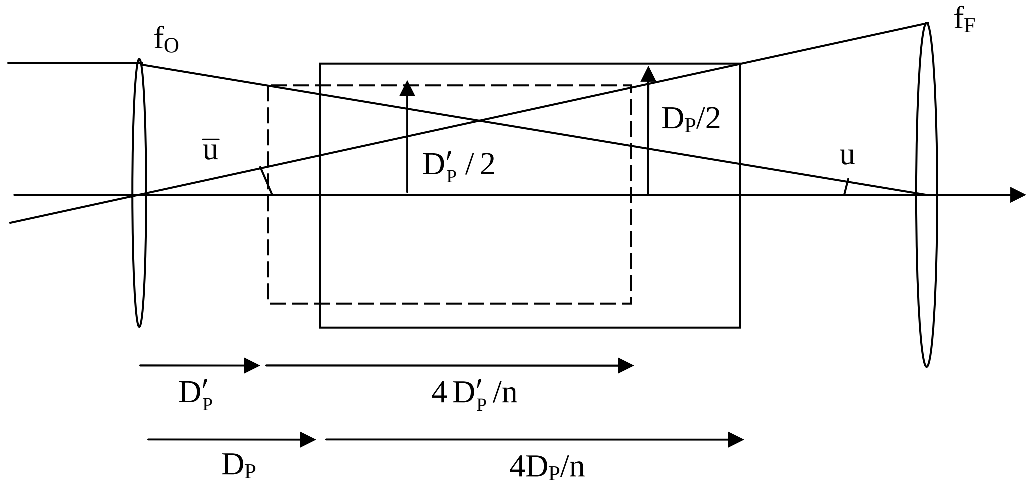


By using the largest possible prism, not only is the maximum half-vignetted FOV obtained, but the maximum unvignetted FOV also results. However, many other prism sizes will produce the same half-vignetted FOV. The prism must be placed as close to the objective lens as possible ($t = D_P$). The chief ray is limited by the upper rear corner of the prism, and the chief ray angle can be then written as

$$\bar{u} = \frac{D_P / 2}{D_P + 4D_P / n} = \frac{0.5}{3.637} = 0.1374$$

$$\text{HFOV} = \tan^{-1}(\bar{u}) = 7.83\,\text{deg}$$

This result is independent of $D_P$. The prism can be scaled to the chief ray height while maintaining the minimum separation of $D_P$ from the objective.




The smallest possible prism occurs when the entrance face size matches the marginal ray height. Any smaller prism would result in the entrance face of the prism becoming the stop of the system and reducing the EP size.

Using z as the distance from the objective lens, the marginal ray height is

$$y = (D_O/2) + uz$$

$$u = -(D_O/2)/f_O$$

$$y = (D_O/2) - (D_O/2)z/f_O = 20\text{mm} - 0.1z$$

At the minimum prism size,

$$z = D'_P \qquad y = D'_P/2$$

$$D'_P/2 = 20\text{mm} - 0.1D_P$$

$$D'_P = 33.33\text{mm} \qquad t = 33.33\text{mm}$$

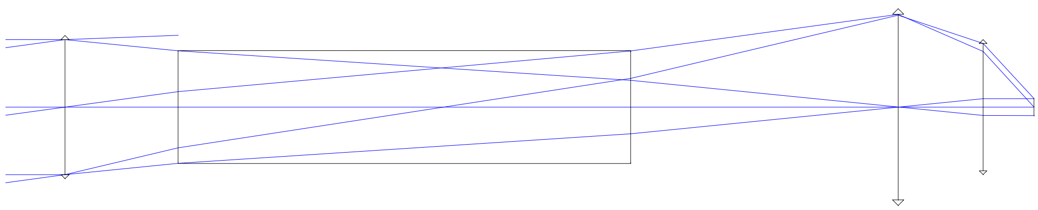
The distance from the prism exit face to the field lens and the overall system length are then

$$t' = f_0 - D'_P - 4D'_P / n = 78.8\text{mm}$$

$$L = D'_P + t' + f_E = 137.1\text{mm}$$

Because the chief ray and marginal rays have not changed, the required field lens and eye lens diameters have not changed from the previous configuration.

The optical design layout of this system is shown:



Note that the marginal ray is just passed by the prism system. This configuration has no unvignetted FOV (is unvignetted only on axis).

Both of these maximum half-vignetted FOV configurations have the dihedral line of the second Porro prism of the Porro prism system in the plane of the objective lens. In the minimum prism size configuration, the Porro prism aperture is less than the aperture size of the objective lens or stop, and the second Porro prism will block a portion of the objective lens. The EP will be clipped on one side and no longer be round. To avoid this clipping, a better minimum prism size would be to match the prism size to the objective lens diameter ($D_P = D_O = 40$ mm). This clipping does not occur with the maximum-size Porro prism system as its aperture is larger than the objective lens.