Design a slide projector.

24 × 36 mm onto 1.2 × 1.2 m at 10 m

Project 36 mm onto 1.2 m

$$m = -1.2\text{ m}/36\text{ mm} = -33.33$$

Projection Lens:

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

$z'/z = -33.33$

$z' = 10$ m

$z = -300$ mm

$f = 291$ mm

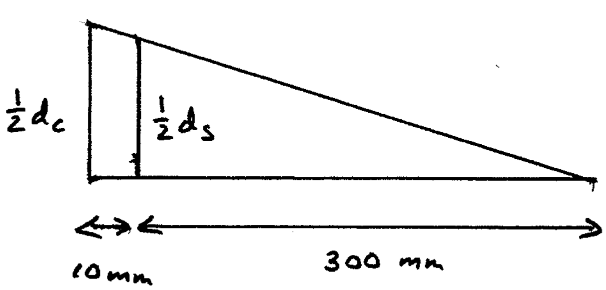
First – Let's do the problem with a "conventional" design:

Condenser Lens – assume Koehler illumination.

The maximum dia of the slide is

$$d_s = \sqrt{24^2 + 36^2} = 43.3\text{ mm}$$

Place the slide 1 cm in front of the condenser lens.
The required diameter is then



$$\frac{\frac{1}{2} d_c}{310\text{ mm}} = \frac{\frac{1}{2} d_s}{300\text{ mm}}$$

$d_c \geq 44.7$ mm

Let's use $d_c = 45$ mm

For an f/1 lens: $f_c = 45$ mm

Filament Imaging

Image the filament into the projection lens.

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f_c}$$

$f_c = 45\text{ mm}$

$z' = 310\text{ mm}$

$z = -52.6\text{ mm}$

(lamp - condenser)

m_c = condenser magnification = $310\text{ mm} / -52.6\text{ mm} = -5.89$

Filament diagonal

$8.0 \times 7.8\text{ mm}$

$$d_f = \sqrt{8.0^2 + 7.8^2} = 11.17\text{ mm}$$

Image of filament

$$d_f' = m_c\, d_f = 65.8\text{ mm}$$

Projection Lens Diameter

The projection lens diameter should equal the size of the filament image:

$$d_p = d_f' = 65.8\text{ mm}$$

$$f/\#_p = 291\text{ mm} / 65.8\text{ mm} = 4.42$$

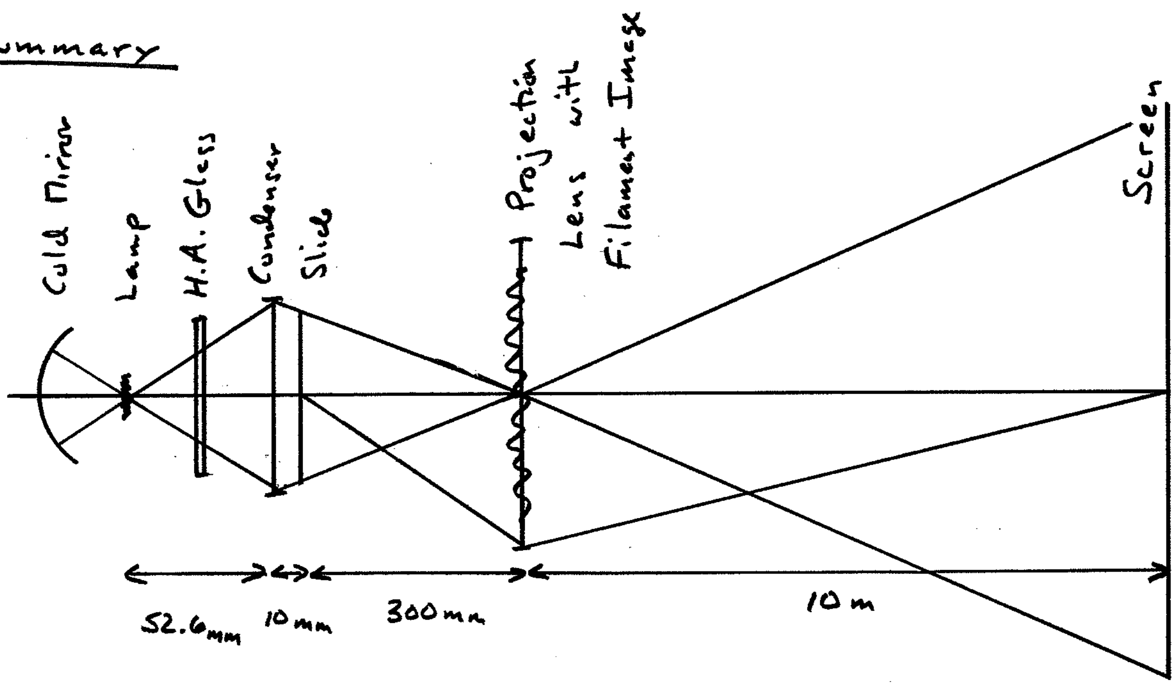
Additions

A concave cold mirror should be placed behind the filament, with the filament at the center of curvature.

$r_{c.m} \approx 2 - 3$ cm (or an alternate mirror configuration)

Heat absorbing glass should be placed between the lamp and the condenser lens. The lamp-condenser spacing will need to be increased slightly to accomodate the glass.

Summary



Condenser: $f_c = 45$ mm, $d_c = 45$ mm   f/1

Projection Lens: $f_p = 291$ mm, $d_p = 65.8$ mm   f/4.42

Note: Other choices could be made to obtain slightly different results

Let's repeat the design with a Faceted Reflector

For simplicity, let's model the source (including the reflector) as a uniform disk equal in size to the aperture of the reflector:

$$d_r \approx 4 \text{ cm}$$

The filament is placed at the focus of the nominally parabolic reflector so that most of the light is directed forward.

This source change does not change the focal length of the projection lens or the condenser diameter. It will change the required condenser focal length.

Since the source is much larger, let's image it at 1:1 magnification into the projector lens:

$$\text{Condenser} \rightarrow \text{Projection Lens} = 310\text{mm} = 2f_c \text{ for } 1:1$$

$$f_c = 155 \text{ mm}$$

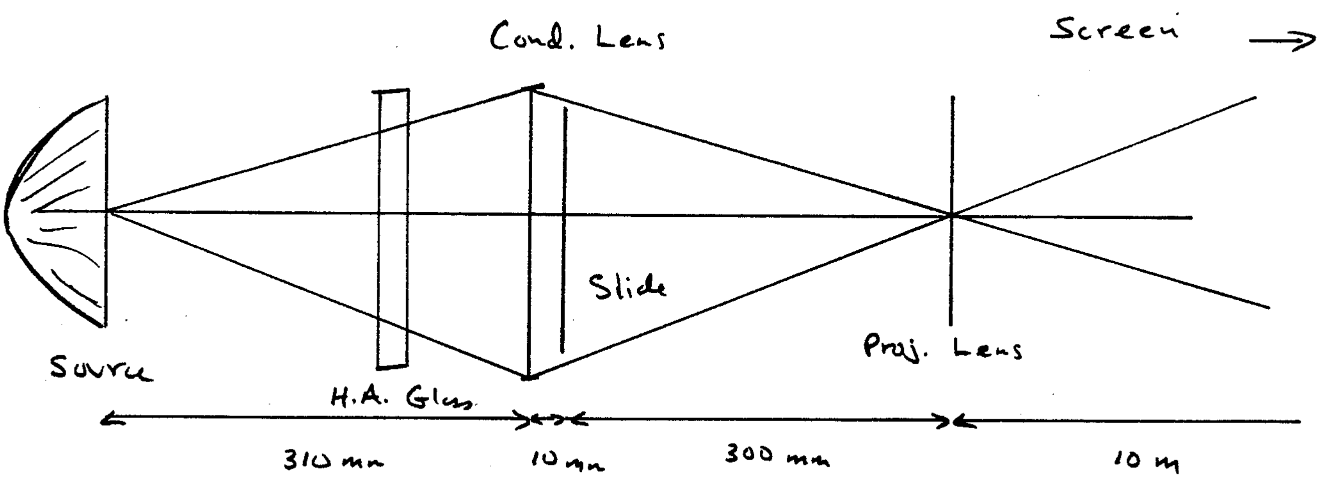
$$f/\# = 155/45 = f/3.4$$

This f/# is much slower than would normally be used. We are designing a system for a small, distant screen and need a long focal length projection lens ($f_p = 291$) A usual projection lens is about 100 mm f.l., and the 1:1 imaging of the source would result in a condenser f.l. of about 50 mm (or f/1) as expected.

The radiometry of this system with a f/3.6 condenser is not bad since the parabolic reflector collects the light over a very large solid angle and directs it through the slide and into the projection lens.

The source-condenser distance z' is also equal to 310 mm. The projection lens diameter is equal to the source size: $d_p = 4$ cm

$$f/\#_p = 291/40 = f/7.2$$



$f_c = 155$ mm
$d_c = 45$ mm
$f/\#_c = f/3.4$

$f_p = 291$ mm
$d_p = 40$ mm
$f/\#_p = f/7.2$

Heat management concerns as before.