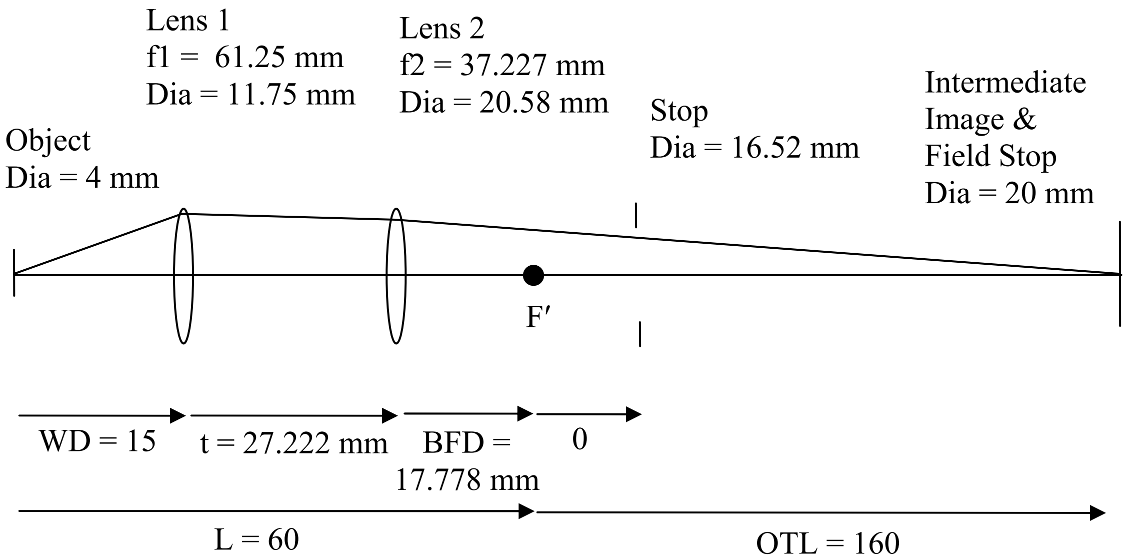
Microscope Objective – Solution Summary

Sections A and B



Objective Focal Length = 32 mm

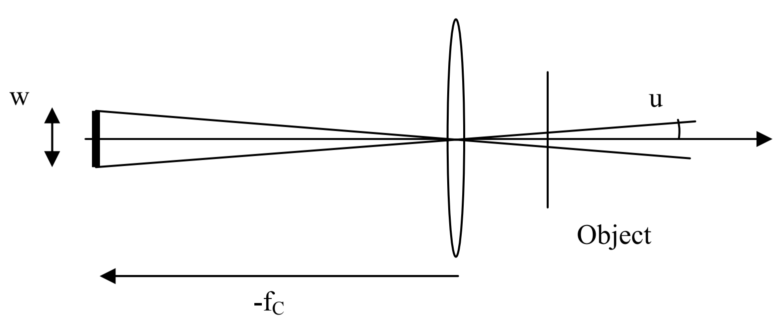
Principal Plane Locations: d = 23.4 mm d′ = -14.222 mm

Section C – Summary only; provide further discussion in the body of the solution.

Conditions: 1) The condenser lens must image the source into the EP or stop of the objective. The source must be located at the front focal point of the condenser.

2) The condenser must provide illumination that fills the NA of the objective. The angular spread of the illumination through the object equals the angular size of the source as seen by the objective.

Sketch:



$$\frac{w}{f_C} = 2u = 2\tan\left(\arcsin NA\right) \approx 2NA$$

Microscope Objective

5X
WD = Working Distance = 15 mm
NA = 0.25

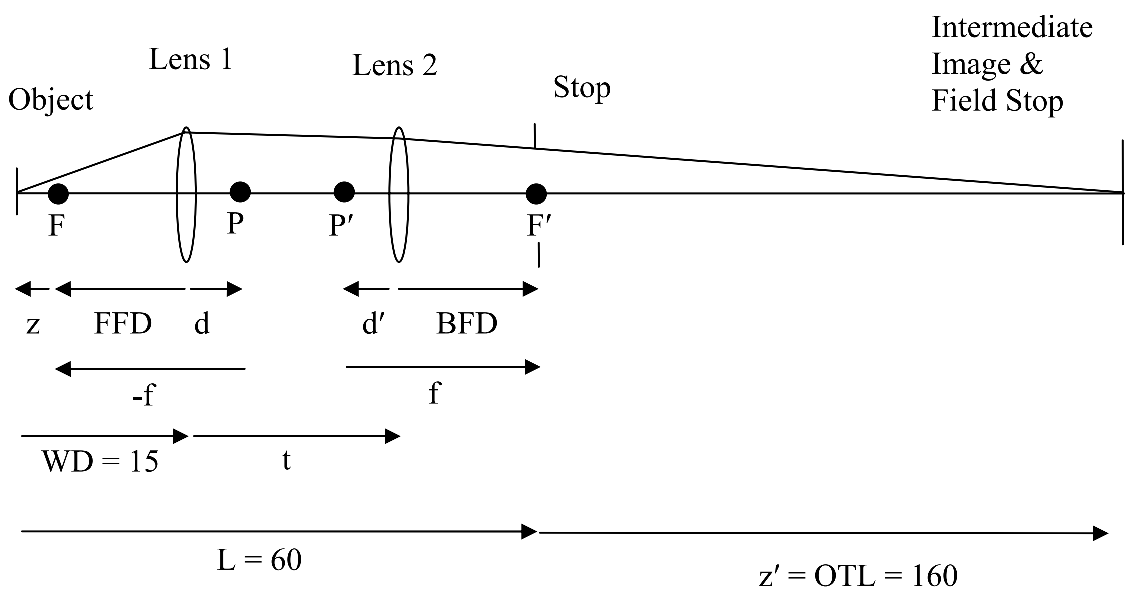
Telecentric in Object Space
OTL = 160 mm
L = Object to F′ Distance = 60 mm
Field Stop Diameter = 20 mm
Two Thin Lenses in Air

Section A – First Order Design

Use Newtonian Distances

For the system to be telecentric in object space, the system stop is located at the rear focal point of the two element objective.

Let f be the objective focal length.



5X implies m = -5

First, determine the focal length and the object distance:

$$z' = -mf = OTL \qquad z = \frac{f}{m}$$

$$f = \frac{OTL}{-m}$$

Now, determine the required principal plane shifts and relate those values to the focal lengths of the two lens elements:

$$FFD = -z - WD$$

$$FFD = -f + d$$
$$BFD = f + d'$$
$$WD = -z - FFD = -z + f - d$$

$$d = -WD - z + f$$

$$d = \frac{\phi_2}{\phi} t = \frac{f}{f_2} t$$

$$f_2 = \frac{ft}{d}$$

$$L = WD + t + BFD$$
$$L = WD + t + f + d'$$

$$d' = L - WD - t - f$$

$$d' = -\frac{\phi_1}{\phi} t = -\frac{f}{f_1} t$$

$$f_1 = -\frac{ft}{d'}$$

But t is not known yet to enable the evaluation of the value of d′. Use the Gaussian reduction relationship to relate the three focal lengths and the element spacing:

$$\phi = \frac{1}{f} = \phi_1 + \phi_2 - \phi_1 \phi_2 t$$
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2}$$
$$\frac{1}{f} = -\frac{d'}{ft} + \frac{d}{ft} + \frac{dd'}{f^2 t}$$

$$t = -d' + d + \frac{dd'}{f} \qquad d' = L - WD - t - f$$

$$t = -L + WD + t + f + d + \frac{d}{f}\left[L - WD - t - f\right]$$

$$0 = -L + WD + F + \frac{d}{f}\left[L - WD - t\right]$$

$$t = -\frac{Lf}{d} + \frac{WDf}{d} + \frac{f^2}{d} + L - WD$$

The above general expressions can be evaluated to give the system specifications:

$$f = 32\ mm$$

$$z = -6.4\ mm$$

$$FFD = -8.6\ mm$$

$$d = 23.4\ mm$$

$$t = 27.222\ mm$$

$$d' = -14.222\ mm$$

$$BFD = 17.778\ mm$$

$$f_1 = 61.25\ mm$$

$$f_2 = 37.227\ mm$$

Section B – Element Diameters

The field stop of 20 mm diameter defines the maximum image size. Since the objective magnification is -5, the maximum object size is also known:

$$h' = 10\ mm$$

$$h = \frac{h'}{m} = -2\ mm$$

Object Diameter $= 4\ mm$

The NA allows the marginal ray angle in object space to be determined:

$$NA = n \sin U = 0.25$$

$$U = .2527\ rad$$

$$u = \tan U$$

$$u = .2582$$

Since the system is telecentric in object space, the chief ray angle in object space is zero and the chief ray height equal h = -2 mm.

A marginal and chief ray can be traced through the system to determine the required aperture sizes. The ray trace is attached.

The stop radius is the marginal ray value at the stop:

$$y_{STOP} = 8.262\ mm$$

$$D_{STOP} = 16.52\ mm$$

The marginal and chief ray values are used along with the condition for no vignetting to determine the required lens element diameters:

$y_1 = 3.873\ mm$ $y_2 = 9.180\ mm$

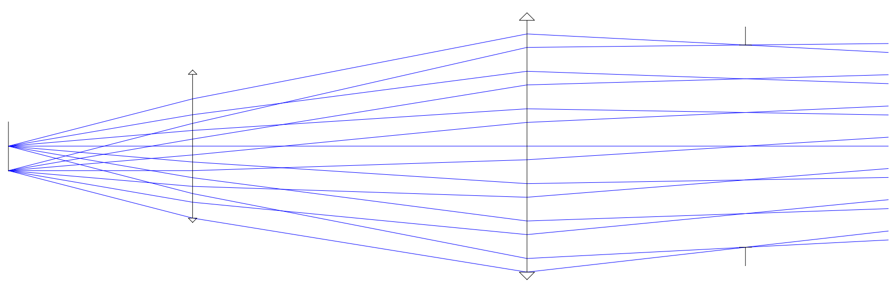
$\bar{y}_1 = -2\ mm$ $\bar{y}_2 = -1.111\ mm$

$$a_i = |y_i| + |\bar{y}_i|$$

$a_1 = 5.87\ mm$ $a_2 = 10.29\ mm$

$D_1 = 11.75\ mm$ $D_2 = 20.58\ mm$

A layout of the microscope objective is shown below:



Section C – Illumination Requirements

The condenser system must do two things:

- it must image the source into the EP or stop of the objective.
- the condenser must provide illumination that fills the NA of the objective.

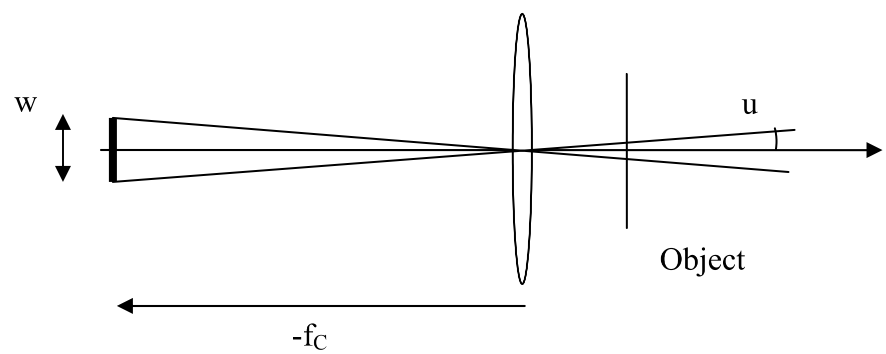
The first condition requires that the source be located at the front focal point of the condenser. Each point on the source will produce a collimated beam through the object which will be imaged at the rear focal point of the objective. Since the system is telecentric, this is the location of the system stop.

The second condition relates to the source size. The angular spread of the illumination through the object equals the angular size of the source as seen by the objective.

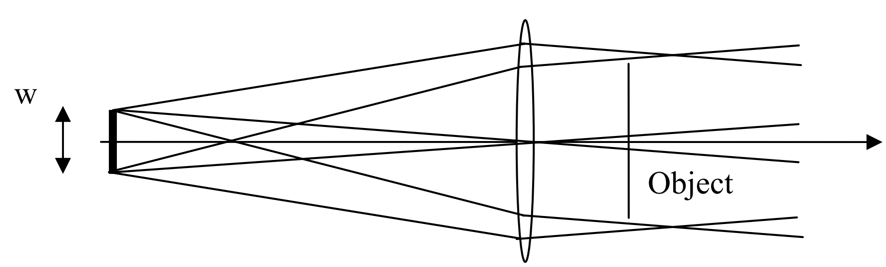
If the source diameter is w and the condenser focal length is f_C, the ratio of these two values is related to the NA of the objective (and the marginal ray angle).

$$u = \tan\left(\arcsin NA\right) \qquad n = 1$$

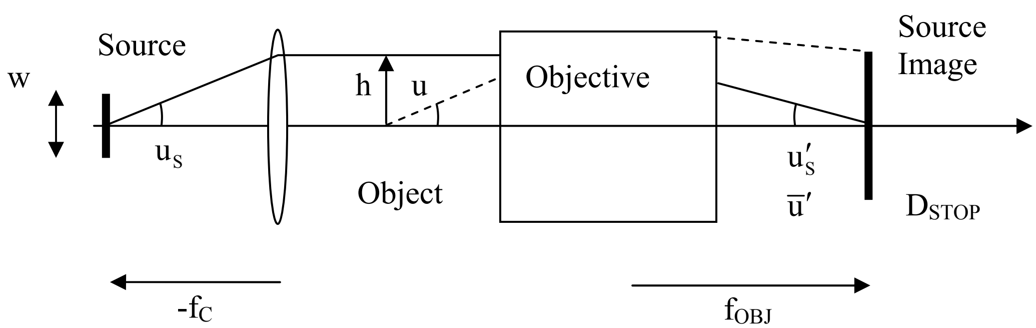
$$\frac{w}{f_C} = 2u = 2\tan\left(\arcsin NA\right) \approx 2NA$$



At first look, it may appear that the object needs to be in close proximity to the condenser lens, but this is not the case. The angular spread of light through the object is independent of the object location. Each point on the source provides a different angle of illumination through the object. Of course, as the separation between the object and the condenser lens increases, the required diameter of the condenser lens also increases.



Another way of meeting the condition of having the illumination fill the NA of the objective is to match the source image size to the stop diameter.



The Lagrange invariant allows the magnification to be determined by the marginal ray angles of the source imaging:

$$m_{SOURCE} = -\frac{D_{STOP}}{w} = \frac{u_S}{u'_S} \qquad u_S = \frac{h}{f_C} \qquad u'_S = -\frac{h}{f_{OBJ}}$$

$$m_{SOURCE} = -\frac{f_{OBJ}}{f_C} \qquad \frac{w}{f_C} = \frac{D_{STOP}}{f_{OBJ}}$$

This last result can be correctly evaluated, but the more general relationship to the NA of the objective is not clear. Note that the marginal ray of the source image is the same ray as the chief ray for the image of the object. The Lagrange invariant relates the source size to the stop size:

$$hu = -\bar{u}' D_{STOP} / 2 \qquad \bar{u}' = u'_S = -\frac{h}{f_{OBJ}}$$

$$hu = \frac{hD_{STOP}}{2 f_{OBJ}} \qquad \frac{D_{STOP}}{f_{OBJ}} = 2u$$

Returning to the above relationship between w and f_C:

$$\frac{w}{f_C} = \frac{D_{STOP}}{f_{OBJ}} = 2u$$

$$\frac{w}{f_C} = 2u = 2\tan\left(\arcsin NA\right) \approx 2NA$$

This derivation makes it even clearer that the relationship between w and f_C is independent of the relative location of the object and the condenser lens.

YNU Method

	Object 0	Lens 1	Lens 2	Stop 3	Image 4	5	6	7	8	9
f ∅		61.25	37.227	—						
t	15	27.222	17.778	160						
n										
-φ		-.016327	-.026862							
t/n	15	27.222	17.778	160						
Marginal —										
y	0	3.873	9.180	8.262	0					
nu										
u	.2582	.1950	-.0516	-.0516						
Chief										
$\bar{y}$	-2	-2	-1.111	0	10.00					
nu										
$\bar{u}$	0	.0327	.0625	.0625						
y										
nu										
u										