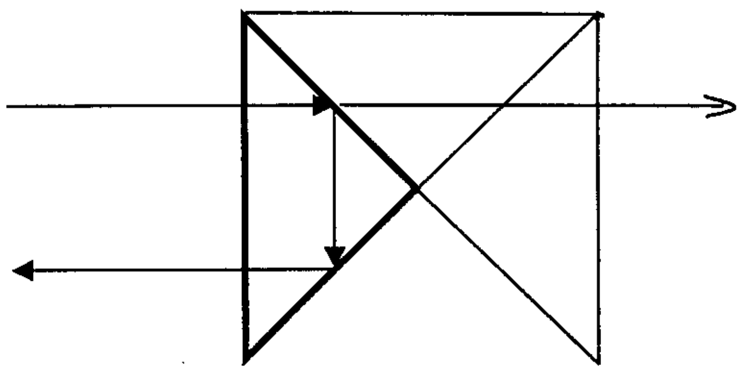
October 26, 2000 Lecture 20 Name Solutions

Closed book; closed notes. Use the back sides if required.
Do not use any pre-stored information or programs in your calculator.
Note any assumptions you make in solving the problems.
Show your work. Present it in a neat and logical fashion.

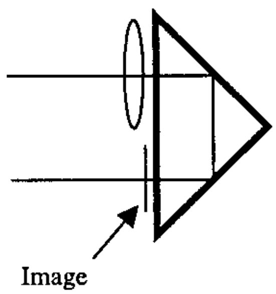
1) (15 points) Given the following right angle prism and ray path:



a) Draw the tunnel diagram. What is the resulting image parity?

Right-Handed (2 reflections - no parity change)

b) This prism is to be placed behind a thin lens to image an object at infinity. The lens covers only half of the entrance face of the prism. What is the shortest focal length lens that will result in an image just outside the prism?



Prism index = 1.5
Entrance face is 50 mm
The other sides are 35.35 mm

Tunnel Diagram Length = $t = 50\,mm$

Reduced Length = $t/n = \tau$

$\tau = 33.33\ mm$

Shortest focal length $f = \tau$

$f_{min} = 33.33\ mm$

2) (10 points) A thin lens of index 1.7 has a rear focal length of 100 mm when immersed in water (n = 1.33). What is the focal length of this lens in air?

In water:

$$\frac{1}{\phi_w} = f_e = f'/n_w \qquad f' = 100 \text{ mm} \quad n_w = 1.33$$

$$\phi_w = 0.0133/\text{mm}$$

For a thin lens:

$$\phi_w = (n - n_w)\Delta C \qquad n = 1.7$$

$$\Delta C = \frac{0.0133/\text{mm}}{(1.7 - 1.33)} = 0.0359/\text{mm}$$

ΔC doesn't change with medium.

In air:

$$\phi_A = (n - n_A)\Delta C \qquad n_A = 1.0$$

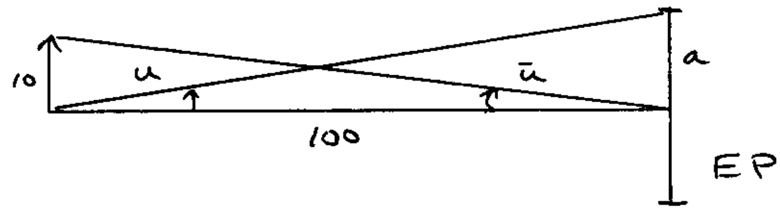
$$\phi_A = 0.0252/\text{mm}$$

$$f_A = 1/\phi_A = 39.7 \text{ mm}$$

3) (10 points) The Lagrange invariant of an optical system in air is -1.

a) A 10 mm high object is located 100 mm to the left of the entrance pupil of the system. What is the diameter of the entrance pupil?

$$\mathcal{K} = n\bar{u}y - nu\bar{y}$$



Object: $y = 0$ $\quad u = a/100$
$\bar{y} = 10$

$\mathcal{K} = -nu\bar{y} = -1$

$\frac{-10a}{100} = -1$

$a = 10$

$D_{EP} = 20$

or

EP: $y = a$ $\quad \bar{u} = \frac{-10}{100} = -.1$
$\bar{y} = 0$

$\mathcal{K} = n\bar{u}y = -1$

$-.1a = -1$

$a = 10$

$D_{EP} = 20$

b) If the entire optical system is scaled longitudinally by a factor of two, what is the new value of the Lagrange invariant? This scaling changes all distances measured along the optical axis by a factor of two.

The object distance change to 200.

Both u and $\bar{u}$ are reduced by 2.

The Lagrange invariant also is reduced by 2

$$\mathcal{K} = -1/2$$

4) (20 points) Do the first order design of an optical system comprised of two thin lenses with the following specifications:

Focal length	-100 mm
Back focal distance	200 mm
Focal length of the first element	100 mm

Provide the focal length of the second element and the element spacing. Sketch the system showing a marginal ray for an object at infinity. The system has a negative focal length yet produces real images. Give a brief explanation of this "curious" system.

$f' = f_E = -100$ mm

$BFD = f' + d'$ $\qquad 200 = -100 + d'$

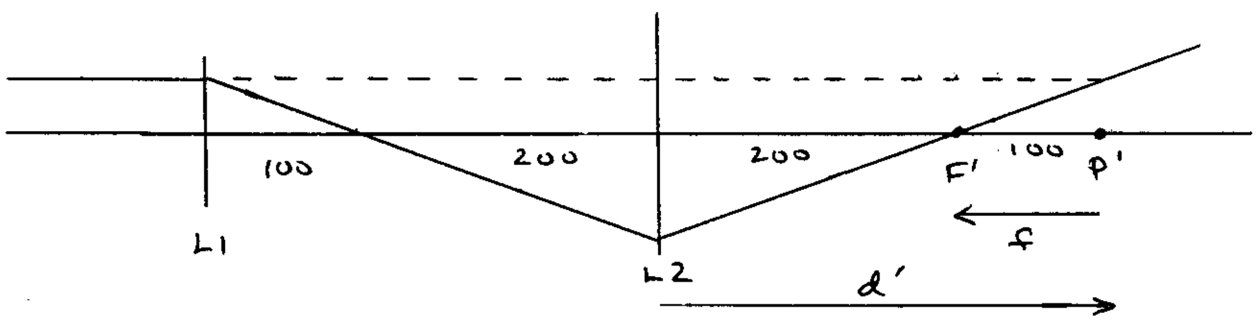
$d' = 300$ mm

$d' = -\frac{\phi_1}{\phi} t$ $\qquad \phi_1 = 1/100 \qquad \phi = -1/100$

$\underline{t = d' = 300 \text{ mm}}$

$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t$ $\qquad \phi_2 = \frac{\phi - \phi_1}{1 - \phi_1 t} = 1/100$

$\underline{f_2 = 100 \text{ mm}}$



The second element acts as a 1:1 relay, changing the angle of the marginal ray in image space. This forces the rear principal plane beyond the focal point.

5) (20 points) The scene illuminance produced by an average-size electronic flash mounted on a camera, as a function of the object distance d, is

$$E_v = 10^7 \text{ lm} / d^2$$

This is, of course, the inverse square law. The duration of the flash is 0.0001 sec. The camera has an f/4 lens, and we can assume that the object distance is much greater than the focal length of the lens. The flash is the only source of light, and we can use the usual assumptions about the scene.

If the detector in the camera requires a minimum exposure of 0.1 mcs, what is the maximum object distance that will produce a properly exposed image?
(mcs = meter-candle-second = lm-sec/m^2)

Assumptions: Lambertian $\rho = .18$

$$L_v = \frac{\rho E_v}{\pi} = \frac{.18 \; 10^7 \, lm/d^2}{\pi} = 5.7 \times 10^5 \, lm/d^2 \, sr$$

Image Plane:

$$E_v = \frac{\pi L_v}{4 (f/\#)^2} = \frac{\pi L_v}{4 (4)^2} = 2.8 \times 10^4 \, lm/d^2$$

Exposure: $\Delta t = .0001$ sec

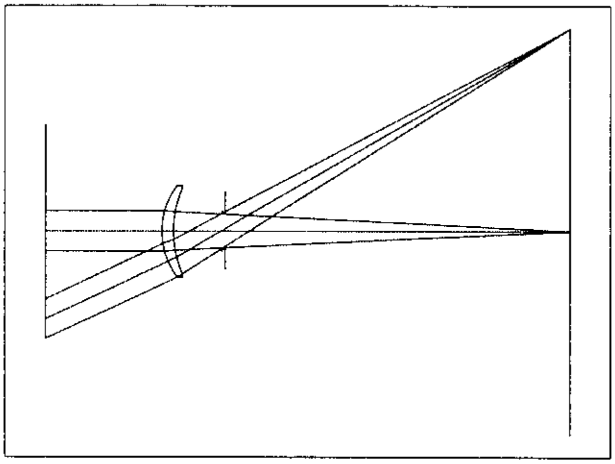
$$Exp = .1 \text{ mcs} = E_v \Delta t = 2.8 \times 10^4 \, lm/d^2 \cdot .0001 \text{ sec}$$

$$.1 \, lm\text{-}sec/m^2 = 2.8 \, lm\text{-}sec/d^2$$

$$d^2 = 28 \, m^2$$

$$d = 5.3 \, m$$

6) (25 points) A landscape lens is a common lens configuration for inexpensive and single use cameras. This lens consists of a positive meniscus element with a separated stop. The lens is relatively slow, but its aberrations are well balanced over its field of view.



For the purpose of this problem, we can consider the lens element to be a thin lens. Since the thickness of the thin lens element is zero, and we are only interested in the first order properties of the system, we do not need to discuss the bending of the lens. (The bending of the lens corrects the aberrations of the system.)

Focal length	38 mm
Lens to stop separation	5 mm
Stop diameter	3 mm
Maximum image height	+/- 18 mm

(to match the long dimension of a 35 mm negative)

a) Determine the location and size of the entrance pupil. What is the f/# of this system?

Stop is to the right of the lens

$\frac{-1}{z'} = \frac{-1}{z} + \frac{1}{f}$ $z = 5\text{mm}$

$z' = 5.76\text{ mm}$ (to the right of the lens)

$m = z'/z = 1.15$ $D_{EP} = 3.45\text{mm}$

$f/\# = f/D_{EP} = f/11$

b) What is the required lens diameter for the system to be unvignetted over this field of view?

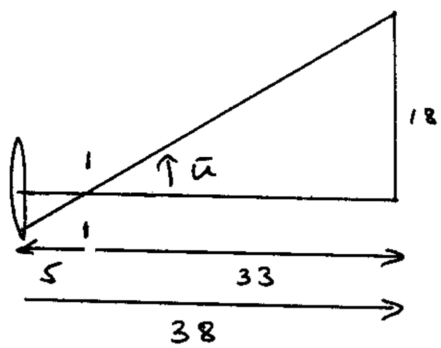
Need the marginal and chief ray heights at the lens.

$$y = D_{EP}/2 = 1.73\,mm$$

At the stop:

$$\bar{u} = \frac{18\,mm}{38\,mm - 5\,mm}$$

$$\bar{u} = .545$$



At the lens:

$$\bar{y} = -5\,\bar{u}$$

$$\bar{y} = -2.73\,mm$$

For unvignetted

$$a \geq |y| + |\bar{y}|$$

$$a \geq 4.46\,mm$$

Dia Lens > $\underline{8.9\ mm}$