

October 16, 2001 Lecture 17

Name Solutions

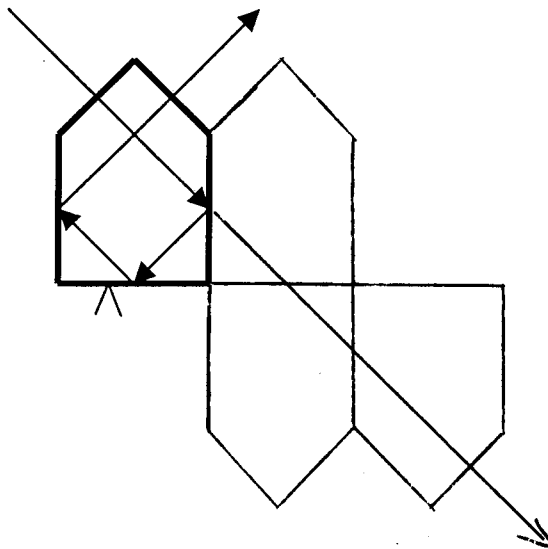
Closed book; closed notes. Use the back sides if required.

Do not use any pre-stored information or programs in your calculator.

Note any assumptions you make in solving the problems.

Show your work. Present it in a neat and logical fashion.

1) (10 points) Draw the tunnel diagram and determine the resulting image parity for this prism and the ray path shown. The bottom surface of the prism is a roof.



4 reflections -

no parity change.

2) (15 points) A 50 mm focal length thin lens is used to image a 250 x 250 mm object onto a 10 x 10 mm detector. What is the overall object-to-image distance?

$$m = \frac{h'}{h} = -\frac{10}{250} = -0.04$$

$$m = \frac{z'}{z} \quad z' = mz$$

Exact: $\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$

$$\frac{1}{mz} - \frac{1}{z} = \frac{1}{f}$$

$$z = \left(\frac{1-m}{m}\right) f$$

$$z = -26 f = -1300 \text{ mm}$$

$$z' = mz = 52 \text{ mm}$$

$$L = z' - z = \underline{1352 \text{ mm}}$$

Approx: z is many times f

$$z' \approx f$$

$$z = \frac{z'}{m} \approx \frac{f}{m}$$

$$L = z' - z \approx f \left(1 - \frac{1}{m}\right)$$

$$L \approx 26 f$$

$$L \approx 1300 \text{ mm}$$

This approximation gets better as $|z|$ increases.
or as $|m|$ decreases.

3) (15 points) A 100 mm focal length thin lens is used to image an object at infinity.

a) If the object has a total angular subtense of 10 milliradians, what is the image size?

$$\text{FOV} = .010 \text{ rad}$$

This is a small angle:

$$\text{HFOV} = .005 \text{ rad}$$

$$\bar{u} = \text{HFOV} = .005 \text{ rad}$$

$$\bar{u} = \tan(.005 \text{ rad}) = .005$$

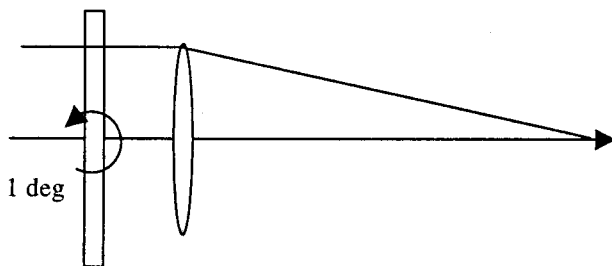
$$h' = \bar{u} f = .5 \text{ mm}$$

$$\underline{D = 1.0 \text{ mm}}$$

Since the object is at infinity, the result is independent of stop position.

(The FOV is measured at the EP).

b) The lens is looking through a 10 mm thick glass plate ($n=1.5$). What is the image shift if the plate is tilted by 1 degree?



No Shift

- Transmitted rays through the plate are parallel to the incident rays.

- Rays parallel to the axis focus at F' (on axis)

Explanation: The tilt of the plate will shift the object by some fraction of the plate thickness. Since the object is at infinity this small lateral shift of the object cannot be seen. This is an important result for windows and filters.

- An image shift will occur for finite conjugate objects.

Spacing: by triangles and $f/\#$'s

$$\text{Primary: } f/\# = 1 = \frac{-(f_p' - x)}{D_s} \quad D_s = -(f_p' - x)$$

$$\text{System: } f/\# = 5 = \frac{100 - x}{D_s} \quad D_s = 20 - x/5$$

$$x - f_p' = 20 - x/5$$

$$1.2x = 20 + f_p' = -180 \text{ mm}$$

$$\underline{x = -150 \text{ mm}}$$

Secondary: Images intermediate image (primary) to

final image: $n = -1$ $n' = 1$

$$z = f_p' - x = -50 \text{ mm} \quad z' = 100 - x = 250 \text{ mm}$$

$$\frac{n'}{z'} = \frac{n}{z} + \phi_{\text{sec}} \quad \phi_{\text{sec}} = -.0160$$

$$\frac{1}{250} = \frac{-1}{-50} + \phi_{\text{sec}} \quad \phi_{\text{sec}} = -2nC = -2n/R_{\text{sec}}$$

$$R_{\text{sec}} = z/\phi_{\text{sec}}$$

$$\underline{R_{\text{SEC}} = -125 \text{ mm}}$$

c) Exit Pupil is the image of the primary by the secondary:

$$z = -x = 150 \text{ mm} \quad n = -1 \quad n' = 1$$

$$\frac{n'}{z'} = \frac{n}{z} + \phi_{\text{SEC}} \quad \frac{1}{z'} = \frac{-1}{150} + \phi_{\text{SEC}} = \frac{-1}{150} - .016$$

$$\underline{z' = -44.11} \quad (\text{to the left of the secondary})$$

$$m = \frac{z'/n'}{z/n} = \frac{-44.11}{-150} = .294$$

$$D_{\text{XP}} = m D_{\text{EP}} = \underline{58.8 \text{ mm}}$$

Alternate solution for t and R_{SEC} by Gaussian Optics:

Let n_2 be the intermediate index $n_2 = -1$

$$\phi_1 = \frac{1}{f_p} = .005 \quad \phi = \frac{1}{f_e} = .001 \quad f_e = 1000 \text{ mm}$$

$$z = \frac{t}{n_2} = -t$$

$$d' = \text{BFD} - f_e \quad (\text{BFD from secondary})$$

$$\text{BFD} = -t + 100$$

$$d' = -t + 100 - 1000 = -t - 900$$

$$d' = -\frac{\phi_1}{\phi} \frac{t}{n_2} = \frac{\phi_1}{\phi} t$$

$$-t - 900 = \frac{\phi_1}{\phi} t = \frac{.005}{.001} t = 5t$$

$$6t = -900$$

$$\underline{t = -150 \text{ mm}}$$

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 z = \phi_1 + \phi_2 + \phi_1 \phi_2 t$$

$$.001 = .005 + \phi_2 + .005 \phi_2 (-150)$$

$$-.004 = \phi_2 (1 - .75)$$

$$\phi_2 = -.0160 \quad \leftarrow \text{secondary}$$

$$\phi_2 = -2n_2 c = -2n_2 / R_{\text{SEC}}$$

$$R_{\text{SEC}} = 2 / \phi_2$$

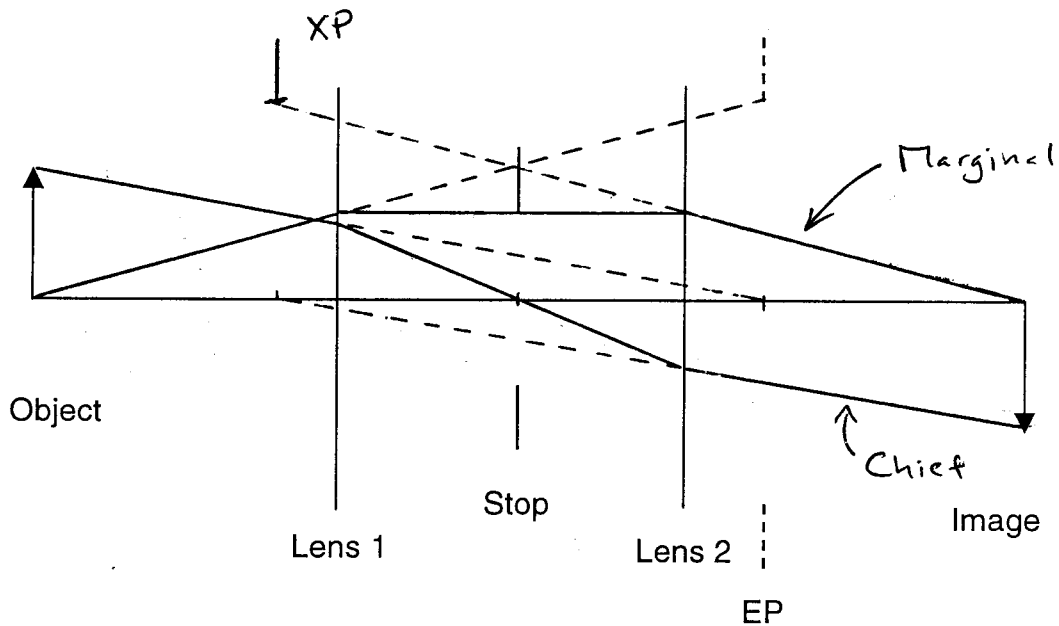
$$\underline{R_{\text{SEC}} = -125 \text{ mm}}$$

5) (15 points) Below is a representation of an optical system showing the locations and sizes of the object, image, two lenses, stop and entrance pupil.

Draw the marginal and chief rays for this system.

Show the location and size of the exit pupil.

This is a graphical problem; no calculations are required.



XP: location - chief ray crosses the axis
size - marginal ray height

6) (15 points) Design a Petzval lens (two positive thin lenses) with a system focal length of 100 mm and a back focal distance of 75 mm. The spacing between the two lens elements is 50 mm. Sketch the system noting the position of P' and F' .

Petzval: P' located between the lenses.

$$\phi = \frac{1}{f_e} = .01/\text{mm} = \phi_1 + \phi_2 - \phi_1 \phi_2 t$$

$$d' = \text{BFD} - f' = \text{BFD} - f_e = 75 - 100 = -25 \text{ mm}$$

$$d' = -25 \text{ mm} = -\frac{\phi_1}{\phi} t \quad t = 50 \text{ mm}$$

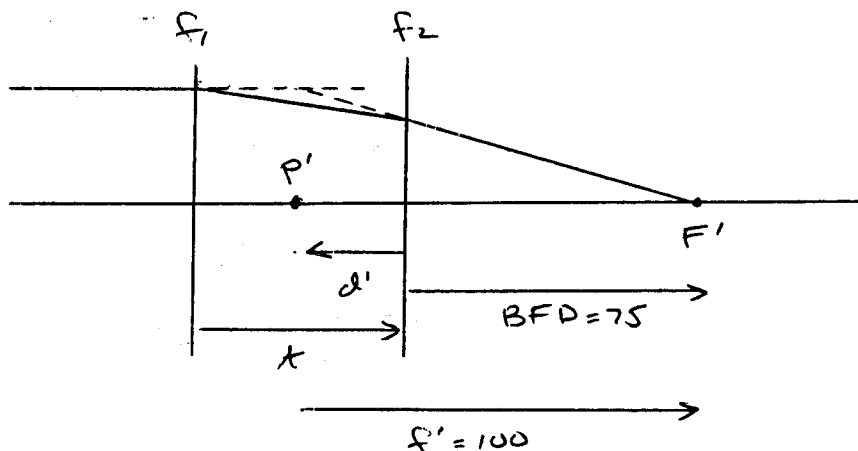
$$\phi_1 = .005/\text{mm}$$

$$f_1 = \underline{200 \text{ mm}}$$

$$\phi = .01/\text{mm} = \phi_1 + \phi_2 - \phi_1 \phi_2 t$$

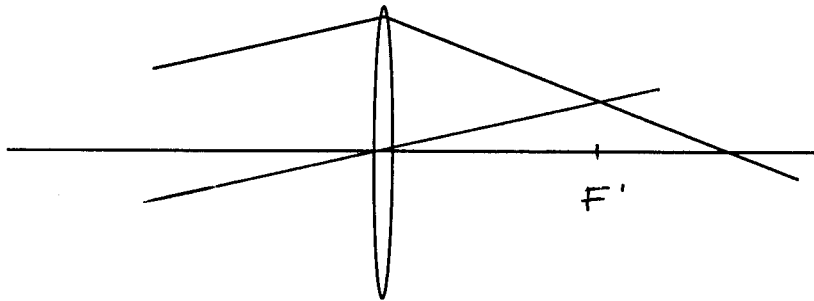
$$\phi_2 = .00666$$

$$f_2 = \underline{150 \text{ mm}}$$



$$t = 50 \quad d' = -25$$

Extra Credit (10 points) The figure shows a ray path through a thin lens. Draw a single line on the figure to locate the rear focal point of the lens. Explain your result.



Parallel rays in object space focus
in the rear focal plane

For a thin lens, the nodal points are
at the lens and this ray is
not deviated.