

October 22, 2002 Lecture 17

Name Solutions

Closed book; closed notes. An equation sheet attached and can be removed.

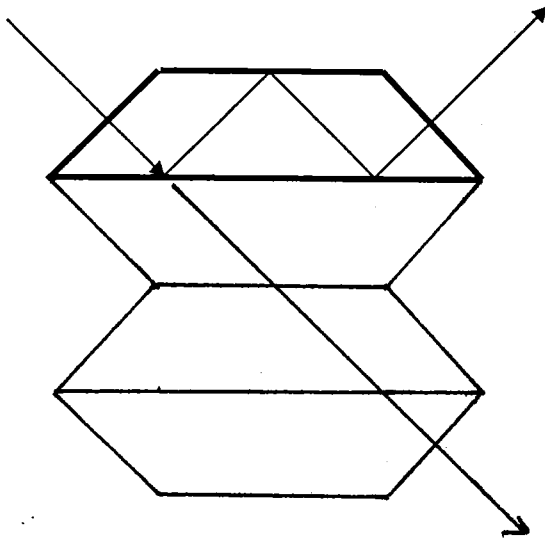
Use the back sides if required.

Do not use any pre-stored information or programs in your calculator.

Note any assumptions you make in solving the problems.

Show your work. Present it in a neat and logical fashion.

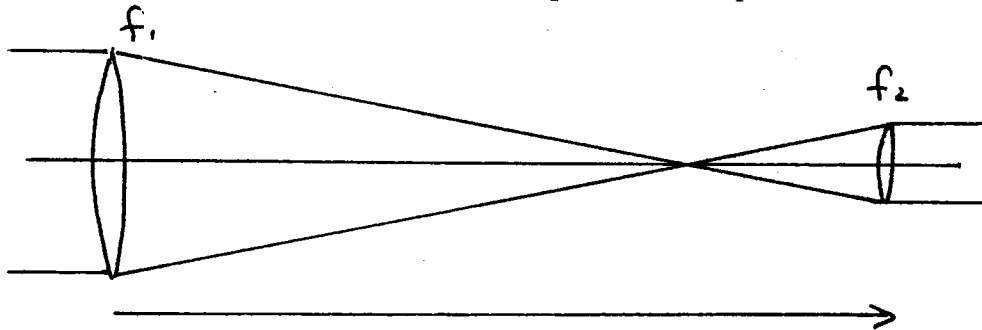
1) (10 points) Draw the tunnel diagram and determine the resulting image parity for this prism and the ray path shown.



3 reflections
- odd parity

2) (10 points) An afocal system in air is constructed out of two positive thin lenses. The separation between the two lenses is 220 mm, and the longitudinal magnification of the system is $\bar{m} = 0.01$. Determine the focal lengths of the two lenses.

This type of afocal system is often called a Keplerian telescope.



$$L = f_1 + f_2 = 220 \text{ mm}$$

$$\bar{m} = m^2 = .01$$

$$m = \pm 0.1$$

must be negative for two pos lenses

$$m = -\frac{f_2}{f_1} = -0.1$$

$$f_2 = 0.1 f_1$$

$$L = f_1 + f_2 = f_1 + 0.1 f_1 = 220 \text{ mm}$$

$$f_1 = 200 \text{ mm}$$

$$f_2 = 20 \text{ mm}$$

3) (15 points) A detector requires a total radiant input of 1 microwatt (10^{-6} Watt) in order to produce a useful signal. The square detector is 2 cm x 2 cm. An $f/2$ lens is used to image a large distant extended object onto the detector. The image of this object overfills the detector.

Assume the object is Lambertian and uniform. The detector integrates all of the light falling on it to produce a single output; it does not produce an image.

What is the required radiant exitance (W/m^2) of the object?

Detector: $\Phi_0 = 10^{-6} \text{ W}$ (required)

$$A_D = (20 \text{ mm})^2 = 400 \text{ mm}^2$$

$$E_D = \frac{\Phi_0}{A_D} = .0025 \text{ W}/\text{m}^2$$

Scene:

$$M_s = \pi L_s$$

Transfer:

$$E_D = \frac{\pi L_s}{4 (f/\#)^2}$$

$$E_D = \frac{M_s}{4 (f/\#)^2}$$

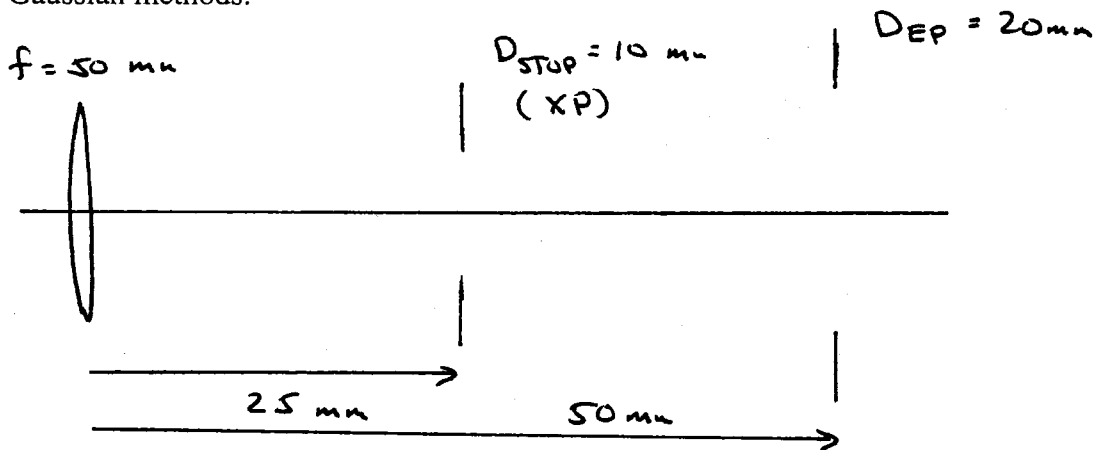
$$M_s = 4 (f/\#)^2 E_D \quad f/2$$

$$M_s = 0.04 \text{ W}/\text{m}^2$$

or

$$M_s = 0.04 \text{ } \mu\text{W}/\text{mm}^2$$

4) (10 points) A 10 mm diameter stop is located 25 mm to the right of a 50 mm focal length thin lens in air. Determine the size and location of the entrance and exit pupils. Use Gaussian methods.



XP: The stop is the XP. $D_{\text{XP}} = 10 \text{ mm}$

EP: The stop is a real object

$$\frac{n'}{z'} = \frac{n}{z} + \frac{1}{f}$$

$$z = 25$$

Light from R \rightarrow L

$$n' = n = -1$$

$$-\frac{1}{z'} = \frac{-1}{25} + \frac{1}{50}$$

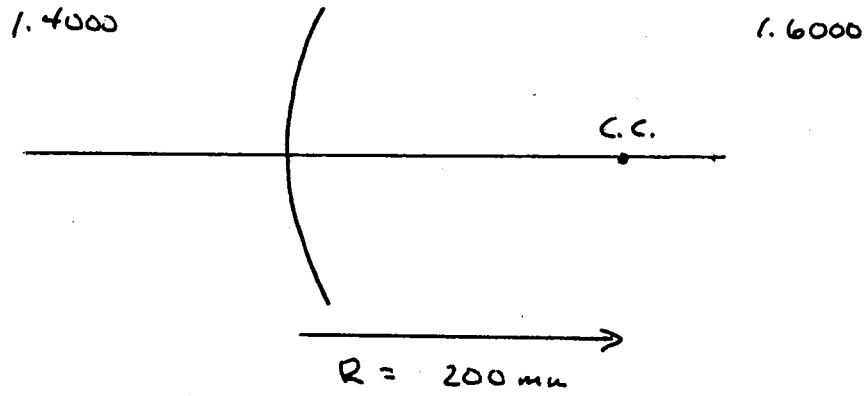
$$z' = 50 \text{ mm}$$

(to the right of the lens)

$$m = \frac{z'/n'}{z/n} = 2$$

$$D_{\text{EP}} = m D_{\text{STOP}} = \underline{20 \text{ mm}}$$

5) (10 points) A convex refractive surface with a radius of 200 mm separates media with indices of 1.4000 (to the left of the surface) and 1.6000 (to the right of the surface). Determine the power of the surface, and the front and rear focal lengths.



$$\phi = (n' - n) C = (n' - n) / R$$

$$\phi = (1.6000 - 1.4000) / 200 \text{ mm}$$

$$\phi = 0.001 / \text{mm}$$

$$f_e = 1 / \phi = 1000 \text{ mm}$$

$$f_F = -n f_e = -1400 \text{ mm}$$

$$f_R' = n' f_e = 1600 \text{ mm}$$

6) (20 points) The Anti-Principal Planes are defined to be the conjugate planes with a unit *negative* lateral magnification ($m = -1$).

An optical system in air is formed with two identical thin lenses of focal length f separated by a distance t .

a) Determine the location of the rear anti-principal plane of the system relative to the second lens. Use Gaussian methods.

Be sure to fully simplify your result. (Hint: Evaluate the result for $t = 0, f, 3f$.)

System Power: $\phi_s = \phi_1 + \phi_2 - \phi_1 \phi_2 t$

$$\phi_1 = \phi_2 = 1/f$$

$$\phi_s = (2 - t/f)/f$$

$$f_s = 1/\phi_s$$

P':

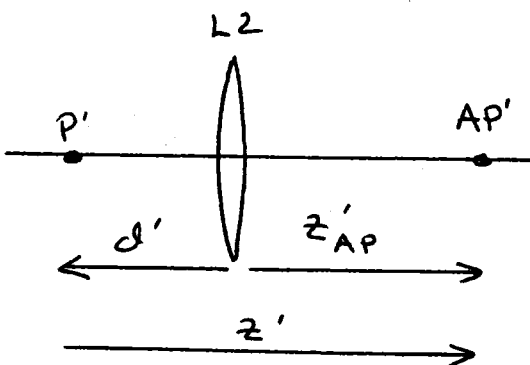
$$d' = -\frac{\phi_1}{\phi_s} t = \frac{-t}{(2 - t/f)}$$

Anti-Principal Plane AP':

$$m = z'/z = -1 \quad z = -z'$$

$$\frac{1}{z'} = \frac{1}{z} + \phi_s \quad z' = \frac{z}{\phi_s} = \underline{2f_s}$$

(But we knew this - it must be a $2f_s - 2f_s$ system)



$$z'_{AP} = z' + d'$$

(continues on next page...)

$$z'_{AP} = z' + d' = 2f_s + d'$$

$$z'_{AP} = \frac{2f}{(2 - t/f)} + \frac{-t}{(2 - t/f)}$$

$$z'_{AP} = \frac{2f - t}{2 - t/f} = f \frac{2f - t}{2f - t}$$

$$z'_{AP} = f$$

The rear anti-principal plane is located at the rear focal point of the second lens - the result is independent of t .

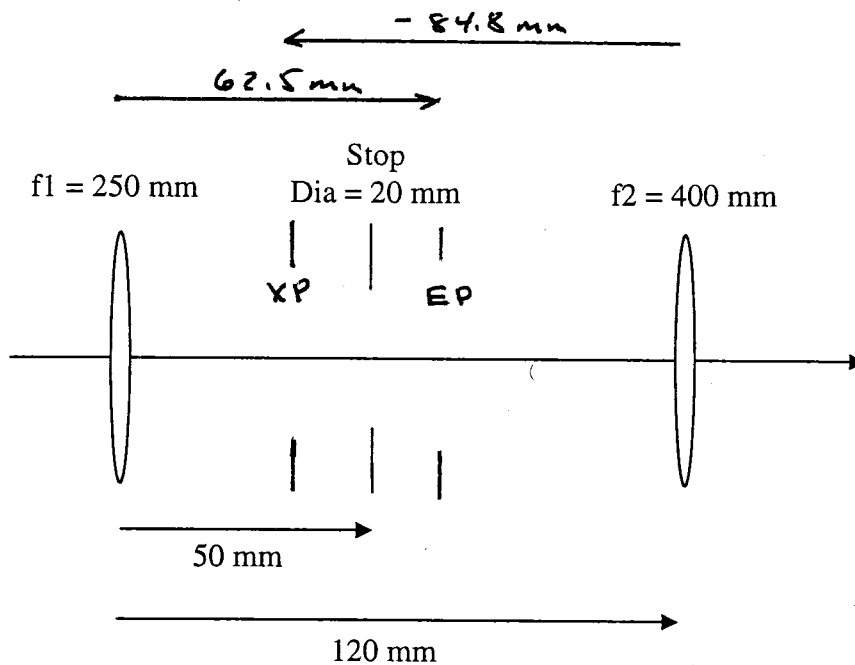
By symmetry, the front anti-principal plane of the system is located at the front principal plane of the first lens.

b) What happens when $t = 2f$? Is the above result valid?

- When $t = 2f$, the system is afocal and does not have principal planes or anti-principal planes.
- The result is not valid because of a division by zero in the derivation.
- This afocal system does, however, have a lateral magnification $m = -1$.

7) (20 points) Given the following optical system in air, use paraxial raytrace methods to determine the location and size of the entrance pupil and the exit pupil. Both lenses are thin lenses.

A raytrace sheet is on the next page. Report your answers on this page and sketch the locations/sizes on the figure.



EP: 62.5 mm to the right of L1

$$D_{EP} = 25 \text{ mm}$$

XP: 84.8 mm to the left of L2
(35.2 mm to the right of L1)

$$D_{XP} = 24.24 \text{ mm}$$

EP L1 Stop L2 XP
 0 1 2 3 4

f t n	250		-		400	
	?	50	70	?		
	1.0	1.0	1.0	1.0		

* φ	-0.004		-		-0.0025	
	-62.5	50	70	-84.8		

①

y	0	-5.0	0	7.0	0
nu	.08	.1*	.1*	.0825	
u					

②

y	12.5	10	10	10	12.12
nu	0.04	0*	0*	-0.025	
u					

y					
nu					
u					

① Arbitrary ray through the center of the stop.

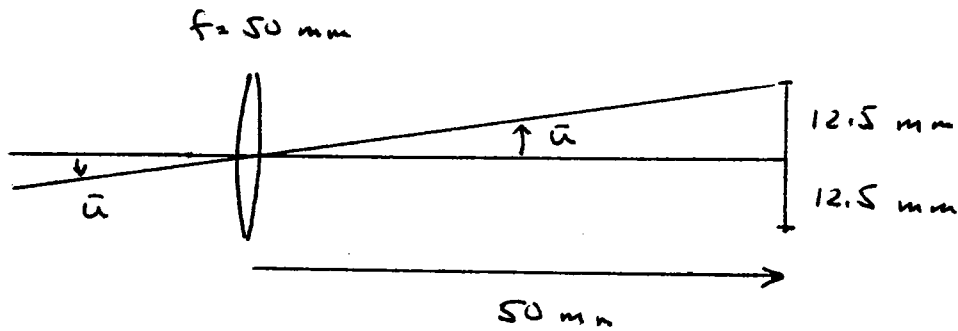
② Ray through the top of the stop (height = 10)

* arbitrary

$r_{EP} = 12.5 \text{ mm}$

$r_{XP} = 12.12 \text{ mm}$

8) (10 points) A 25 mm diameter sensor is used with a 50 mm focal length thick lens to image a distant scene. What is the angular FOV of the system?



$$\bar{u} = \frac{h'}{f} = \frac{12.5 \text{ mm}}{50 \text{ mm}}$$

$$\bar{u} = 0.25$$

$$\text{HFOV} = \tan^{-1}(\bar{u})$$

$$\text{HFOV} = 14.04^\circ$$

$$\text{FOV} = 28.07^\circ$$

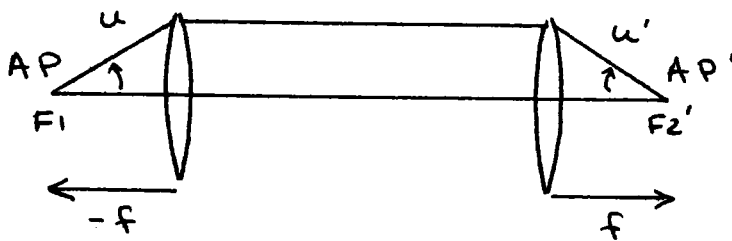
Extra Credit (10 points) Knowing the result for problem #6 (anti-principal planes), can you provide a *simple* explanation/proof of the result?

I'll be the judge of simple.....Just a few sentences or phrases, a single figure and no more than an equation or two.

AP is at F of Lens 1

AP' is at F' of Lens 2

Sketch a "marginal" ray. The light will be collimated between the lenses (independent of t)



$$u = -u'$$

$$m = \frac{w}{w'} = \frac{u}{u'} = -1$$