

October 23, 2002 Lecture 18

Name Solutions

Closed book; closed notes. An equation sheet is attached and can be removed.

Use the back sides if required.

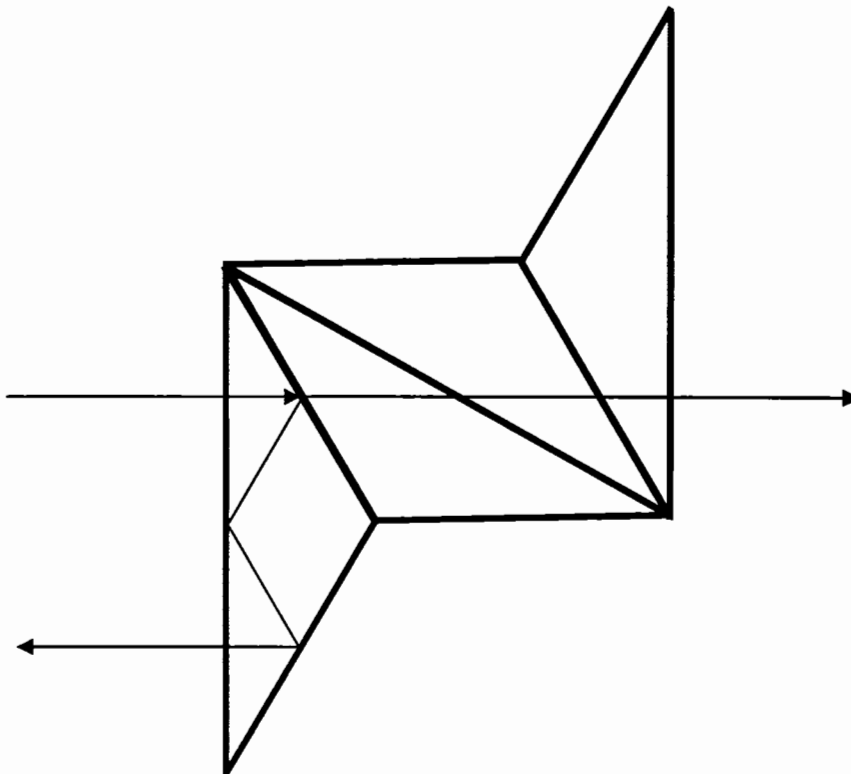
Do not use any pre-stored information or programs in your calculator.

Note any assumptions you make in solving the problems.

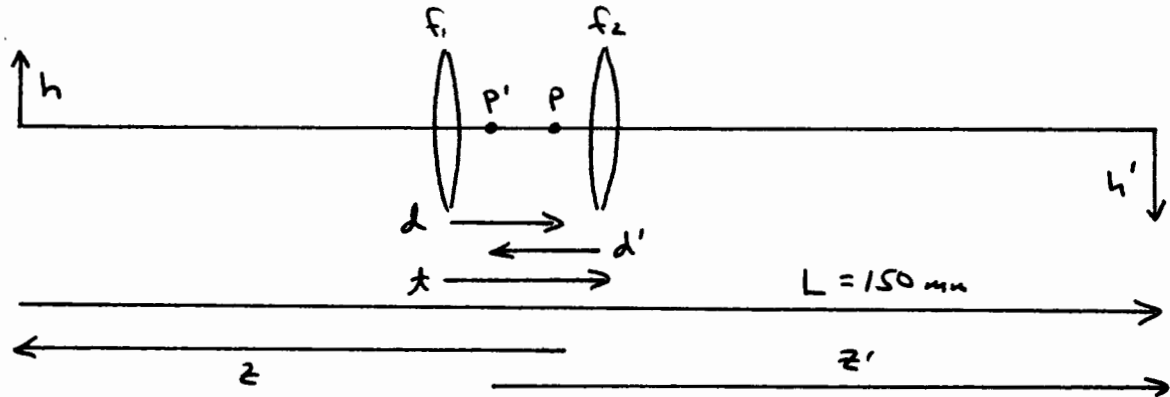
Show your work. Present it in a neat and logical fashion.

Distance Students: Please return the original exam only; do not FAX an additional copy.

1) (10 points) Draw the tunnel diagram for this prism and the ray path shown.



2) (30 points) You have a lens assembly made up of two identical thin lenses separated by 25 mm, and the focal length of each thin lens is 50 mm. Fixed object and image planes are separated by 150 mm. What are the two possible image magnifications that can be obtained with your lens assembly?



$$f_1 = f_2 = 50 \text{ mm}$$

$$t = 25 \text{ mm}$$

$$\phi_1 = \phi_2 = .02/\text{mm}$$

$$d = \frac{\phi_2}{\phi} t \quad d' = -\frac{\phi_1}{\phi} t$$

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t$$

$$\phi = .03/\text{mm}$$

$$d = -d' = \frac{\phi_2}{\phi} t$$

$$F = 33.333 \text{ mm}$$

$$d = 16.667 \text{ mm}$$

$$d' = -16.667 \text{ mm}$$

Object to image distance:

$$L = -(z+d) + (z'+d') + t = -z + z' + \overline{PP'}$$

$$L = -z + z' - 8.333 \text{ mm}$$

$$\overline{PP'} = -8.333 \text{ mm}$$

$$L' = -z + z' = L + 8.333 \text{ mm}$$

$$L' = -z + z' = 158.333 \text{ mm}$$

continues.....

Imaging

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

$$z' = L' + z$$

$$L' = 158.333$$

$$\frac{1}{L' + z} = \frac{f + z}{z f}$$

$$f = 33.333$$

$$(L' + z)(f + z) = z f$$

$$z^2 + L' z + f L' = 0$$

$$z = \frac{-L' \pm \sqrt{L'^2 - 4L'f}}{2}$$

$$z = -47.709 \text{ mm} \quad \text{or} \quad -110.624 \text{ mm}$$

$$z' = 110.624 \text{ mm} \quad \text{or} \quad 47.709 \text{ mm}$$

$$m = z'/z$$

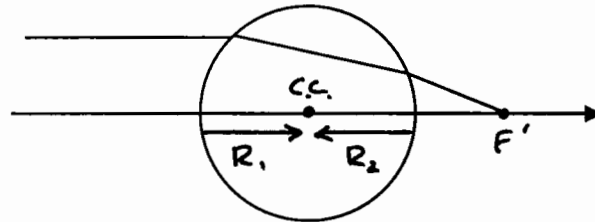
$$m = \underline{-2.319} \quad \text{or} \quad \underline{-.431}$$

Note that these are reciprocal magnifications

$$m_1 = \frac{1}{m_2}$$

3) (15 points) A spherical crystal ball has a diameter of 50 mm and an index of refraction of 1.5. The ball is used as a lens (in air).

Use paraxial raytrace methods to determine the power of the ball, its back focal distance, and the location of the rear principal plane. A raytrace sheet is on the next page.



$R_1 = 25 \text{ mm}$   
 $R_2 = -25 \text{ mm}$   
 $t = 50 \text{ mm}$   
 $n = 1.5$

			$F'$	
	0	1	2	3
C t n		.04	-.04	
	$\infty$	50	?	
	1.0	1.5	1.0	
$-\phi$ t/n		-.02	-.02	
	$\Delta$	33.333	12.5	
y nu u	1	1	.3333	0
	0	-.02	-.026667	

BFD = 12.5 mm

$\phi = -\frac{u'}{y'} = -\frac{u'}{y'} = +.026667$

$d' = \text{BFD} - f_R'$

$\phi = .026667/\text{mm}$

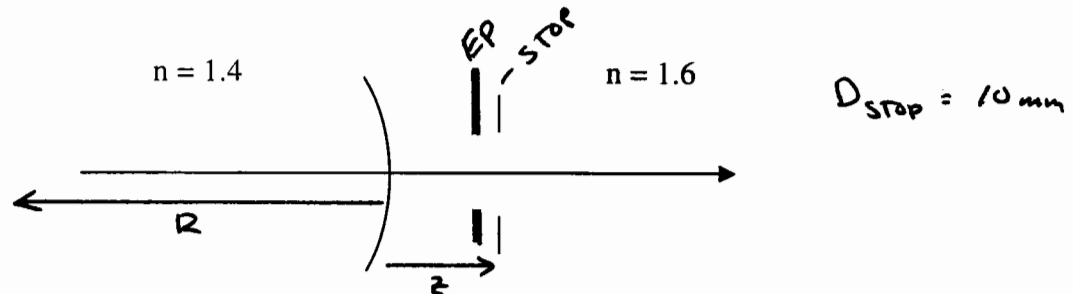
$d' = -25.0 \text{ mm}$

$f = f_R' = 37.50 \text{ mm}$

$P'$  is at the center of curvature of the ball. This result could be found by inspection since both nodal points of the ball must be at its center of curvature.

4) (15 points) A concave single refracting surface of radius 100 mm separates indices of refraction 1.4 and 1.6. A 10 mm diameter stop is located 25 mm to the right of the surface. Determine the size and location of the entrance pupil.

Use Gaussian methods only.



Surface Power:

$$R = -100 \text{ mm}$$

$$\phi = (n' - n) / R = (1.6 - 1.4) / -100$$

$$\phi = -0.002 / \text{mm}$$

Imaging the Stop - light from Right  $\rightarrow$  Left

for this imaging

$$n = -1.6$$

$$n' = -1.4$$

$$\frac{n'}{z'} = \frac{n}{z} + \phi$$

$$z = 25 \text{ mm}$$

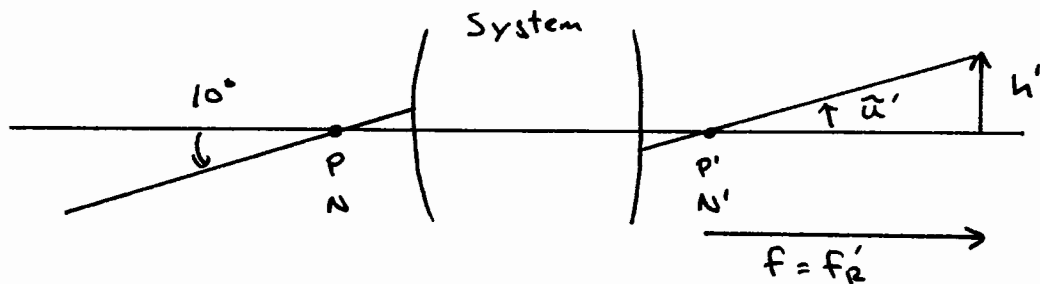
$$\frac{-1.4}{z'} = \frac{-1.6}{25} - 0.002$$

$$\underline{z' = 21.21 \text{ mm}} \quad \text{to the right of the surface}$$

$$m = \frac{z'/n'}{z/n} = \frac{21.21/-1.4}{25/-1.6} = 0.9696$$

$$\underline{D_{EP} = m D_{STOP} = 9.696 \text{ mm}}$$

5) (10 points) An optical system images a distant scene onto a 10 mm diameter detector. The angular field of view of the system is  $\pm 10^\circ$ . What is the required focal length for this FOV to fill the detector?



The nodal ray is parallel to the object space chief ray (distant object)

$$\bar{\alpha}' = \tan(10^\circ) = .1763$$

$$\bar{\alpha}' = h'/f$$

$$h' = 5 \text{ mm}$$

$$f = h'/\bar{\alpha}'$$

$$\underline{f = 28.36 \text{ mm}}$$

There is no need to assume a thin lens with a stop at the lens.

6) (10 points) The image in the eye is formed in an index of refraction of 1.336. The rear focal length of the eye is 22.4 mm. An object is 1 m in front of the eye (in air) and has a height of 20 mm. What is the height of the image formed in the eye (on the retina)?

Assume that the eye changes length to keep the image in focus.

$$f_R' = 22.4 \text{ mm} \quad n' = 1.336$$

$$f = f_R' / n' = 16.77 \text{ mm}$$

$$\phi = .0596 / \text{mm}$$

Imaging:  $z = -1000 \text{ mm} \quad n = 1.00$

$$\frac{n'}{z'} = \frac{n}{z} + \phi = -.001 + .0596$$

$$\frac{n'}{z'} = .0586 / \text{mm}$$

$$m = \frac{z'/n'}{z/n} = \frac{1/.0586}{-1000} = \underline{\underline{-.0171}}$$

$$h' = m h \quad h = 20 \text{ mm}$$

$$\underline{\underline{h' = -.341 \text{ mm}}} \quad (\text{inverted})$$

7) (10 points) Returning to our spherical ball lens of the earlier problem (diameter = 50 mm;  $n = 1.5$ ). A physical aperture stop with a diameter of 10 mm has been imbedded in the glass and is located at the center of curvature of the ball. Determine the locations and sizes of the entrance pupil and the exit pupil.

Use the method of your choice. A raytrace sheet is attached on the next page if needed.

By Inspection - Both nodal points of a single surface are at its C.C. The pupils are therefore located at the C.C. with the magnification of the nodal points.

$$m_N = \frac{n}{n'} \quad n = 1.5 \text{ (where the stop is)}$$

$$n' = 1.0$$

$$m_N = 1.5$$

$$\underline{D_{EP} = D_{XP} = 15 \text{ mm}}$$

Gaussian - do the XP

$$z = -25 \text{ mm} \quad n = 1.5 \quad n' = 1.0 \quad R = -25 \text{ mm}$$

$$\phi = (n' - n)/R = .02 / \text{mm}$$

$$\frac{n'}{z'} = \frac{n}{z} + \phi$$

$$\underline{z' = -25 \text{ mm}} \quad (\text{at the C.C.})$$

$$m = \frac{z'/n'}{z/n} = 1.5$$

$$\underline{D_{XP} = 15 \text{ mm}}$$

By symmetry, the EP has the same location and size.

$$D_{EP} = 15 \text{ mm} \quad \text{at C.C.}$$

continues....

Ray Trace

	EP 0	1	STOP 2	3	XP 4
C		.04	-	-.04	
t		?	25	25	?
n		1.0	1.5	1.5	1.0

$-\phi$		-.02	-	-.02	
t/n		-25	16.667	16.667	-25

## Pseudo Chief Ray

y	0	-1.6667	0	1.6667	0
nu		.06666	.1*	.1*	.06666
u					

## Pseudo Marginal Ray

y	7.5	5	5	5	7.5
nu		.1	0*	0*	-.1
u					

\* arbitrary

EP: 25mm to the right of surface 1

XP: 25mm to the left of surface 2

- both at C.C.

$$D_{EP} = D_{XP} = 15 \text{ mm}$$