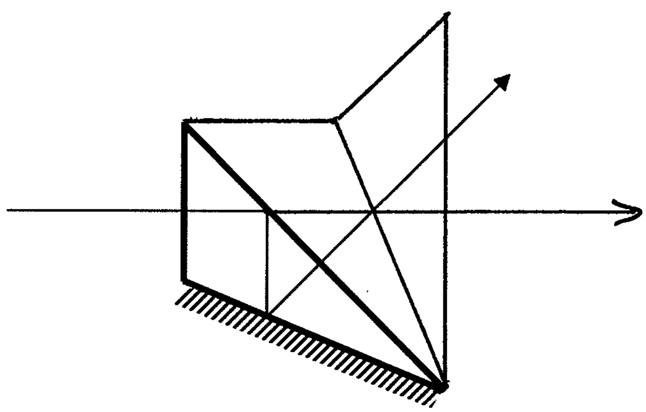
October 11, 2007 Lecture 16 Name Solutions

Closed book; closed notes. Time limit: 75 minutes.
An equation sheet is attached and can be removed.
Use the back sides if required.
Do not use any pre-stored information or programs in your calculator.
Note any assumptions you make in solving the problems.
Show your work. Present it in a neat and logical fashion.
If a method of solution is specified, that method must be used.

Distance Students: Please return the original exam only; do not FAX or scan an additional copy.

1) (10 points) Draw the tunnel diagram for this 45 degree deviation prism with the ray path shown.



2) (20 points) An afocal system is constructed out of two positive thin lenses. The first lens has a focal length of 200 mm, and the magnitude of the lateral magnification is 0.1:

$$|m| = 0.1 \qquad \text{Two Positive Lenses} \rightarrow m = -0.1$$

a) Determine the focal length of the second lens and the spacing between the two lenses.

$$m = -\frac{f_2}{f_1} = -\frac{f_2}{200} = -0.1$$

$$f_2 = 20\ \text{mm}$$

$$t = f_1 + f_2 = 220\ \text{mm}$$

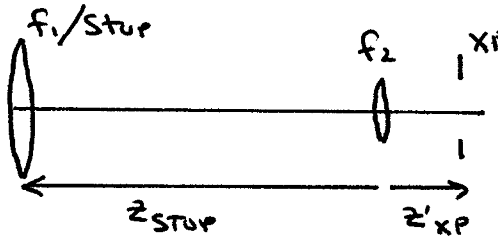
f_2 = 20 mm

t = 220 mm

b) The first lens serves as the aperture stop of this afocal system, and the diameter of the first lens (or stop) is 50 mm. Determine the locations and diameters of the Entrance Pupil and the Exit Pupil. Use Gaussian methods.

EP: is at the first lens with the same diameter. $D_{EP} = 50\ \text{mm}$

XP: Image the stop (first lens) through the second lens.



$$z_{STOP} = -t = -220\ \text{mm}$$

$$\frac{1}{z'_{XP}} = \frac{1}{z_{STOP}} + \frac{1}{f_2}$$

$z'_{XP} = 22$ mm to the right of f_2

$$m_{XP} = \frac{z'_{XP}}{z_{STOP}} = \frac{22\ \text{mm}}{-220\ \text{mm}} = -0.1$$

$$D_{XP} = |m_{XP}|\, D_{STOP} = 0.1 \cdot 50\ \text{mm}$$

$$D_{XP} = 5.0\ \text{mm}$$

or

Use the m of the afocal system:

$$D_{XP} = |m|\, D_{EP}$$

$$D_{XP} = 5.0\ \text{mm}$$

EP: D_{EP} = 50 mm; Located 0 mm to the —— of the first lens.

XP: D_{XP} = 5.0 mm; Located 22 mm to the Right of the second lens.

3) (10 points) A 3 m tall elephant is to be imaged onto a 1 cm detector. The elephant is about 30 m away, and the image of the elephant fills the detector. Approximately what focal length lens is required?

$$m = -\frac{1\ \text{cm}}{3\ \text{m}} = -\frac{1}{300}$$

$$z = -30\ \text{m}$$

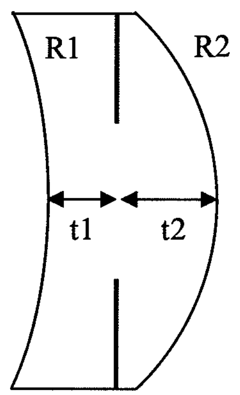
$$m = \frac{z'}{z} = \frac{z'}{-30\ \text{m}} = -\frac{1}{300}$$

$$z' = 0.1\ \text{m} = 100\ \text{mm}$$

$$f \approx z' = 100\ \text{mm}$$

$f \approx$ 100 mm

4) (30 points) A plano-concave lens and a plano-convex lens have been glued together to create a thick lens. When the lenses were glued togeter, the system stop was placed between the two lenses. The lens is used in air.



R1 = -100 mm

R2 = -50 mm

t1 = 10 mm

t2 = 15 mm

n = 1.5 (both lenses)

Stop Diameter = 20 mm

a) Use Gaussian methods to determine the location and diameter of the Entrance Pupil of the system.

Front Surface: $\phi_1 = (1.5 - 1.0)/R_1$ $\phi_1 = -0.005/mm$

EP: Image the stop into object space.
Light going from Right to Left.

$$\frac{n'}{z'_{EP}} = \frac{n}{z_{STOP}} + \phi_1$$

$z_{STOP} = t_1 = 10$ mm

$n = -1.5$ $n' = -1.0$
- For EP imaging

$$\frac{-1}{z'_{EP}} = \frac{-1.5}{10} - 0.005$$

$z'_{EP} = 6.45$ mm
to the Right of V_1

$$m_{EP} = \frac{z'_{EP}/n'}{z_{STOP}/n} = \frac{-6.45}{10/-1.5}$$

$m_{EP} = 0.968$ $D_{EP} = m_{EP} D_{STOP} = 19.35$ mm

EP: D_{EP} = 19.35 mm; Located 6.45 mm to the Right of the front vertex.

b) Use paraxial raytrace methods to determine the system Focal Length, the Back Focal Distance, the Exit Pupil location, and the Exit Pupil daimeter. Note that the results from part (a) are not needed for this solution.

A potential chief ray must be traced from the stop to the XP.

	Obj	R1	Stop	R2	XP	F'	
Surface	0	1	2	3	4	5	6
~~C~~ R		-100	–	-50			
t	∞	10	15	z'_{XP}	$z'_{XP\to F'}$		
n	1.0	1.5	1.5	1.0	1.0		
				-11.11			
$-\phi$		0.005	–	-0.01	–		
t/n	∞	6.667	10.0	z'_{XP}	$z'_{XP\to F'}$		

Potential Chief Ray - zero at Stop and XP

	0	1	2	3	4	5	6
$\bar{y}$			0	1.0	0		
$n\bar{u}$		0.1*	0.1*	0.09			
u							

Potential Marginal Ray (196.8)

	0	1	2	3	4	5	6
$\tilde{y}$	1	1	1.0333	1.0833	1.148	0	
$n\tilde{u}$	0	0.005	0.005	-.005833	-.005833		
u							

Marginal Ray: Scale to Stop Radius $r_{STOP}/\tilde{y}_{STOP} = 10/1.0333 = 9.678$

	0	1	2	3	4	5	6
y	9.678	9.678	10.0	10.48	11.11	0	
nu	0	.0484	.0484	-.0565	-0.565		
u							

	0	1	2	3	4	5	6
y							
nu							
u							

	0	1	2	3	4	5	6
y							
nu							
u							

$z'_{XP} = -11.11$ $\qquad z'_{XP\to F'} = 196.8$

$n'u' = -0.565$ $\qquad y_1 = 9.678$

$n'\tilde{u}' = -.005833$ $\qquad \tilde{y}_1 = 1.0$

Continues...

* arbitrary

$y_{XP} = 11.11$

$$\phi = -\frac{n'u'}{y_1} = -\frac{n'\bar{u}'}{\bar{y}_1} = .005833/mm$$

$$f = 1/\phi = 171.4 \text{ mm}$$

$$BFD = z'_{XP \to F'} + z'_{XP} = 196.8 \text{ mm} - 11.11 \text{ mm}$$

$$BFD = 185.7 \text{ mm}$$

XP: $z'_{XP} = -11.11$ (to the left of V_2)

$$r_{XP} = y_{XP} = 11.11 \text{ mm}$$

$$D_{XP} = 2 r_{XP} = 22.22 \text{ mm}$$

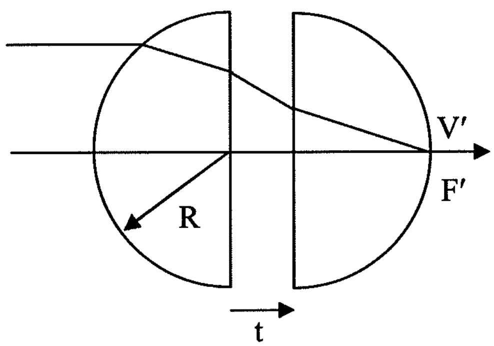
Focal Length = 171.4 mm BFD = 185.7 mm

XP: D_{XP} = 22.22 mm; Located 11.11 mm to the Left of the rear vertex.

$z'_{XP} = -11.11$

5) (20 points) In the homework, we found that a sphere with an index of 2.0 is needed for light from a distant object to be focused by the front surface of the sphere onto the opposite side of the sphere. Unfortunately an index of 2.0 is difficult to obtain.

A solution to obtain this same effect is to split a sphere into two identical hemispheres. What separation between the hemispheres is required to have the light from a distant object focus on the vertex of the second hemisphere? F′ is to be coincident with V′. The radii of the hemispheres are 50 mm, their indices are 1.5, and the hemispheres are in air.



$\phi_1 = (1.5 - 1.0)/R$

$\phi_1 = 0.01/mm$

The curved surface of the second hemisphere has no optical effect for this problem.

Several solution methods possible.

Solution 1 Determine the BFD of the first hemisphere.

$\phi_2 = 0$ $\phi = \phi_1 = 0.01/mm$ $f = f_R' = 1/\phi = 100$ mm

For the first hemisphere, $n' = 1$

$$d' = -\frac{\phi_1}{\phi}\frac{t}{n} = -\frac{t}{n} = -\frac{R}{n} = -\frac{50}{1.5} = -33.33 \text{ mm}$$

$$BFD = f_R' + d' = 100\text{mm} - 33.33\text{ mm} = 66.67 \text{ mm}$$

The second hemisphere looks just like a block of glass of thickness R placed after the first hemisphere. Use reduced thickness: For F′ at V′:

$$BFD = t + \frac{R}{n} = 66.67 \text{ mm}$$

$$t = 66.67 - 33.33 = 33.33\text{mm}$$

Required separation t = 33.33 mm

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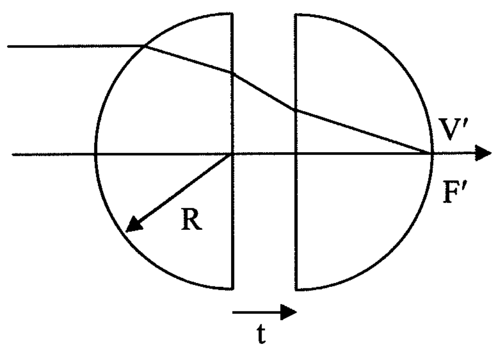

$\phi_1 = (1.5 - 1.0)/R$

$\phi_1 = 0.01/mm$

Solution 2

Consider this situation to be a single refracting surface with a inserted "block" of air.

$f_1 = 1/\phi_1 = 100\ mm$ Here $n' = 1.5$ for the refracting surface.

$f_R' = n f_1 = 150\ mm$

For F′ at V′: $f_R' = R + nt + R = 150\ mm$

$nt = 50\ mm$ $t = 33.33\ mm$

or solve in reduced thicknesses:

$f = f_R'/n = R/n + t + R/n = 100\ mm$

$t = 33.33\ mm$

Required separation t = 33.33 mm

6) (10 points) An optical system in air is comprised of two thin lenses:

$f_1 = 50$ mm
$f_2 = -50$ mm
$t = 20$ mm

Use Gaussian methods to determine the system Focal Length, the Back Focal Distance, and the location of the Rear Principal Plane.

$\phi_1 = 1/50 = 0.02\,/mm$ $\phi_2 = -1/50 = -0.02/mm$

$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t = -\phi_1 \phi_2 t$

$\phi = 0.008\,/mm$

$f = 1/\phi = 125$ mm

$d' = -\dfrac{\phi_1}{\phi} t = -50$ mm

(to the Left of L_2)

$BFD = f + d' = 75$ mm

Focal Length = 125 mm BFD = 75 mm

Rear Principal Plane is located 50 mm to the Left of the second lens.

$d' = -50$ mm