

October 20, 2009 Lecture 17

Name Solutions

Closed book; closed notes. Time limit: 75 minutes.

An equation sheets is included. A spare raytrace sheet is also attached

Use the back sides if required.

Do not use any pre-stored information or programs in your calculator.

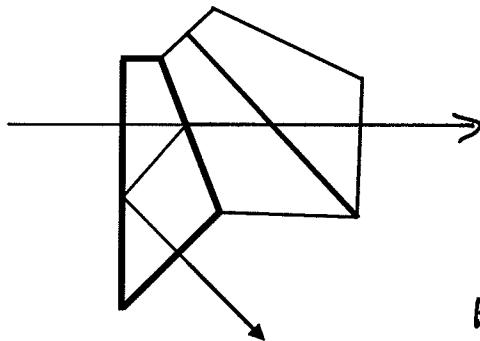
Assume thin lenses in air if not specified.

If a method of solution is specified in the problem, that method must be used.

You must show your work and/or method of solution in order to receive credit or partial credit for your answer.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy.

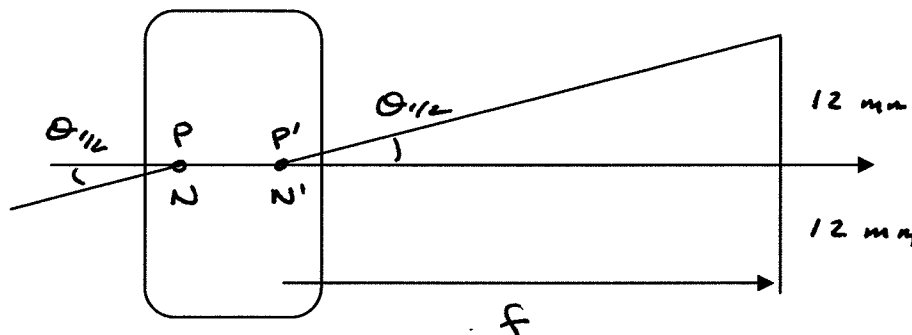
1) (10 points) Draw the tunnel diagram for this 45 degree deviation prism with the ray path shown. The tunnel diagram must be drawn to the same scale as the prism drawing.



Exit face of  
tunnel diagram  
is perpendicular  
to the ray.

2) (10 points) Many medium-format digital single lens reflex (DSLR) cameras use an image sensor that is about 2/3 the size of the 35 mm film format. The sensor size is 24 x 16 mm. This is called the APS-C or DX format.

What focal length lens is required to obtain a full field of view of  $50^\circ$  (i.e.  $\pm 25^\circ$ )? This FOV corresponds to the 24 mm dimension of the sensor. Assume a distant object.



$$\theta_{1/2} = 25^\circ \quad \tan \theta_{1/2} = .466 = \frac{12 \text{ mm}}{f}$$

$$f = 25.7 \text{ mm}$$

In air, the principal planes and nodal planes are coincident.

For an object at infinity, the chief ray is parallel to this nodal ray - independent of pupil position.

$$\text{Focal Length} = \underline{25.7} \text{ mm}$$

3) (25 points) An imaging system or objective is comprised of two thin lenses. It is to be used in conjunction with a cube of glass. What is the size of the largest glass cube that will fit between the rear element of this objective and the image plane? Consider that the image is formed on the rear surface of the cube, and the cube is in contact with the rear element of the objective.

Gaussian methods must be used.

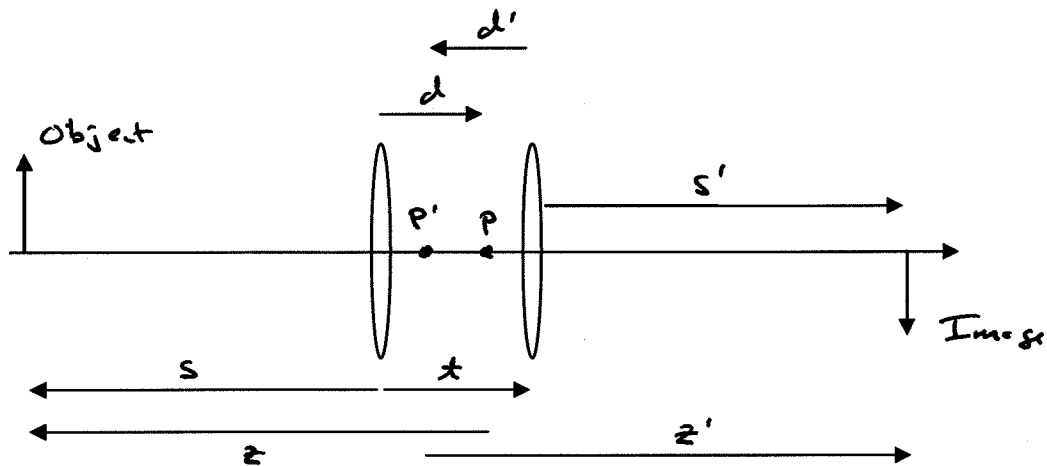
Details:

Glass Cube: Index of refraction = 1.6

Objective: Two 40 mm focal length thin lenses (in air) separated by 20 mm

Object: The object is 100 mm to the left of the front element of the objective.

Consider the optical system without the glass cube



$$s = -100 \text{ mm} \quad t = 20 \text{ mm}$$

$$f_1 = f_2 = 40 \text{ mm} \quad \phi_1 = \phi_2 = 0.025 / \text{mm}$$

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t = 0.0375 / \text{mm}$$

$$f = 26.67 \text{ mm}$$

Continues...

$$d' = -\frac{\phi_1}{\phi} t = -13.33 \text{ mm} \quad d = -d' = 13.33 \text{ mm}$$

$$z = s - d = -113.33 \text{ mm}$$

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \quad z' = 34.87 \text{ mm}$$

$s'$  is the distance from the second lens to the image plane

$$s' = z' + d' = 21.54 \text{ mm}$$

Glass Cube Size - the reduced thickness of the cube must equal  $s'$  for the largest possible cube.

$$\frac{L}{n} = s'$$

$L =$  Cube Dimension

$$n = 1.6$$

$$L = 34.46 \text{ mm}$$

$$\text{Glass Cube Dimension} = \underline{34.46} \text{ mm}$$

4) (15 points) A two-element imaging system is comprised of two thin lenses in air:

$$f_1 = -20 \text{ mm}$$

$$t_1 = 10 \text{ mm}$$

$$f_2 = 20 \text{ mm}$$

The system stop has a diameter of 10 mm and is located halfway between the first two thin lenses (5 mm behind the first lens). Use Gaussian methods to determine the locations and sizes of the system Entrance and Exit Pupils.

Note: No raytrace analysis is permitted for this problem.

XP:

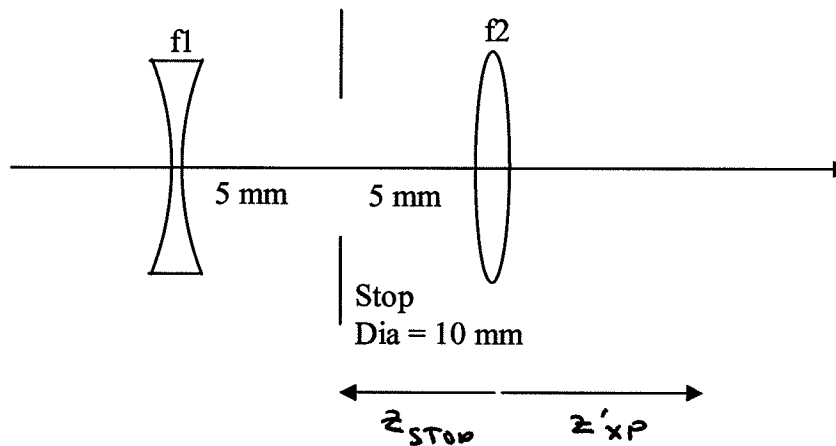


Image the stop through Lens 2:

$$z_{\text{STOP}} = -5 \text{ mm}$$

$$f_2 = 20 \text{ mm}$$

$$\frac{1}{z'_{\text{XP}}} = \frac{1}{z_{\text{STOP}}} + \frac{1}{f_2}$$

$$z'_{\text{XP}} = -6.67 \text{ mm}$$

Virtual (to the left of  $f_2$ )

$$m_{\text{XP}} = \frac{z'_{\text{XP}}}{z_{\text{STOP}}} = \frac{6.67}{5} = 1.33$$

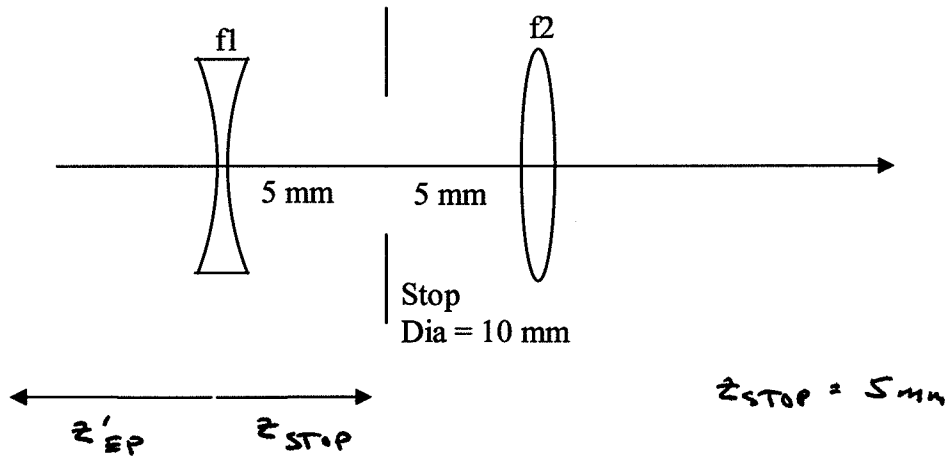
$$D_{\text{STOP}} = 10 \text{ mm}$$

$$D_{\text{XP}} = m_{\text{XP}} D_{\text{STOP}} = \underline{13.33 \text{ mm}}$$

Continues...

EP: Image Stop through  $f_1$ . Light goes from R  $\rightarrow$  L

$$n = n' = -1$$



$$\frac{n'}{z'_{EP}} = \frac{n}{z_{STOP}} + \frac{1}{f_1}$$

$$\frac{-1}{z'_{EP}} = \frac{-1}{5 \text{ mm}} + \frac{1}{-20 \text{ mm}}$$

$$\underline{z'_{EP} = 4 \text{ mm}}$$

Virtual - (to the right of  $f_1$ )

$$m_{EP} = \frac{z'_{EP}/n'}{z_{STOP}/n} = \frac{4}{5} = 0.8$$

$$D_{EP} = m_{EP} D_{STOP}$$

$$D_{STOP} = 10 \text{ mm}$$

$$\underline{D_{EP} = 8.0 \text{ mm}}$$

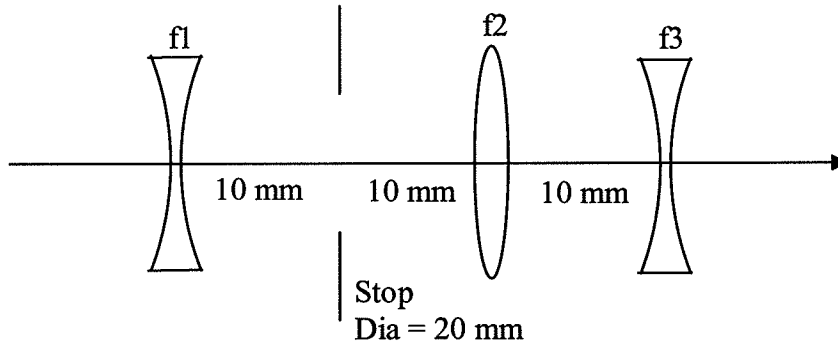
EP:  $D_{EP} = \underline{8.0}$  mm; Located  $\underline{4.0}$  mm to the R of the first lens.

XP:  $D_{XP} = \underline{13.33}$  mm; Located  $\underline{6.67}$  mm to the L of the second lens.

5) An air-spaced triplet is comprised of three thin lenses in air:

$$\begin{array}{ll} f_1 = -50 \text{ mm} & t_1 = 20 \text{ mm} \\ f_2 = 20 \text{ mm} & t_2 = 10 \text{ mm} \\ f_3 = -20 \text{ mm} & \end{array}$$

The system stop is located halfway between the first two lenses. The diameter of the stop is 20 mm.



This problem is to be worked using raytrace methods only. Gaussian imaging methods may not be used for any portion of this problem.

For grading purposes, the values for your final rays must be entered into the raytrace sheets that follow. Use the raytrace sheets noted for the marginal ray and the chief ray.

Be sure to clearly label your rays, and note which ray is used to determine each answer.

a) (10 points) Determine the locations of the entrance and exit pupils.

Trace a potential chief ray starting at the center of the stop. The EP and XP are located where this ray crosses the axis in object space and image space.

$$A1: \quad EP \rightarrow L1 = -8.33 \text{ mm} \quad (EP \text{ to the R of } L1)$$

$$A2: \quad L3 \rightarrow XP = -12.0 \text{ mm} \quad (XP \text{ to the L of } L3)$$

Entrance Pupil: 8.33 mm to the R of the first lens.

Exit Pupil: 12.0 mm to the L of the third lens.

b) (10 points) Determine the focal length and back focal distance of this optical system.

Trace a potential marginal ray for an axial object at infinity  $\tilde{y}_0 = 1$   $u_0 = 0$

The rear focal point  $F'$  is located where this ray crosses the axis.

$$\tilde{u}' = -.005 \quad B1 = XP \rightarrow F' = 192 \quad L3 \rightarrow XP = -12$$

$$\phi = -\frac{\tilde{u}'}{\tilde{y}_0} = .005$$

$$BFD = L3 \rightarrow F' = \underline{180 \text{ mm}}$$

$$f = \frac{1}{\phi} = 200 \text{ mm}$$

$$f = \underline{200} \text{ mm}$$

$$BFD = \underline{180} \text{ mm}$$

c) (10 points) Determine the diameters of the Entrance and Exit Pupils.

Scale the potential marginal ray to the stop radius

$$D_{STOP} = 20 \text{ mm} \quad a_{STOP} = 10 \text{ mm}$$

$$\tilde{y}_{STOP} = 1.2 \text{ mm}$$

$$\text{Scale Factor} = \frac{a_{STOP}}{\tilde{y}_{STOP}} = \frac{10 \text{ mm}}{1.2 \text{ mm}} = 8.33$$

$$y_{EP} = 8.33 \text{ mm}$$

$$D_{EP} = 16.67 \text{ mm}$$

$$y_{XP} = 8.0 \text{ mm}$$

$$D_{XP} = 16.0 \text{ mm}$$

$$\text{Entrance Pupil Diameter} = \underline{16.67} \text{ mm}$$

$$\text{Exit Pupil Diameter} = \underline{16.0} \text{ mm}$$

d) (10 points) The system has a Field of View of +/- 15 degrees. What is the image height corresponding to this FOV?

$$\bar{u}_o = \tan 15^\circ = 0.2680$$

$$\tilde{u}_o = 0.12$$

Extend the chief ray to  $F'$  (image plane) and scale the potential chief ray to  $\bar{u}_o$  in object space

$$\text{Scale Factor} = \frac{\bar{u}_o}{\tilde{u}_o} = \frac{0.2680}{0.12} = 2.233$$

$$\text{Image height at } F' = \bar{y}' = 53.59 \text{ mm}$$

$$\text{Image Height} = \pm \underline{53.59} \text{ mm}$$

Thin Lens YNU Method

	EP	L1	STOP	L2	L3	VP	Imag F'
	1	2	3	4	5	6	7
f	-	-50	-	20	-20	-	
$\phi$	-	.02	-	-.05	.05	-	
t	-8.33	10	10	10	-12.0	81	

Potential Marginal Ray

y	1.0	1.0	1.2	1.4	0.9	0.96	0
u	0	0	.02	.02	-.05	-.005	-0.005

Marginal Ray

y	8.33	8.33	10.0	11.67	7.5	8.0	0
u	0	0	.167	.167	-.417	-.0417	-.0417

y							
u							

$B1 = 192 \text{ mm}$   
 $\omega' = -.005$

Scale Factor = 8.33

Thin Lens YNU Method

EP	L1	STOP	L2	L3	XP	Image F'
0	1	2	3	4	5	6

-	-50	-	20	-20	-	
-	.02	-	-.05	.05	-	
	A1	10	10	A2	B1	112

Potential

Chief Ray

0	-1.0	0	1.0	1.5	0	
	.12	0.1*	0.1*	.05	.125	

Chief Ray

0	-2.233	0	2.233	3.339	0	53.59
	.2680	.2233	.2233	.1116	.2791	.2791

y						
u						

A1 = -8.33 mm

A2 = -12.0 mm

Scale Factor = 2.233

\* Arbitrary