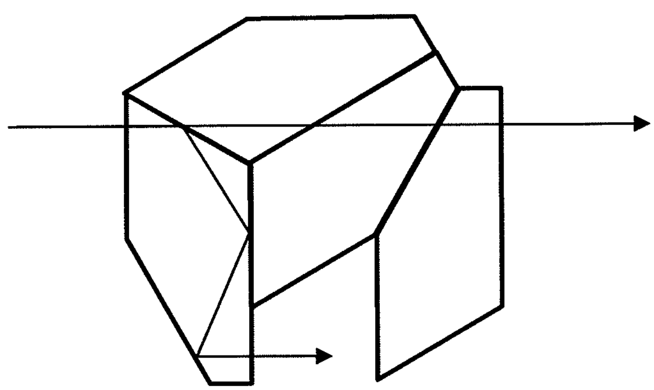
October 19, 2010 Lecture 17 Name Solutions

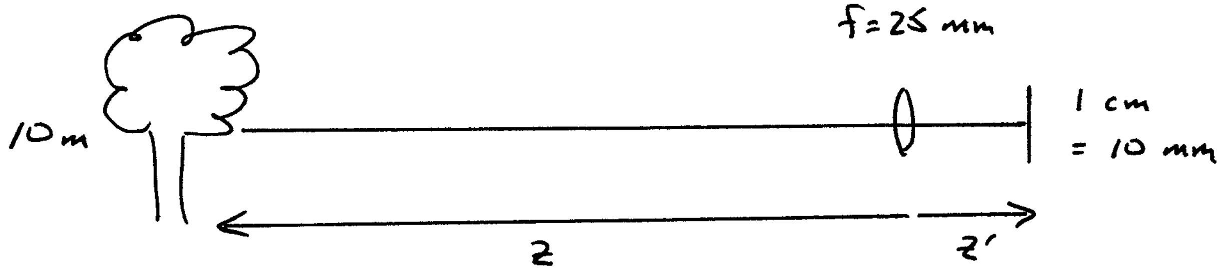
Closed book; closed notes. Time limit: 75 minutes.
An equation sheet is included. A spare raytrace sheet is also attached
Use the back sides if required.
Assume thin lenses in air if not specified.
If a method of solution is specified in the problem, that method must be used.
You must show your work and/or method of solution in order to receive
credit or partial credit for your answer.
Only a basic scientific calculator may be used. This calculator must not have
programming or graphing capabilities. An acceptable example is the TI-30
calculator. Each student is responsible for obtaining their own calculator.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy.

1) (10 points) Draw the tunnel diagram for this prism with the ray path shown. The tunnel diagram must be drawn to the same scale as the prism drawing.



2) (10 points) A 10 m tall tree is to be photographed with a camera that has a 25 mm focal length lens and an image sensor that is 1 x 1 cm. At approximately what object distance will the image of the tree fill the sensor? (The image size equals 1 cm.)



$z' \approx f$

m: 10 m : 10 mm → 1000 : 1

$m = \frac{z'}{z} \approx \frac{f}{z}$ $\qquad z \approx 1000 f = 25{,}000\ mm$

$z \approx 25\ m \quad (z = -25\ m)$

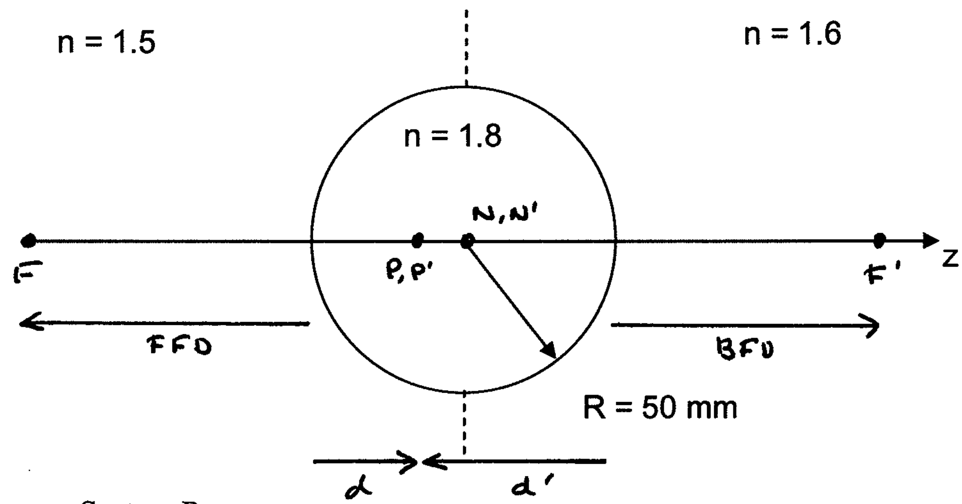
Exact: $m = -0.001 = \frac{z'}{z}$ $\qquad z = -1000 z'$

$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$ $\qquad z' = 25.025\ mm$

$z = -25.025\ m$

Object Distance ≈ 25 m

3) (20 points) A spherical ball of index 1.8 is mounted between two media with indices of refraction of 1.5 and 1.6. The radius of the ball is 50 mm.



Determine: System Power
Locations of the Principal Planes relative to the respective vertices
Front Focal Distance
Back Focal Distance
Principal Plane to Nodal Point Separation

Sketch the approximate locations of F, F', P, P', N, N' on the above figure.

NOTE: Only Gaussian methods may be used for this problem.

$n = n_1 = 1.5$ $n_2 = 1.8$ $n' = n_3 = 1.6$ $t = 100\ mm$

$R_1 = 50\ mm$ $R_2 = -50\ mm$

$\phi_1 = \frac{n_2 - n_1}{R_1} = \frac{1.8 - 1.5}{50} = .006/mm$ $\phi_2 = \frac{n_3 - n_2}{R_2} = \frac{1.6 - 1.8}{-50} = .004/mm$

$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 \frac{t}{n_2} = .006 + .004 - (.006)(.004)\frac{100}{1.8}$

$\phi = \overline{0.00867}/mm$

$f_E = 1/\phi = 115.4\ mm$

$f_F = -n_1 f_E = -173.1\ mm$ $f_R' = n_3 f_E = 184.6\ mm$

Continues...

Principal Planes:

$\delta' = \frac{d'}{n'} = -\frac{\phi_1}{\phi}\frac{t}{n_2}$

$\delta = \frac{d}{n} = \frac{\phi_2}{\phi}\frac{t}{n_2}$

$\frac{d'}{1.6} = -\frac{.006}{.00867}\frac{100}{1.8}$

$\frac{d}{1.5} = \frac{.004}{.00867}\frac{100}{1.8}$

$d' = -61.5\ mm$

$d = 38.5\ mm$

$BFD = d' + f_R' = 123.1\ mm$

$FFD = d + f_F = -134.6\ mm$

$PP' = t + d' - d = 0$ P and P' are physically coincident

$PN = P'N' = f_F + f_R' = -173.1\ mm + 184.6\ mm = 11.5\ mm$

The nodal points are 11.5 mm to the right of the principal planes

N: Location from front surface $= d + 11.5\ mm = 50\ mm$

N': Location from rear surface $= d' + 11.5\ mm = -50\ mm$

The nodal points are physically coincident at the center of curvature of the sphere.

This last result can be obtained by inspection. The principal planes must also be coincident since PN = P'N'

Power = 0.00867 mm^{-1}

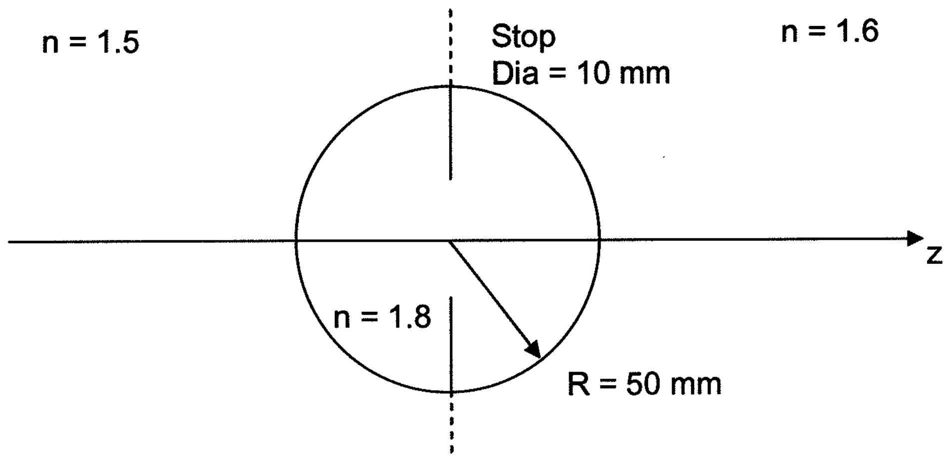
P: Located 38.5 mm to the R of the front vertex.

P': Located 61.5 mm to the L of the rear vertex.

FFD = -134.6 mm BFD = 123.1 mm

Principal Plane to Nodal Point Separation = 11.5 mm

4) (15 points) A 10 mm diameter stop is now inserted at the center of the spherical ball from the last problem: A spherical ball of index 1.8 is mounted between two media with indices of refraction of 1.5 and 1.6. The radius of the ball is 50 mm.



Determine the entrance pupil and exit pupil locations and diameters.

NOTE: Only Gaussian methods may be used for this problem.

From earlier problem: $\phi_1 = .006/mm$ $\phi_2 = .004/mm$

$n_1 = 1.5$ $n_2 = 1.8$ $n_3 = 1.6$

XP : $z_{STOP} = -50\ mm$

$$\frac{n_3}{z'_{XP}} = \frac{n_2}{z_{STOP}} + \phi_2 \qquad \frac{1.6}{z'_{XP}} = \frac{1.8}{-50} + .004$$

$\underline{z'_{XP} = -50\ mm}$ at C.C.

$$m_{XP} = \frac{z'_{XP}/n_3}{z_{STOP}/n_2} = \frac{-50/1.6}{-50/1.8} = 1.125$$

$D_{XP} = m_{XP}\, D_{STOP} = \underline{11.25\ mm}$

Continues...

EP: Light from R → L

$z_{STOP} = 50$ mm

$$\frac{-n_1}{z'_{EP}} = \frac{-n_2}{z_{STOP}} + \phi_1 \qquad \frac{-1.5}{z'_{EP}} = \frac{-1.8}{50} + .006$$

$$\underline{z'_{EP} = 50 \text{ mm}} \quad \text{at C.C.}$$

$$m_{EP} = \frac{z'_{EP}/-n_1}{z_{STOP}/-n_2} = \frac{50/-1.5}{50/-1.8} = 1.20$$

$$D_{EP} = m_{EP}\, D_{STOP} = \underline{12.0 \text{ mm}}$$

Can also be solved using the principles of nodal points: The nodal points of both refracting surfaces are at the C.C. along with the stop. Both the EP and the XP must therefore be at the stop.

$$m_N = \frac{n}{n'}$$

XP: $m_{XP} = 1.8/1.6 = 1.125$ $\qquad D_{XP} = 11.25$ mm

EP: $m_{EP} = -1.5/-1.8 = 1.2$ $\qquad D_{EP} = 12.0$ mm

The relative sizes of the EP and XP can be found

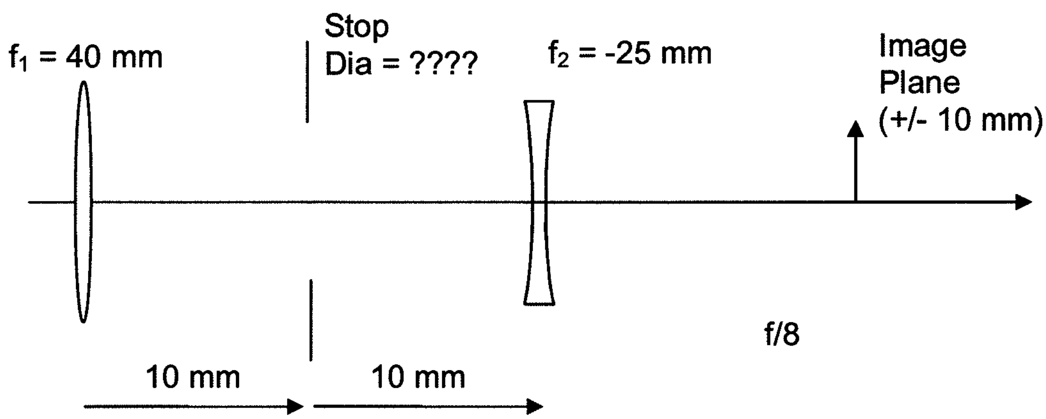
$m_{EP \to XP} = 1.5/1.6 = .9375$ $\qquad D_{XP} = .9375\ D_{EP}$

EP: D_{EP} = 12.0 mm; Located 50 mm to the R of the front surface of the sphere.

XP: D_{XP} = 11.25 mm; Located 50 mm to the L of the rear surface of the sphere.

5) (30 points) The following diagram shows the design of a telephoto objective that is comprised of two thin lenses in air. The system stop is located between the two lenses.

The system operates at f/8.
The object is at infinity.
The maximum image size is +/- 10 mm.



Determine the following:
- System focal length.
- Back focal distance
- Entrance pupil and exit pupil locations and sizes.
- Stop Diameter.
- Angular field of view (in object space).

NOTE: This problem is to be worked using raytrace methods only. Gaussian imaging methods may not be used for any portion of this problem. Be sure to clearly label your rays on the raytrace form.

Your answers must be entered below. Be sure to provide details on the pages that follow to indicate your method of solution (how did you get your answer: which ray was used, analysis of ray data, etc.)

Entrance Pupil: 13.33 mm to the R of the first lens. D_{EP} = 25.0 mm

Exit Pupil: 7.143 mm to the L of the second lens. D_{XP} = 13.40 mm

Stop Diameter = 18.75 mm

System Focal Length = 200 mm Back Focal Distance = 100 mm

FOV = +/- 2.86 deg in object space

	Obj	EP	L1	Stop	L2	XP	F' Image
Surface	0	1	2	3	4	5	6
f		–	40	–	-25	–	
$-\phi$		–	-.025	–	.04	–	
t		-13.333	10	10	-7.143	107.143	

Potential Chief Ray

	0	1	2	3	4	5	6
$\hat{\bar{y}}$		0	-1.0	0	1.0	0	
$\hat{\bar{u}}$	.075	.075	.1 *	.1 *	.14		

Potential Marginal Ray 107.143

	0	1	2	3	4	5	6
$\hat{y}$	1 *	1	1	0.75	0.5	0.5357	0
$\hat{u}$	0	0	-.025	-.025	-.005	-.005	

Potential Chief Ray - Extended

	0	1	2	3	4	5	6
$\hat{\bar{y}}$		0	-1.0	0	1.0	0	15.00
$\hat{\bar{u}}$	.075	.075	.1	.1	.14	.14	

Marginal Ray - Scale Factor = 12.5

	0	1	2	3	4	5	6
y	12.5	12.5	12.5	9.375	6.25	6.70	0
u	0	0	-.3125	-.3125	-.0625	-.0625	

Chief Ray - Scale Factor = .667

	0	1	2	3	4	5	6
$\bar{y}$		0	-.667	0	.667	0	10.0
$\bar{u}$	.05	.05	.0667	.0667	.0933	.0933	

y							
u							

y							
u							

Continues...

* arbitrary

Provide Method of Solution:

EP/XP Location: Trace a potential chief ray starting at the center of the stop. The pupils are located where this ray crosses the axis in object/image space.

$L1 \rightarrow EP = 13.333$ mm (Right of L1)

$L2 \rightarrow XP = -7.143$ mm (Left of L2)

Focal Length: Trace a potential marginal ray parallel to the axis in object space ($\bar{y} = 1$). The rear focal point is located where this ray crosses the axis.

$XP \rightarrow F' = 107.143$ mm

$BFD = (L2 \rightarrow XP) + (XP \rightarrow F') = -7.143 + 107.143$

$BFD = 100.0$ mm

$\phi = -\dfrac{u'}{y_1}$ $\quad u' = -.005 \quad y_1 = 1$

$\phi = 0.005$ /mm

$f = 1/\phi = 200$ mm

Extend the potential chief ray to F'

Entrance Pupil: $f/\# = f/8 = f/D_{EP}$

$D_{EP} = f/8 = 200\text{mm}/8 = 25.0$ mm

Continues...

Provide Method of Solution:

<u>Pupil / Stop Sizes:</u> $r_{EP} = 12.5$ mm

Scale the marginal ray to the proper r_{EP}

Scale Factor = $12.5/1.0 = 12.5$

$r_{STOP} = 9.375$ mm $r_{XP} = 6.70$ mm

$D_{STOP} = \underline{18.75}$ mm $D_{XP} = \underline{13.40}$ mm

<u>FOV</u>: Scale the potential chief ray to the desired image height of 10 mm (from the current value of 15.0 mm)

Scale Factor = $10.0/15.0 = 0.667$

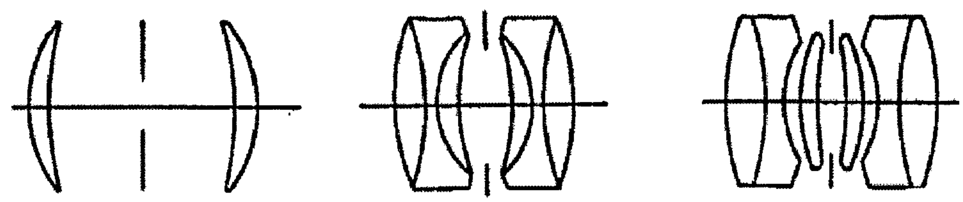
Object Space Chief Ray:

$\bar{u}_0 = 0.05$

$HFOV = \tan^{-1}(0.05) = 2.86°$

$FOV = 5.72°$ or $\pm 2.86°$

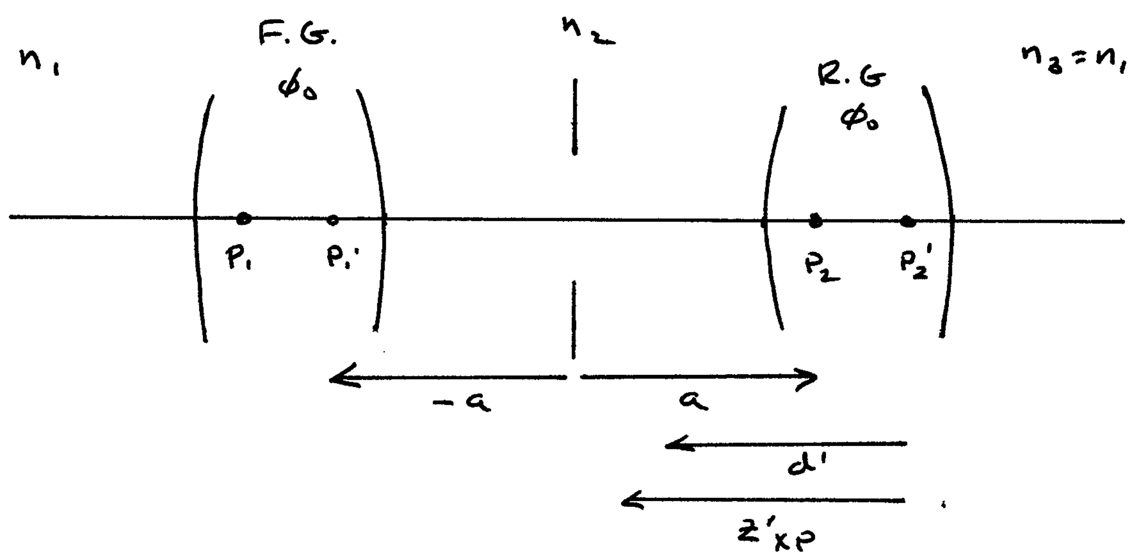
6) (20 points) An optical system is symmetric about its stop. This means that are curvatures, spacings and indices are all symmetric about the stop. To help clarify, here are a few of examples of symmetrical optical systems:



For an arbitrary symmetrical system, prove that the exit pupil of the system is coincident with the rear principal plane of the system and that the entrance pupil is coincident with the front principal plane.

Be sure to provide your proof in a logical and easy to follow manner.

Model the system as a front group and a rear group symmetric about the stop:



P_1' of the Front Group corresponds to P_2 of the rear group.

Both d' and z'_{XP} are measured from P_2'

Blank Page Follows...

$t = 2a$

System Rear Principal Plane:

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 \frac{t}{n_2} = 2\phi_0 - \phi_0^2 \left(\frac{2a}{n_2}\right)$$

$$\delta' = \frac{d'}{n_3} = -\frac{\phi_1}{\phi}\frac{t}{n_2} = -\frac{\phi_0}{2\phi_0 - 2a\phi_0^2/n_2} \cdot \frac{2a}{n_2}$$

$$d' = \frac{-n_3 a}{n_2 - a\phi_0}$$

XP: Image stop through Rear Group

$$\frac{n_3}{z'_{XP}} = \frac{n_2}{z_{STOP}} + \phi_0 \qquad z_{STOP} = -a$$

$$\frac{n_3}{z'_{XP}} = \frac{n_2 - a\phi_0}{-a}$$

$$z'_{XP} = \frac{-n_3 a}{n_2 - a\phi_0}$$

$$\underline{d' = z'_{XP}}$$

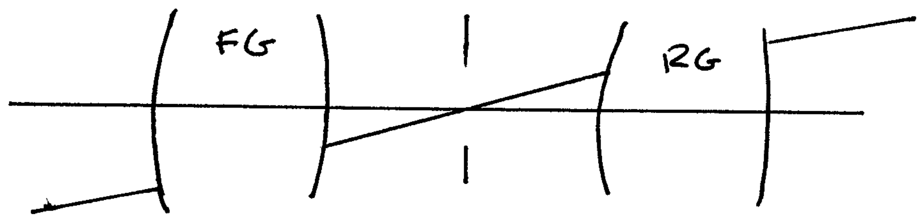
The system rear principal plane and the XP are coincident!

By symmetry, the same condition must hold for the EP and the system front principal plane.

Alternate: By symmetry, the EP in object space and the XP in image space must be the same size. The EP and the XP are also conjugate elements. The two pupils must therefore be located at the object space/image space planes associated with a magnification of 1 — the Principal Planes of the System.

Alternate Ray Trace Solution

Launch a potential chief ray through the center of the stop and propagate it into object and image space.



By symmetry of the system, these object and image space rays must have the same angle. They are rays defining the nodal points.

When these rays cross the axis:

- the EP and XP are located
- N and N' are located

Since the object space and image space indices are equal, the system principal planes are coincident with the system nodal points.

Therfore EP is coincident with P and N
XP is coincident with P' and N'