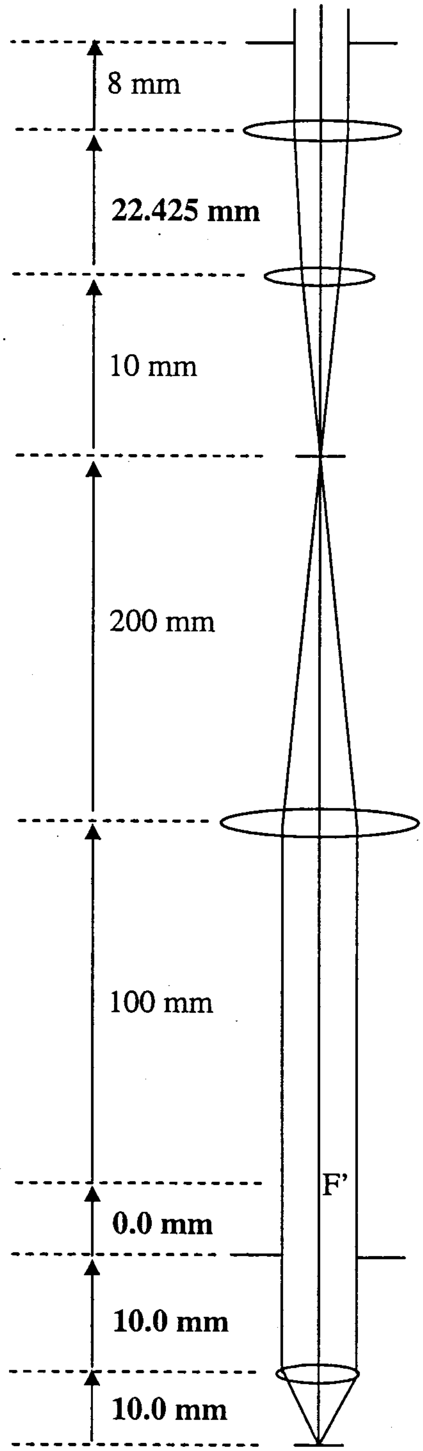
<u>200X Infinity Corrected Microscope</u> — <u>Solutions</u>



Exit Pupil: Dia = 1.0 mm

Eye Lens: f = **37.375 mm**
Dia = **7.40 mm**

Field Lens: f = **30.20 mm**
Dia = **20.9 mm**

Intermediate Image Plane
Dia = **20.0 mm**

Tube Lens: f = 200 mm
Dia = **18.0 mm**

Rear Focal Point of Objective Lens

Stop: Dia = **8.0 mm**

Objective Lens: f = **10.0 mm**
Dia = **9.0 mm**

Object Dia = 1.0 mm

System NA = **0.400**

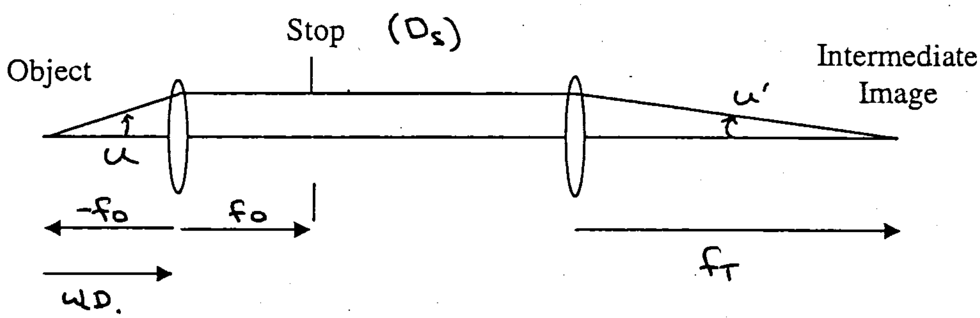
200X Infinity Corrected Microscope

Part A – Objective

The objective magnification is due to the combination of the Objective Lens f_O and the Tube Lens f_T.

Magnification = 20 X   m = -20

The system is telecentric in object space. This implies that the system stop is at the rear focal point of the objective lens.



$$u = \frac{D_S/2}{f_o} \qquad u' = -\frac{D_S/2}{f_T}$$

$$m = u/u'$$

$$m = -f_T/f_o = -20$$

$$f_T = 200 \text{ mm (given)}$$

$$f_o = 10 \text{ mm}$$

Working Distance = f_O = 10 mm

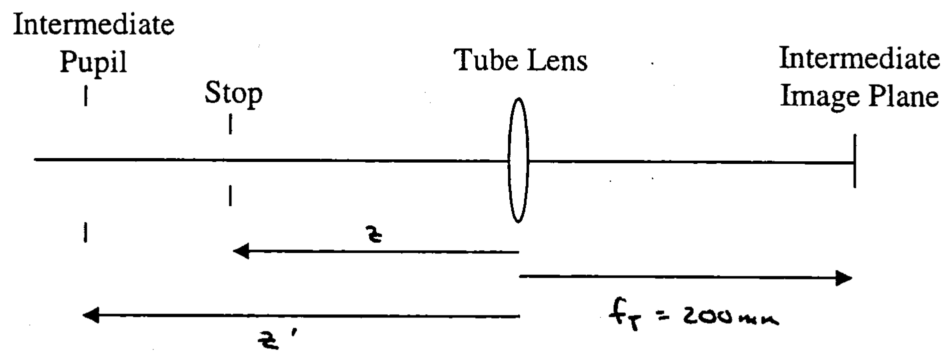
Stop Position = f_O from the objective lens = 10 mm

Part B – Eyepiece

Since the Field Lens f_F is not at the intermediate image plane, and therefore not at the front focal point of the Eye Lens, the focal length of the Eyepiece f_{EYE} is not equal to the focal length of the Eye Lens f_E. Since a relaxed eye is assumed, the focal length of the eyepiece can be found by treating it as a magnifier:

MP = 10X $\qquad f_{EYE} = \dfrac{250\,mm}{MP} \qquad f_{EYE} = 25\,mm$

The system must also produce the correct eye relief ER. We must be able to calculate the location of the XP by imaging the stop through the tube lens and the eyepiece (field lens and eye lens). We could reduce all three of these elements, but it is probably easier to first image the stop through the tube lens to produce a pupil in the space of the intermediate image (object space of the eyepiece):



$z = -100\,mm$

$f_T = 200\,mm$

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f_T}$$

$$z' = -200\,mm$$

The intermediate pupil is 400 mm to the left of the intermediate image plane.

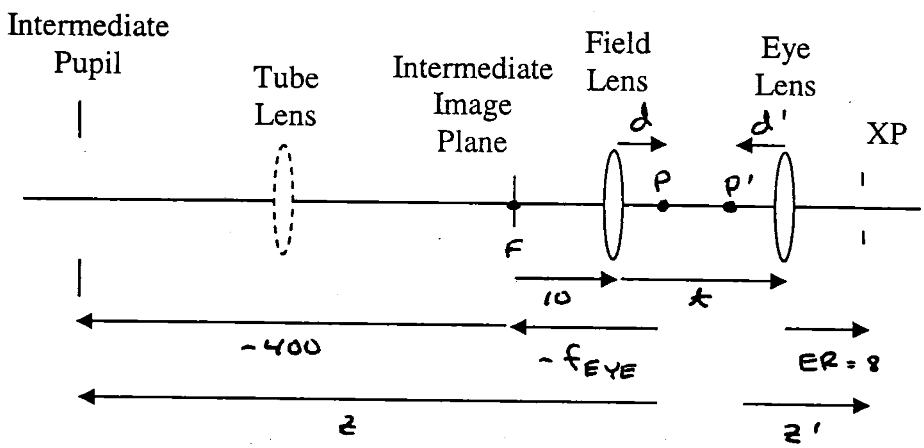
For later reference the intermediate pupil magnification is

$$m_1 = z'/z = 2.00$$

The eyepiece design must satisfy three conditions:

- The focal length of the eyepiece gives the proper MP
- The front focal plane of the eyepiece is at the intermediate image plane
- The intermediate pupil is imaged to the proper eye relief

The field lens is 10 mm to the right of the intermediate image plane.



① $$\phi_{EYE} = \phi_F + \phi_E - \phi_F \phi_E t = 1/f_{EYE}$$

$$d = \frac{\phi_E}{\phi_{EYE}} t \qquad d' = -\frac{\phi_F}{\phi_{EYE}} t$$

② $$f_{EYE} = 10 + d$$

③ $$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f_{EYE}}$$

$$z' = 8 - d'$$

$$z = -400 - 10 - d$$

All three conditions must be met.

From (2)

$$f_{EYE} = 10 + d = 25$$

$$d = 15\ mm$$

$$d = \frac{\phi_E}{\phi_{EYE}} t \qquad \phi_{EYE} = 1/25$$

$$\phi_E t = d\ \phi_{EYE}$$

$$\phi_E t = 0.6 \qquad (4)$$

From (3)

$$z = -400 - 10 - d \qquad z = -425\ mm$$

$$\frac{1}{z'} = \frac{1}{z} + \phi_{EYE} = \frac{1}{-425} + \frac{1}{25}$$

$$z' = 26.5625\ mm$$

$$z' = 8 - d'$$

$$d' = -18.5625\ mm$$

$$d' = -\frac{\phi_F}{\phi_{EYE}} t$$

$$\phi_F t = -d'\ \phi_{EYE}$$

$$\phi_F t = 0.7425 \qquad (5)$$

Note that the magnification from the intermediate pupil to the XP is

$$m_2 = z'/z = -.0625$$

3 Equations: ①, ④ and ⑤

3 Unknowns: ϕ_F, ϕ_E and t

① $\phi_{EYE} = \phi_F + \phi_E - \phi_F \phi_E t$

④ $\phi_E t = 0.600$

⑤ $\phi_F t = 0.7425$

$$\phi_{EYE} t = \phi_F t + \phi_E t - (\phi_F t)(\phi_E t) = t/f_{EYE}$$

and $f_{EYE} = 25$ mm

$t = 22.425$ mm

Then $\phi_E = .02676$

$f_E = 37.375$ mm

$\phi_F = .03311$

$f_F = 30.20$ mm

Part C – Stop Size and NA

The required XP Diameter is 1.0 mm. Can be worked by Gaussian or Raytrace Methods:

Gaussian:

Use the magnifications determined between the stop and the XP

$$m(\text{stop} \rightarrow XP) = m_1 m_2 = (2.00)(-.0625) = -.1250$$

$$|m| = \frac{D_{XP}}{D_{STOP}} = \frac{1\text{ mm}}{D_{STOP}} \qquad D_{STOP} = 8.00 \text{ mm}$$

Raytrace

All of the element focal lengths and spacings are known. Trace a marginal ray from the XP back to the stop plane (the reverse raytrace is attached). The marginal ray is parallel to the axis in eye space:

At the XP $y = 0.5$ mm
$u = 0.0$

At Stop: $y = 4.00\text{ mm}$

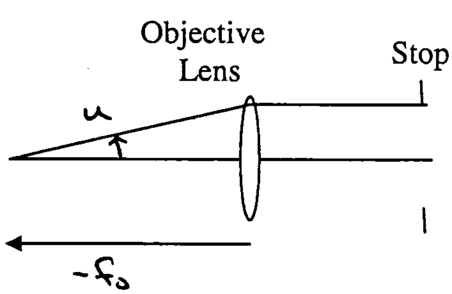
$$D_{STOP} = 8.00 \text{ mm}$$

NA

Since the object is at the front focal point of the objective lens, the marginal ray height at this lens is equal to the stop radius:

$r_{STOP} = 4.00$

$u = 4.00/f_o$



$f_o = 10.0 \text{ mm}$

$u = .400$

$NA = n|u| = 0.400$

Part D – Required Diameters

Trace marginal and chief rays through the system. The raytrace is attached. The marginal ray starts at the XP as described in Part C. The chief ray can start at the object. Since the system is telecentric in object space, the chief ray is parallel to the axis with a height equal to half the object diameter:

At the object $\bar{y} = 0.5$ mm
$\bar{u} = 0.0$

The required apertures are then found: $a \geq |y| + |\bar{y}|$ $D = 2a$

Objective Lens

$y = -4.0$ $\bar{y} = 0.5$ $a \geq 4.5$ $D \geq 9.0$ mm

Tube Lens

$y = -4.0$ $\bar{y} = -5.0$ $a \geq 9.0$ $D \geq 18.0$ mm

Intermediate Image

$y = 0.0$ $\bar{y} = -10.0$ $a > 10.0$ $D \geq 20.0$ mm

Field Lens

$y = 0.2$ $\bar{y} = -10.25$ $a \geq 10.45$ $D \geq 20.9$ mm

Eye Lens

$y = 0.5$ $\bar{y} = -3.20$ $a \geq 3.7$ $D \geq 7.40$ mm

YNU Method

	Object 0	f_o 1	Stop 2	f_T 3	Inter. Image 4	f_F 5	f_E 6	XP 7	8	9
f φ		10	—	200	—	30.20	37.375	—		
t	10	10	100	200	10	22.425	8			
n										
-φ		-.10	0	-.005	0	-.03311	-.02676			
t/n	10	10	100	200	10	22.425	8			
Marginal (Start at XP)										
y	0	-4.0	-4.0	-4.0	0.0	0.2	0.5	0.5		
nu	-0.4	0.0	0.0	0.02	0.02	0.01338	0			
u										
Chief (Start at Object										
y	0.5	0.5	0.0	-5.0	-10.0	-10.25	-3.200	0		
nu	0	-0.05	-0.05	-.025	-.025	0.3144	0.400			
u										
y										
nu										
u										

Discussion:

Since this is an infinite tube microscope system, many of the equations in the notes for microscopes do not apply. Those equations are derived for a finite tube microscope.

Many references describe a Ramsden eyepiece as being constructed of two lenses of equal focal length (or approximately equal focal length) separated by 2/3 to 3/4 of the element focal length. What is required here is simply a Ramsden-style eyepiece as defined in the class notes. The field lens is displaced behind the intermediate image plane. The system is fully constrained by the eyepiece magnifying power, the use of a relaxed eye and the specified eye relief. Assumptions about the relative focal lengths and spacing are unnecessary (and will result in a solution that will not meet the required specifications). The solution obtained here is consistent with the standard definitions of a Ramsden eyepiece.

Another method to relate the stop size to the XP diameter is to note that the combination of the tube lens and the eyepiece forms an afocal system. The magnification between the stop and the XP is the lateral magnification of this afocal system (i.e. the ratio of the two focal lengths $25/200 = 0.125$).

The working distance obtained here is longer than that usually found for 20X objectives, but real objectives are not thin lenses. The objective focal length will be about 10 mm (depending on the tube lens), but the working distance is reduced by the multiple elements that make up a real objective and are needed for aberration correction. Long working distance objectives are available and are designed with increased working distance.

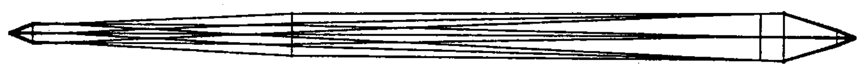
It is interesting to consider what happens if the infinite optical tube length varies. Since the objective is actually the combination of the objective lens and the tube lens, these two elements must be designed to work together. This is why objectives should only be used with a microscope from the same manufacturer.

If the infinite tube length is changed, the location of the intermediate pupil will also change. If the same eyepiece is used, the amount of eye relief will change.

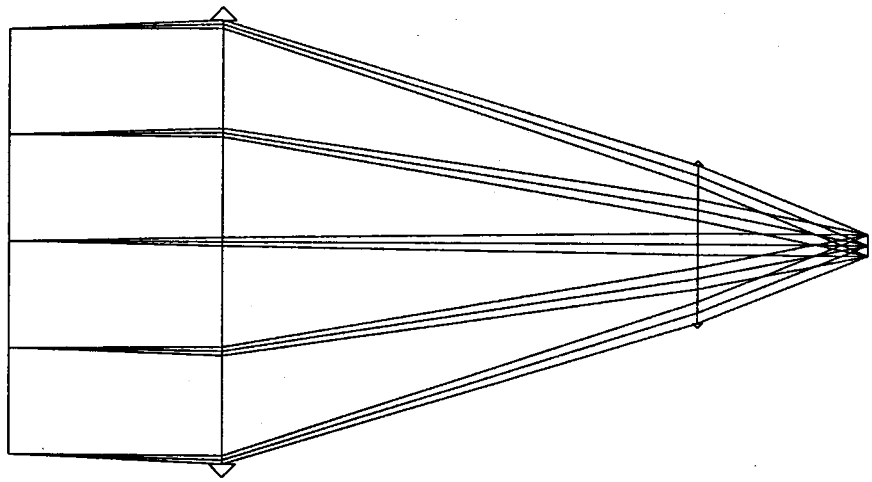
An interesting condition occurs if the infinite optical tube length equals the focal length of the tube lens. The objective lens/tube lens combination becomes afocal as well as double telecentric. The advantage here is that the chief rays at the intermediate image plane are now parallel to the axis. This would be useful with the use of a reticle (image scale does not change with defocus).

To demonstrate the system, the prescription was entered into lens design software.

The full system:



A close-up of the eyepiece starting at the intermediate image plane and going to the XP:



A close-up of the objective showing the stop. Note that in the intermediate space, the rays for each object point are parallel.