WRITTEN PRELIM EXAM – FIRST DAY

FALL 1998

September 28, 1998
8:30 a.m. to 12:30 p.m.

Please answer two questions from each of the four categories (for a total of 8 questions).

Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

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$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

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$2\cos A \cos B = \cos(A - B) + \cos(A + B)$

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$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

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$\nabla \times \nabla\phi = 0$

$\oint_{\mathrm{S}} (\mathbf{F} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_{\mathrm{C}} \mathbf{F} \cdot d\ell = \int_{\mathrm{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

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A-1-1

1. Write the expression for the intensity reflection and intensity transmission through a plane, transparent glass plate. (20%)

2. Plot these expressions for low and high reflectivity. (20%)

3. Derive and explain the meaning of finesse. (20%)

4. In the presence of absorption, what is the expression for the transmission? (20%)

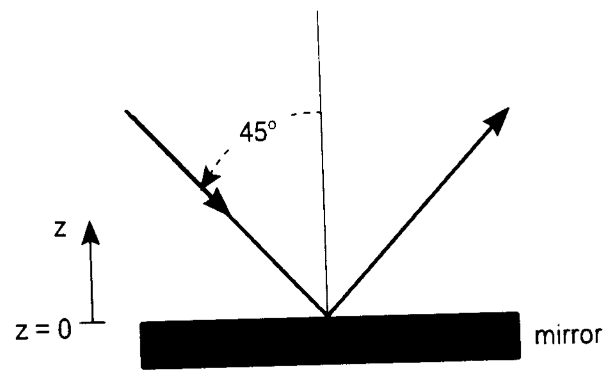
5. Describe briefly an application of a device consisting of a plane, transparent glass plate. (20%)

A-1-2

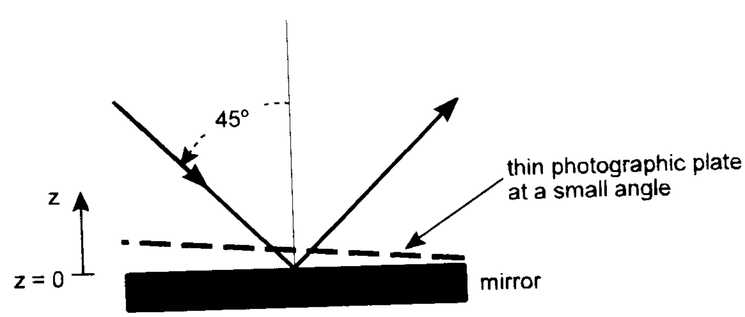
1) A monochromatic point source P emits light of wavelength 600 nm, which falls onto an aperture A 10 cm away and then onto a screen 20 cm beyond A.
   a) (25 %) What is the value of r, the radius of the first Fresnel half-period zone at aperture A?
   b) (25 %) The aperture A is a circle of radius 1 cm. How many Fresnel half-period zones does it contain?

   A perfect thin lens of 5 cm focal length is placed next to aperture A.
   c) (10 %) What is the distance from the lens to the on-axis point of maximum irradiance?
   d) (40 %) What is the distance from the on-axis point of maximum irradiance to the point of zero on-axis irradiance?

1.) In the figure below, a linearly polarized plane wave is incident upon a mirror with an angle of incidence of 45°. Write a simple expression as a function of z, λ, and the amplitude A of the incident field that describes the fringe pattern observed when the incident wave and the reflected wave interfere. Assume that the incident light is s polarized, and assume a perfect mirror.



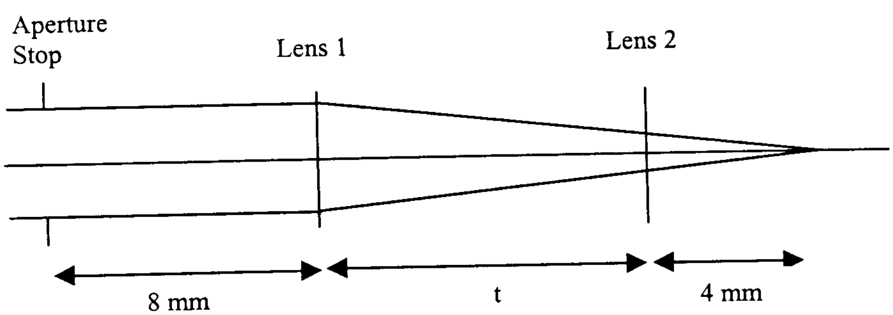
2.) The figure below is similar to the figure described in part (1). However, a thin photographic plate has been added that is placed near the mirror at a small angle. Assume that the plate does not influence the fringe pattern; it only records the bright and dark fringes so that they may be easily observed. Two experiments are performed with different photographic plates. In the first experiment, the incident light is s polarized. A pattern of bright and dark bands are observed on the plate after development. In the second experiment, the incident light is p polarized. The darkening is now uniform over the plate, and no banding is observed. Explain these results, and describe how the results can be used to prove that the electric field rather than the magnetic field is responsible for observable interference phenomena.



The following are the requirements for a thin-lens optical system in air that is designed for an optical disc application:

- The system is composed of two separated thin lenses.
- The system focal length equals 12 mm.
- The system is telecentric in image space.
- The object is at infinity.
- The aperture stop is 8 mm to the left of the first lens.
- The image space working distance (from lens 2 to the image plane) is 4 mm.

Schematic Diagram



Determine the focal lengths of the two lenses and the required separation t.

B-1-2

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The following is the design of a thick lens achromat in air. All units are mm.

Surface	Radius	Spacing	Glass
1	10.32745		
		10.0	BK7
2	-25.00000		
		10.0	F2
3	6.308569		

Glass Data:

	F	d	C
BK7	1.52238	1.51680	1.51432
F2	1.63208	1.62004	1.61503

This thick lens achromat has been designed to produce a common focal point for the F and C wavelengths.

Use Gaussian Reduction to determine the focal lengths of the lens at the F and C wavelengths, compare this result to what is expected for a thin lens achromat, and explain the differences.

To aid in the calculation, the magnitudes of the powers of the individual surfaces are provided. You need to determine the sign of the powers.

Surface Powers

	F	d	C
$\phi 1$	+/- 0.050582	+/- 0.050042	+/- 0.049802
$\phi 2$	+/- 0.004388	+/- 0.004130	+/- 0.004028
$\phi 3$	+/- 0.100191	+/- 0.098283	+/- 0.097489

A 3 mm diameter cooled detector, equally responsive at all wavelengths, is placed at the focal point of a 1 meter diameter, F/2 paraboloidal telescope mirror. The mirror has a reflectance of 0.98, and the detector is geometrically baffled by a cold shield such that it can only see the mirror. A distant (10^{10} km), hot (5000K) star with an intensity of 10^{25} W/sr is observed. The power received from this star is "signal" S and the power received from mirror radiation is "noise" N. Assume an atmospheric transmission of one.

a) What is the temperature of the mirror for a S/N of 10? (50%)

b) Is this feasible? (10%)

c) How else might you achieve the desired S/N? (40%)

Let φ_x and φ_y be the quantum states corresponding to photons that are linearly polarized along the x-axis and the y-axis respectively.

(a) Write down the quantum state ψ_μ for a photon which is linearly polarized at an angle μ relative to the x-axis, in terms of φ_x, φ_y and μ. (20%)

(b) For a photon in the state ψ_μ find the probabilities that a measurement will show it to be polarized along x and y respectively. (20%)

A light source produces pairs of photons traveling left and right along the z axis. The quantum state for a photon pair can be expressed in terms of *product states* of the form $\varphi_i^L \varphi_j^R$, $i, j = x, y$, where the single-photon states φ_x^L, φ_y^L, φ_x^R and φ_y^R correspond to x- or y-polarized photons traveling in the left or right direction. The source is rigged to produce photon pairs in the state

$$\psi = \frac{1}{\sqrt{2}}\left(\varphi_x^L \varphi_y^R + \varphi_y^L \varphi_x^R\right)$$

(c) For this state calculate the probability for each of the four possible outcomes when the polarization is measured simultaneously on the left and right hand sides. Find the probabilities of detecting x and y polarization on one side, irrespective of the polarization detected on the other side. Show also that the detection of x polarization on the left hand side always implies the detection of y-polarization on the right hand side. (35%)

(d) What do the results of (c) imply for the polarizations of a pair of photons? (25%)

(a) When a monochromatic beam of light with initial irradiance I_0 is passed through a sample, the equation,

$$I = I_0\, e^{-\alpha z} \tag{1}$$

often referred to as Beer's Law, is not always valid. Discuss the conditions under which it will fail and explain why the failure occurs.

(30%)

(b) Suppose conditions are such that Eq. (1) holds for a steady beam of light passing through some sample, but now the light beam is turned on as step function at $t = 0$. Will Eq. (1) be valid for all times $t > 0$, for only some times $t > 0$ (if so, specify which times), or no times $t > 0$?

(10%)

(c) Suppose we have a cell containing atomic hydrogen at 0.1 atmosphere pressure, and the absorption due to the H_α line (2s $\rightarrow$ 3p transition) at 656 nm is homogeneously broadened. The steady-state polarization induced in the sample by a monochromatic laser beam near resonance with the H_α line is given by

$$P = \frac{\wp^2 E_0}{2\hbar}(N_b^0 - N_a^0)\left[\frac{\omega_0 - \omega}{\mathcal{R}^2 + \gamma^2} + i\,\frac{\gamma}{\mathcal{R}^2 + \gamma^2}\right] e^{-i(\omega t - kz)} \; + \text{ c.c.}$$

Here $\mathcal{R} = \sqrt{(\omega_0 - \omega)^2 + (\wp E_0/\hbar)^2}$ is the generalized Rabi-flopping frequency, $\wp$ is the transition dipole matrix element, γ is the linewidth, and N_i^0 is the population of state i when no light is present. Find an expression for the absorption coefficient α produced by this polarization.

(20%)

(d) What changes could be made to the hydrogen sample of part (c) which would cause the H_α transition to become inhomogeneously broadened?

(10%)

(e) Suppose the hydrogen sample of part (c) is changed so that the H_α line becomes inhomogeneously broadened. Find an appropriate expression (which may contain integrals) for the steady-state polarization in this case.

(30%)

Consider a first-class dipole-allowed optically created Wannier exciton in a direct-gap bulk semiconductor such GaAs.

(a) Write down the time-independent Schroedinger equation for the center-of-mass motion (not the relative motion!) of the exciton. Give the solution for the center-of-mass wave function $\Phi_{CM}(\vec{R})$ and the center-of-mass energy ε_{CM}, i.e. the energy contribution due to the center-of-mass motion. No formal proof is requested here.

(50 %)

(b) Write down the conservation laws that determine the center-of-mass wavevector in terms of the wavevector of the exciting light field $\vec{q}_i$ inside the crystal and the total exciton energy of the 1s-exciton in terms of the light frequency given as $\hbar\omega$. Neglect any surface and polariton effects. Illustrate the energy conservation law with a sketch of the exciton dispersion and the light dispersion. Indicate in your sketch the bandgap energy E_g, the exciton binding energy E_b, and plot the light dispersion as $\hbar\omega$ vs q_i inside the crystal, assuming that the refractive index of the crystal n is frequency independent.

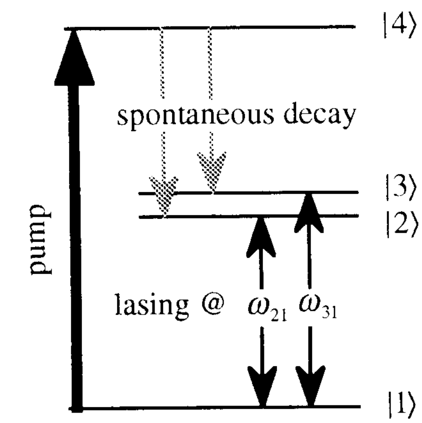
Assume that you want to make the center-of-mass energy equal to the binding energy by changing the bandgap energy E_g appropriately. Also, assume that you can change the bandgap energy E_g without changing any other parameter. How large would you have to choose E_g? Hint: determine first the appropriate value for q_i and use that value to determine E_g.

If needed, use the following parameter values: m_e=0.05m_0, m_h=0.35m_0 (where m_0 is the electron mass in vacuum), E_b=4 meV, and n=4. You may also use the following *approximate values* for $\hbar^2/m_0 \approx 8 \times 10^{-16}$ eV cm^2 and $\hbar c \approx 2 \times 10^{-5}$ eV cm.
Just in case you don't remember: the light dispersion $\omega = \omega(q_i)$ in the medium is less steep than the light dispersion in vacuum if $n > 1$.

Is the resulting value for E_g typical and realistic for bulk III-V compounds? If not, is it larger or smaller than typical values?

(50 %)

Consider a laser with the following level scheme. There are two lasing levels, $|2\rangle$ and $|3\rangle$, separated in frequency by a small splitting $\Delta\omega = \omega_{31} - \omega_{21} << \omega_{21}, \omega_{31}$. Each of the levels $|2\rangle$ and $|3\rangle$ have the same natural linewidth Γ, and the branching ratio of the decay from $|4\rangle$ is such that each is populated at equal rate due to the pumping mechanism. The gain medium is Doppler broadened, and the Doppler broadened linewidth ω_D of each of the transitions $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$ is comparable to the splitting $\Delta\omega$. This gain medium is placed in a standing wave resonator with a free spectral range much larger than any other relevant frequency scale in the problem, $\omega_{FSR} >> \Delta\omega, \omega_D, \Gamma$. By adjusting the resonator length so there is only one resonator mode in the vicinity of the lasing transitions the laser can then be made single mode.



(a) The laser output frequency ω_L is now tuned across the interval at which lasing occurs. Draw a sketch showing how the laser output power changes with frequency. Explain the physical origin of all significant features, and indicate the frequency scale associated with each. (25%)

(b) In (a) the laser output frequency was tuned by accurately changing the resonator length, i. e. by changing the frequency ω_R of the resonator mode. Draw a sketch showing how the frequency difference $\omega_L - \omega_R$ changes as a function of ω_R. Explain the origin of any significant features. What determines the magnitude of the frequency difference (if any)? (25%)

We now change the laser design, making the resonator longer so there is a mode exactly at frequency ω_{21}, and another exactly at frequency ω_{31}. We also cool the gain medium so Doppler broadening becomes negligible. By design the resonator losses are larger at ω_{21} than at ω_{31}.

(c) First assume that $\Gamma >> \Delta\omega$. What frequencies are present in the laser output?
Then assume $\Gamma << \Delta\omega$. What frequencies are present in the laser output? (20%)

The laser resonator consists of two mirrors with identical reflectivity R at frequency ω_{31}. There are no other losses in the system.

(e) Let N_i be the number density of atoms in state $|i\rangle$ and let $\Gamma << \Delta\omega$. Find an expression for the threshold inversion $\Delta N = N_3 - N_1$ for lasing exactly at frequency ω_{31}, in terms of the resonator length L, the reflectivity R and the frequency ω_{31}. (30%)

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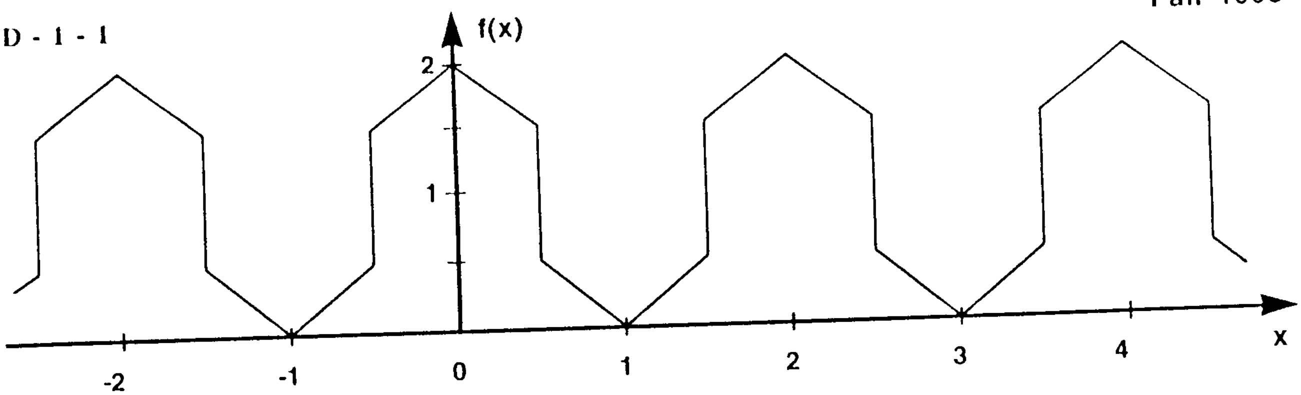
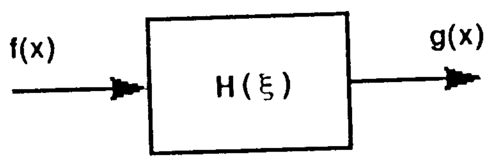
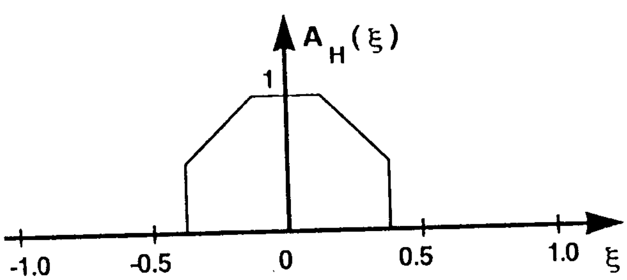
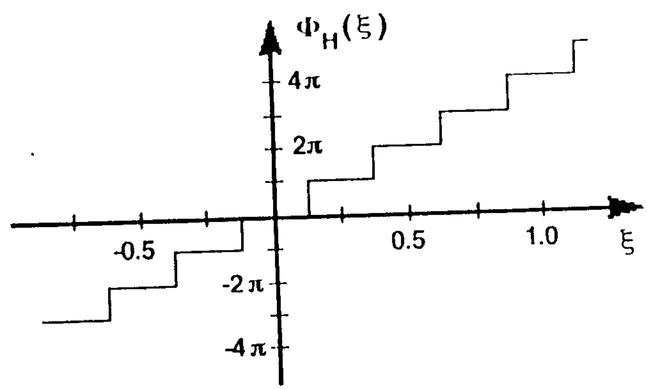
D - 1 - 1





$$H(\xi) = A_H(\xi)\, e^{-j\Phi_H(\xi)}$$





The input signal to a low-pass filter is a periodic function of amplitude 2 and period 2 as shown in the figure. The amplitude- and phase-transfer functions of the system are as shown.

Find an expression for the output signal g(x) and sketch your result.

Answer the following as generally as possible, defining all terms you use.

(a) State Kolmogoroff's three axioms of probability.

(b) How is the probability density for a *complex* event $x + iy$, $i = \sqrt{-1}$, x, y real, represented?

(c) Define the conditional probability $P(B|A)$.

(d) What is the 'additivity' property of probability?

(e) What is the 'additivity' property of Shannon information?

(f) Define the characteristic function.

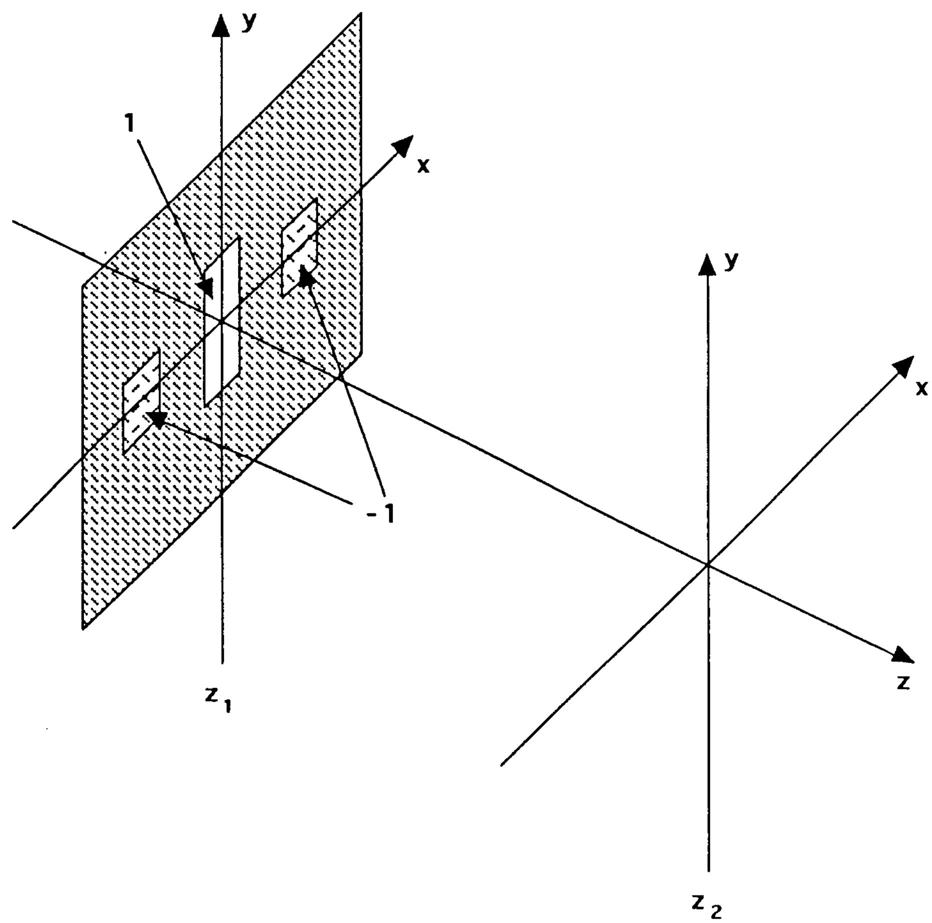
(g) What is a Markov events sequence?

(h) Define what is meant by a stochastic process. Cite an optical example.

(i) What is meant by wide-sense stationarity? Cite an optical example.

(j) What is a superposition (stochastic) process? Cite an optical example.

Three rectangular apertures are located in an opaque screen in the $z = z_1$ plane as shown. The larger aperture has dimensions of 4 mm x 2 mm, is centered at the origin, $(0, 0, z_1)$, and has a transmittance of 1 (it is a clear aperture). The two smaller apertures are square, with dimensions of 2 mm x 2 mm, are centered at $(-4\text{ mm}, 0, z_1)$ and $(+4\text{ mm}, 0, z_1)$, respectively, and each has a transmittance of -1 (they are covered with half-wave phase plates). These three apertures are illuminated with a normally incident, coherent plane wave of $\lambda = 500$ nm and amplitude A, where $|A|^2 = 100\text{ mW / cm}^2$.



(a) Appproximate the minimum distance between the z_1 and z_2 planes that will ensure the latter is in the Fraunhofer region of the aperture combination. (30%)

(b) Find an expression for the irradiance in the z_2 plane assuming that the distance between the two planes is 10,000 m and that the Fraunhofer condition is satisfied. (40%)

(c) **Either:** make a rough, normalized sketch of the irradiance along the x axis, indicating pertinent features such as locations of peaks and zeros, etc.; **or**, find the value of the irradiance at the point $(0, 0, z_2)$. (30%)

WRITTEN PRELIM EXAM – SECOND DAY

FALL 1998

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$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

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$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

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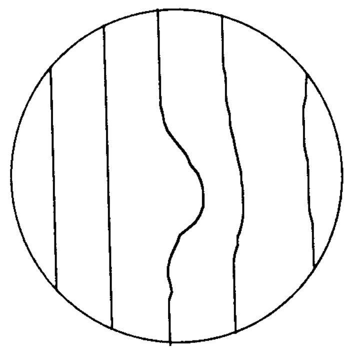
$\oint_{\mathrm{S}} (\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}} (\nabla \times \mathbf{F})\, d^3x$

1) The following interferogram was obtained using a two-beam interferometer to test a 2-cm diameter sample at a wavelength of 633 nm.

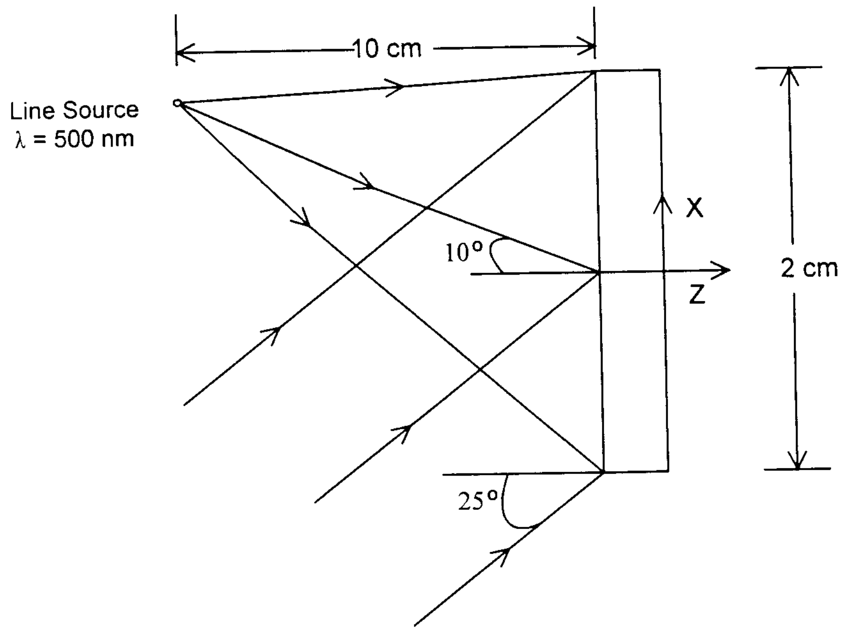


Let the sample being tested be a plane parallel plate of refractive index 1.5.

a) (25 %) What is the maximum thickness variation if the interferogram were obtained testing the sample using a Twyman-Green interferometer?
b) (25 %) When the length of the interferometer arm containing the sample is shortened the fringes move to the right. Is the center portion of the sample too thick or too thin? Explain.
c) (25 %) Before the sample was placed in the interferometer a null fringe existed. What is the wedge in the sample?

Let the sample being tested be a nearly flat mirror.

d) (25 %) What is the maximum departure from flatness if the sample is tested at an angle of incidence of 45 degrees in a Mach-Zehnder interferometer?



A hologram is formed with light from a line source (i.e. light focused with a cylindrical lens) 10° from an axis through the center of the film and a collimated beam 25° to the film normal.

a) Compute the maximum and minimum spatial frequencies recorded on the surface of the hologram.

b) Compute the magnitude and direction of the grating vector at the center of the hologram.

c) If the source wavelength is changed to 510 nm with a line source in the original line source position, calculate the direction of the diffracted beam.

Note: Assume that the refractive index of the film is the same as the surrounding space so that Snell refraction effects can be neglected.

A distributed Bragg reflector (DBR) laser operating at 1.55 μm consists of a gain section and two grating mirrors. The gain section is 3 μm wide and 300 μm long, with a modal internal loss of 20 cm^{-1}. The grating sections are 100 μm and 400 μm with a coupling coefficient of 50 cm^{-1}. The coupling efficiency between the gain and grating section is 90% at each interface. The transverse confinement factor for the 0.1 μm thick active layer is 0.1. The internal quantum efficiency is 1 and the group index in all sections is 4. The gain versus carrier density is considered linear with a transparency carrier density of N_{tr} = 1.5×10^{18} cm^{-3} and a differential gain of 5×10^{-16} cm^2.

a) What is the power reflection from each grating section at the Bragg wavelength?

b) What is the corresponding modal threshold gain ?

c) What is the threshold carrier density?

d) What is the minimum mode spacing?

e) What are the advantages and disadvantages of DBR lasers over Fabry-Perot lasers?

The normal-incidence reflectance for a simple quarterwave antireflection coating is given by::

$$R = \frac{[y_0 - y_f^2/y_{sub}]^2}{[y_0 + y_f^2/y_{sub}]^2}$$

where y_o, y_f and y_{sub} are the admittances of the medium, film, and substrate.

a) Derive an expression relating the admittances at the point where R=0.

b) Does the expression you derived above in A) hold true for:
   i) all wavelengths of light?
   ii) Wavelengths that are an even number of quarterwaves of the coating?
   iii) Wavelengths that are an odd number of quarterwaves of the coating?

c) For a substrate material of admittance 1.52, what admittance coating would give minimum reflectance when operated at normal incidence in vacuum?

d) To broaden the reflection characteristic, one type of two-layer antireflectance coating is the W-coat. For the W-coat, what effect does the additional high-index half-wave coating between the quarterwave coating and the substrate have at the design wavelength?

You are designing an experiment to find the surface reflectance of a test site from satellite imagery. In order to locate your target in an image, you place large tarpaulins at the corners of the test site. Assume the following:

- Incident solar spectral irradiance at the top of the atmosphere is 1300 $W\text{-}m^{-2}\text{-}\mu m^{-1}$ in the spectral band of interest
- Incident solar zenith angle is 60 degrees
- The digital numbers (Dns) reported by the sensor viewing the background surface is 220 Dns
- The sensor has 8-bit resolution, an offset of 10 DN and a gain of 2.54 $DN/(W\text{-}m^{-2}\text{-}sr^{-1}\text{-}\mu m^{-1})$
- No sensor or atmospheric MTF effects
- The ground instantaneous field-of-view (or footprint) of the sensor is 30 m × 30 m.
- The tarpaulin is in the center of the sensor's field-of-view and its hemispherical-directional reflectance when viewed at nadir is 0.05 in the band of interest.

a) (70%) In order to detect the presence of the tarpaulin in the image, you want it to be large enough to reduce the Dns reported by the sensor by 30 DN. Compute the area of the tarpaulin needed to do this, stating all assumptions.

b) (20%) If atmospheric scattering effects are included, would you need a smaller or larger tarpaulin to see the same 30 DN reduction. Explain your answer.

c) (10%) Sensor MTF effects are now present. How must the size of the tarpaulin change if it is no longer in the center of the field-of-view, but still fully enclosed within the 30 m × 30 m footprint? Explain your answer.

B-2-2 Fall 1998

1. Given the y,ybar coordinates (0,-10), (10,0), (0,5) and a focal length of 100 mm, draw the y,ybar diagram and determine the distances involved. (20%)

2. Replace the element in problem 1 with a pair of elements having focal lengths of 100 mm and 80 mm, preserving the original object and image rays. Determine the distances. (30%)

3. If an element with a focal length f_0 is to be replaced with two elements with focal lengths f_1 and f_2, show that the direction numbers α_2 and β_2 of the intermediate ray can be obtained from

$$\alpha_2 = \left(\frac{f_0}{f_2}\right)\alpha_1 + \left(\frac{f_0}{f_1}\right)\alpha_3$$

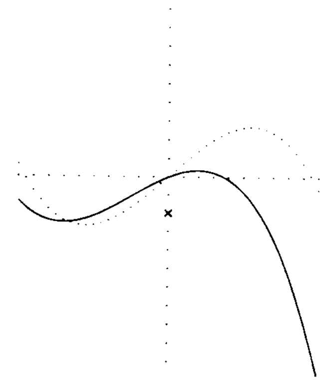
and

$$\beta_2 = \left(\frac{f_0}{f_2}\right)\beta_1 + \left(\frac{f_0}{f_1}\right)\beta_3$$

where α_1 and β_1 are the direction numbers of the input ray and α_3 and β_3 are for the output ray. (50%)

A

1



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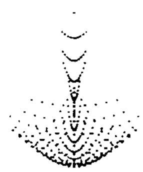

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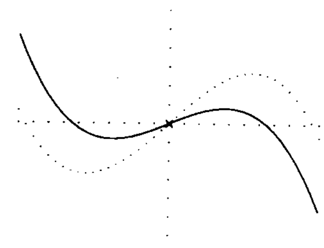

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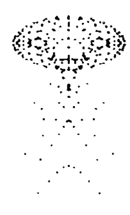

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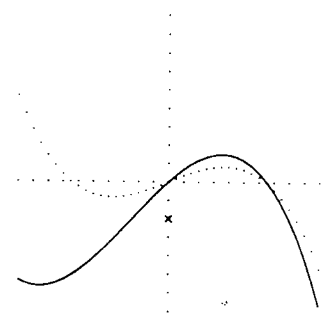
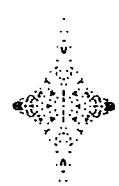


D



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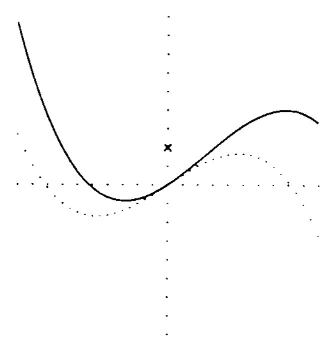
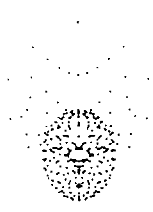
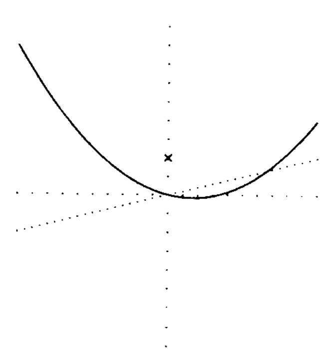


E



5



B-2-3 Fall 1998

1. Match up the spot diagrams and the ray fan plots. (30%)
2. Identify the aberrations (30%)
3. Estimate the relative magnitudes of the wave aberration coefficients. (40%)

B-2-4

Fall 1998

Discuss the strategy you would use to design (1) a telescope doublet, (2) a Cooke triplet, and (3) a double Gauss camera lens. What are the considerations in the design of a microscope objective? An eyepiece?

Consider optical propagation in a nonlinear slab waveguide as described by the following equation for the electric field envelope $\mathcal{E}(x, z)$

$$\frac{\partial \mathcal{E}}{\partial z} = ik_0 n_2 |\mathcal{E}|^2 \mathcal{E} + \frac{i}{2k} \frac{\partial^2 \mathcal{E}}{\partial x^2}, \tag{1}$$

where $k = n_0 k_0 = n_0 \omega / c$, ω being the frequency, n_0 the linear refractive-index, and n_2 is the nonlinear coefficient. Here z is the propagation distance along the waveguide and x the transverse coordinate perpendicular to the slab waveguide structure.

a) First consider Eq. (1) ignoring diffraction and an initial beam $\mathcal{E}(x, 0) = \mathcal{E}_0(x)$. Obtain an expression for the nonlinear phase-shift $\phi(x, L)$ accumulated by the field over a short distance L due to self-phase modulation (20%).

b) For an initial Gaussian beam $\mathcal{E}_0(x) = \sqrt{I_0} \exp(-x^2/x_0^2)$ calculate the local spatial frequency $\Delta K(x, L) = \partial \phi(x, L)/\partial x$, and sketch its variation with transverse coordinate x for a distance L (20%).

c) Next consider the case that the incident field is a truncated Gaussian beam $\mathcal{E}_0(x) = \theta(x)\sqrt{I_0} \exp(-x^2/x_0^2)$, with $\theta(x)$ the Heaviside step function. After propagating through a short length L of nonlinear waveguide the field evolves in free-space under the action of linear diffraction (you may neglect diffraction in the y-direction). Give a physical explanation of why the beam is deflected away from $x = 0$ with propagation past the waveguide, stressing the difference between the self-focusing and self-defocusing cases (30%).

d) Obtain an approximate formula for the beam deflection angle (30%).

(A) The emission of many types of lasers exhibits intensity spikes associated with relaxation oscillations. Briefly explain why this occurs. (10)

In order to describe relaxation oscillations in the output of a laser, consider a homogeneously broadened, ideal, 4-level laser and answer the following:

(B) Write down the rate equations for the population inversion and the density of photons in the cavity mode, and solve these expressions for the condition of equilibrium. (20)

(C) Assume a slight increase in pumping that perturbs the system so the population inversion and photon density are slightly above the threshold values. Solve the rate equations under this condition to show that the density of photons in the cavity is described by a damped harmonic oscillator expression. (50)

(D) Solve the expression derived in (C) to show that the laser output appears as a decaying oscillation with a decay constant and oscillation frequency that both increase as the pump rate increases. (20)

Prelim Question

This question is about normal-mode coupling (NMC) in a semiconductor microcavity. Let the cavity decay rate be κ, the quantum-well exciton homogeneous decay rate γ, and the dipole coupling rate between them Ω.

1. What is the condition for seeing NMC?

2. Describe an experiment for seeing NMC. Sketch what you expect to see.

3. Describe a method for measuring κ.

4. Sketch the change in transmission for an NMC microcavity (with a large splitting-to-linewidth ratio) as the carrier density is increased.

5. Sketch the change in exciton absorption coefficient with increased density, and explain how this causes the NMC transmission change in (d).

6. Describe an experiment to determine whether your NMC microcavity is operating semiclassically or quantum statistically?

7. Suppose that the cavity peak is tuned on the high energy side of the exciton peak by a few exciton linewidths. Sketch the photoluminescence intensities I_{lower}^{PL} and I_{upper}^{PL} as a function of carrier density, from zero density to above the lasing threshold density.

8. Approximating the exciton resonance by a homogeneously broadened Lorenztian of width γ and the Fabry-Perot cavity transmission peak by a Lorentzian of width κ, what is the NMC linewidth of the lower branch, δ_{lower}, and the upper branch, δ_{upper}?

9. Structural disorder makes the actual lineshape of a quantum well exciton resonance inhomogeneously broadened and asymmetric. Sketch such a lineshape. What effect does this lineshape have upon δ_{lower} and δ_{upper}?

10. Write down the susceptibility of a QW including only the 1s exciton and explain all of the parameters.

C-2-4 Fall/1998

Answer all three problems:

Problem 1:
Assume that TE polarized light having a vacuum wavelength of $\lambda_0 = 0.65\ \mu m$ encounters a glass plate with $n = 1.5$. If the plate has a thickness $d = 1.5\ \mu m$, and is used as a planar waveguide, with vacuum being the cover and substrate regions, will the light propagate as a guided mode in the waveguide if its bounce angle is 22.558°? Justify your answer (33%).

Problem 2:
Assume the plate of Problem 2 is used as a one-dimensional resonator instead of a waveguide. Its surfaces are mirrored and the light propagates back and forth across the thickness of the plate ($d = 1.5\ \mu m$). If the mirror reflectances are $\Re_1 = \Re_2 = 0.99$, and the loss coefficient is $\alpha_s = 1\ cm^{-1}$, calculate: i) The spacing between resonance frequencies; ii) The overall distributed loss coefficient; and iii) The finesse for this resonator (33%).

Problem 3:
Consider a 20 cm long He-Ne laser operating at its central frequency. Mirror 1 has a reflectance of 100%. The refractive index, $n = 1$, the absorption coefficient of the medium is zero, and the saturation photon flux density is 10^{19} photons/cm^2/sec. The total effective distributed loss coefficient of the resonator is $5.051 \times 10^{-4}\ cm^{-1}$.

a) Calculate the reflectance of mirror 2 (17%).
b) If, under steady-state conditions, the internal flux density is measured to be 10^{21} photons/cm^2/sec, then calculate the value of the small-signal gain coefficient (17%).

Some equations you might need are given below (not in any particular order):

$c_0 = 3 \times 10^{10}$ cm/sec $\qquad c = \lambda\nu \qquad k = 2\pi/\lambda$

$n_1\sin\theta_1 = n_2\sin\theta_2 \qquad \theta_B = \tan^{-1}(n_2/n_1)$

$(4\pi d/\lambda)\sin\theta = 2\pi m \qquad \tan(\varphi_r/2) = [(\sin^2\bar{\theta}_C)/\sin^2\theta) - 1]^{1/2}$

$\tan[(\pi d/\lambda)\sin\theta - m\pi/2] = [(\sin^2\bar{\theta}_C)/\sin^2\theta) - 1]^{1/2} \qquad I = I_0\sin^2(m\varphi/2)/\sin^2(\varphi)$

$\nu_q = qc/2d \qquad M(\nu) = 4/c \qquad \nu_F = c/2d$

$I = I_{max}/[1 + (2\Im/\pi)^2\sin^2(\pi\nu/\nu_F)] \qquad I_{max} = I_0/(1-r)^2 \qquad \delta\nu \approx \nu_F/\Im$

$\alpha_r = \alpha_s + [1/(2d)]\ln[1/(\Re_1\Re_2)] \qquad \Im = \pi\exp(-\alpha_r d/2)/[1 - \exp(-\alpha_r d)]$

$\Im \approx \pi/\alpha_r d \qquad M(\nu) = 8\pi\nu^2/c^3 \qquad p_{sp} = c\sigma(\nu)/V$

$P_{st} = nc\sigma(\nu)/V \qquad P_{sp} = 1/t_{sp} \qquad \sigma(\nu) = \lambda^2 g(\nu)/8\pi t_{sp}$

$W_i = \phi\sigma(\nu) \qquad \gamma(\nu) = N\sigma(\nu) \qquad G(\nu) = \exp[\gamma(\nu)d]$

$N = N_0/(1 + \tau_s W_i) \qquad \tau_s = \tau_2 + \tau_1(1 - \tau_2/\tau_{21}) \qquad N_0 \approx R_2 t_{sp}$

$\gamma(\nu) = \alpha_r = \gamma_o(\nu)/(1 + \phi/\phi_s) \qquad g(\nu_o) = 2/(\pi\Delta\nu) \qquad g_o = 2\gamma_o d$

$T_{op} = (g_o L)^{1/2} - L \qquad L = 2(\alpha_s + \alpha_{m2})d \qquad M(\nu) = 4\pi\nu/c^2$

$I = I_1 + I_2 + 2(I_1 I_2)^{1/2}\cos\varphi \qquad \tau_p = 1/(\alpha_r c) \qquad M \doteq 2dNA/\lambda_o$

$NA = (n_1{}^2 - n_2{}^2)^{1/2} \qquad M \doteq 2d\sin\bar{\theta}_c/\lambda \qquad I = I_o\exp(\gamma d)$

$N_t = 8\pi t_{sp}/[\lambda^2 c\tau_p g(\nu)] \qquad g(\nu) = \Delta\nu/\{2\pi[(\nu - \nu)^2 + (\Delta\nu/2)^2]\}$

$\alpha_r = \alpha_s + \alpha_{m1} + \alpha_{m2}$

1) a) (20 %) What is the OTF of a circular pupil optical system in terms of the MTF of an aberration free optical system, if there are 3 fringes of x-tilt across the aperture?

b) (20 %) An aberration of the form $Ay(x^2+y^2)$ is present. What can be said about the phase of OTF[ν_x, 0]?

A 50-mm diameter lens operating at a wavelength of 500 nm has a cutoff frequency of 100 lines/mm in image space when the lens is used to image an object at a magnification of 3.

c) (20 %) What is the cutoff frequency in object space?
d) (20 %) What is the focal length of the lens?
e) (20 %) What will the cutoff frequency in image space become if the lens has a central obscuration of 10-mm diameter?

(a) What is the stochastic process called a "superposition" process? Define all necessary parameters and functions, noting which are random and which deterministic.

(b) What is the particular superposition process called a "Shot noise" process?

(c) Define the particular superposition process, describing a photographic emulsion, called the "poker chip" or "overlapping circular grain" model.

(d) What are, then, the mean, variance, and autocorrelation function of the photographic density, based upon this model?

Ground-based telescopes are severely limited by atmospheric turbulence. Even if the mirror is diffraction limited, a 5 m telescope gives about the same resolution as a much smaller one. In 1970 Antoine Labeyrie published a method to overcome this limitation, basically by taking a large number of images, each with a short exposure time, and subsequently processing them to get a single high-resolution image. This method is usually referred to as stellar speckle interferometry. The main objective of this problem is to walk through the steps in this method and give a pictorial explanation of each.

Assume the following parameters: a 5 m diffraction-limited mirror with unspecified focal length (you won't need the focal length if you work in angular units), wavelength of 500 nm, atmospheric scintillation time of 50 msec. You can model the atmosphere as a thin phase screen in the pupil. For the interferometry data, assume that 1000 images are recorded, each with an exposure time of 20 msec, with an image taken every 100 msec. For comparison, a single image with an exposure time of 20 seconds is also recorded. The short-exposure images are recorded on movie film, and all photographic processes can be assumed to yield an amplitude transmittance that is linear in exposure.

(a) Compute the diffraction-limited angular resolution of the telescope.

(b) Suppose that the observed angular resolution in the 20 sec image is 2 arc-seconds. Estimate the size of a turbulence cell. Estimate the angular size of a speckle blob in a short-exposure image.

(c) Suppose only a single bright star is in the field of view of the telescope. Sketch a 20 msec image (a speckle pattern). Sketch another 20 msec image taken 200 msec after the first one. Sketch the 20 sec image. Label relevant dimensions.

(d) Now suppose there are two stars of equal brightness with a separation of 0.1 arc-second. Repeat the sketches of part (c). Again, label relevant dimensions.

(e) In Labeyrie's method, an optical Fourier transformer is used to compute the squared-modulus of the Fourier transform of each image. Sketch this function for the 20 sec image of the double star and for one of the 20 msec images.

(f) Labeyrie suggested running the movie film through the input plane of the optical Fourier transformer while integrating the transform by a time exposure on a stationary piece of film in the transform plane. Discuss this operation. What is the effect of the motion of the movie film? Sketch the final integrated transform for the case of a double star.

(g) Now suppose the film with the integrated transform is developed and inserted into the input plane of the transformer. What does its Fourier transform look like for the double star. What information about the double star can you get readily?