WRITTEN PRELIM EXAM – FIRST DAY
SPRING 1998

February 24, 1998
8:30 a.m. to 12:30 p.m.

Please answer two questions from each of the four categories (for a total of 8 questions).

Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$

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$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2 \cos^2 A - 1$

$\cos 2A = 1 - 2 \sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\boldsymbol{\ell} = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

If monopoles, exist Maxwell's equations would require two more terms. These are introduced to account for the magnetic charges and magnetic currents. Maxwell's equations take the form:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}, \quad \nabla \times \vec{E} = -\vec{J}^* - \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = \rho^*, \quad \nabla \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

where ρ^* is the magnetic charge density, expressed in webers /meter^3 and $\vec{J}^*$ is the magnetic current density, in webers/second-meter^2.

1. Obtain the equation of continuity for the magnetic charge.
2. Show that, by analogy with electric fields, near a point magnetic charge Q*, the magnetic induction field is in the radial direction and that it depends on $1/r^2$.
3. Obtain the energy law of the electromagnetic field including the dissipative terms. Give a physical interpretation of your equation.

[Equal weight for all parts]

1. Discuss the continuity of each one of the four electromagnetic fields across a boundary of two a homogeneous media (40%)

2. What are the wave equations of the electromagnetic field for homogeneous media? (30%)

3. Discuss the phase and group velocities of an electromagnetic wave, and explain under what conditions do they differ (30%)

1) A Michelson Stellar interferometer is used to measure the separation of binary stars.
   a) (50%) What is the separation of the binary stars, in micro-radians, if the first minimum of fringe visibility is obtained with a 2-meter mirror separation? The wavelength is 500 nm.
   b) (40%) An unresolved single star is observed using the Michelson Stellar interferometer having the 2-meter mirror separation. A spectral filter is used that passes a spectrum that extends from 500 nm to 550 nm. How many interference fringes are observed between the zero-order fringe and the first minimum of the fringe visibility function. State any assumptions being made.
   c) (10%) Repeat Part b for a 3-meter mirror separation.

<u>B-1-1</u> <u>Spring 1998</u>

A plano-convex lens has a focal length of 100 mm, a thickness of 15 mm and an index of refraction of 1.5. This lens is now immersed into an environment of index n.

Determine the power of the lens and the location of its cardinal points as a function of n. All distances must be measured relative to the plano surface of the lens. Evaluate these results for several values of n between 1.0 and 2.0.

An achromatic thin wedge prism is to be designed out of glasses LaK N12 (678552) and SF56A (785261). The prism angle of the lower dispersion prism is 5 degrees. Determine the other prism angle and the net deviation of the prism system.

A noiseless diode laser ($\lambda = 670$ nm) is modulated in the voice band (200-2000 Hz). To receive the signal, a photodiode ($\eta = 0.8$, $T = 300K$) will be interfaced to a noiseless transimpedance amplifier (TIA) with the feedback resistor set equal to the detector resistance. The power on the detector is 10^{-10} watts.

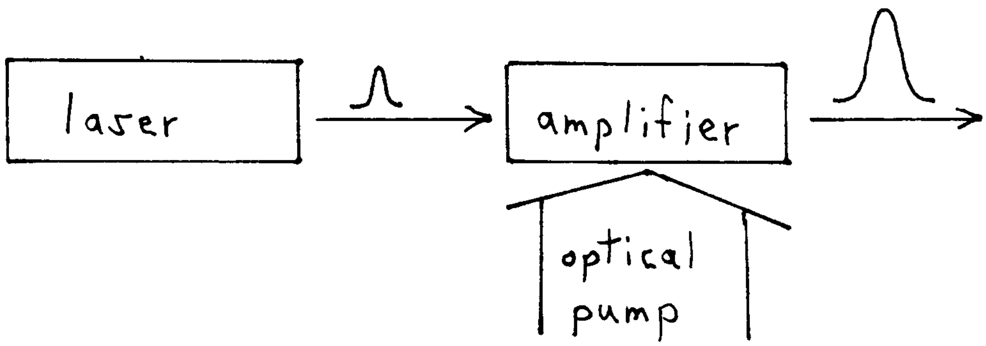
a) Identify the "white" noise sources. (10%)

b) Discuss the selection of an appropriate noise bandwidth required to pass the desired frequency range? (20%)

c) For each of the following four detector types listed in the table below, determine the signal current, the ("white") noise current, the output voltage of the TIA and the SNR. (70%)

DETECTOR	Io (amps)	Eg (eV)
Ge	10^{-8}	0.7
Si	10^{-10}	1.1
GaAs	10^{-11}	1.5
GaP	10^{-12}	1.9

We want to consider the behavior of an amplifier (i.e., a gain medium without mirrors around it) for a pulsed laser.



(a) Suppose that the energy level structure of the amplifier material is that of a "3-level laser". Sketch the appropriate energy level diagram for such an amplifier and discuss the characteristics that the energy levels and the transitions between them should have in order for the amplifier to be as efficient as possible (i.e., to provide maximum gain per unit of input pump energy).

(25%)

(b) Write down an appropriate set of rate equations for the amplifier, assuming that the amplifier is being optically pumped by an external flashlamp. Make sure you explicitly show the form of the pump terms for this situation.

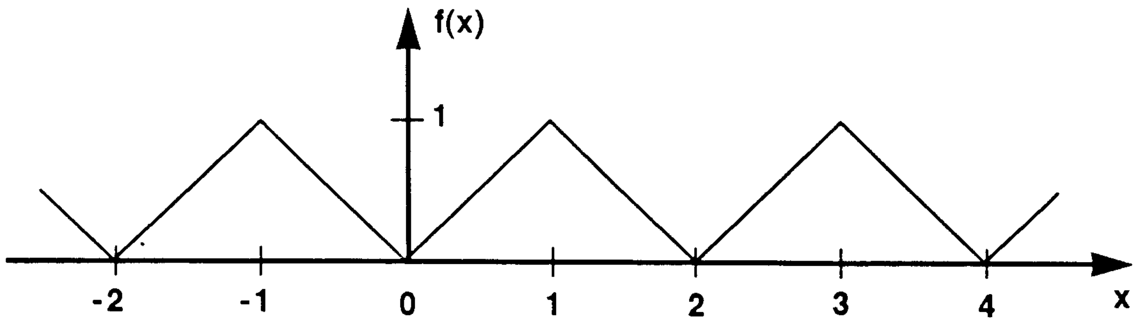
(30%)

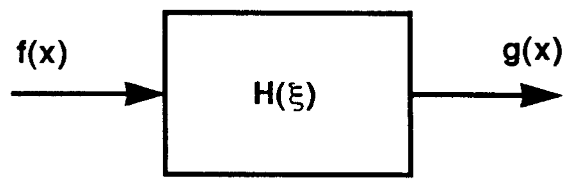
(c) By increasing the amplifier's length, its gain can be increased. Is there a fundamental limit to how high we can make the gain, or is it just limited by practical considerations such as how big an amplifier we can construct and pump? Explain your answer.

(20%)

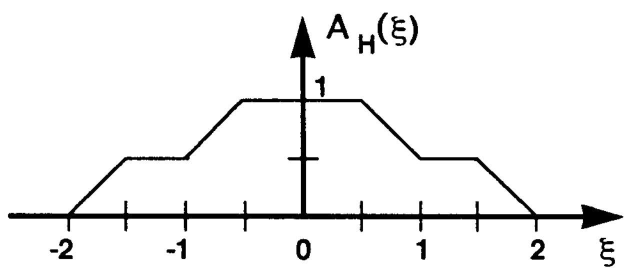
(d) Suppose that the pulse produced by the laser is resonant with the transition being used in the amplifier, and that the pulse is shorter than the decay time for the upper state of the amplifier's transition. If the total energy of the pulse is just right, the pulse will pass through the amplifier and experience virtually no amplification even though the steady state gain of the amplifier may be very high. Explain what is happening inside the amplifier to produce this phenomenon.

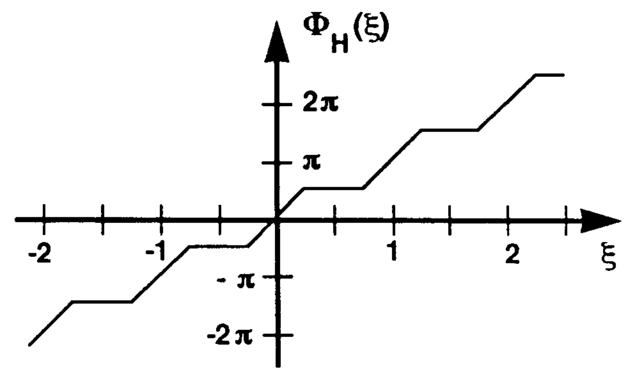
(25%)





$$H(\xi) = A_H(\xi)\, e^{-j\Phi_H(\xi)}$$





The input signal to a linear, shift-invariant filter is a periodic triangle-wave function of amplitude 1 and period 2. The amplitude- and phase-transfer functions of the system are as shown.

(a) Find an expression for the output signal g(x). (70%)

(b) Sketch a few periods of g(x), showing important features. (30%)

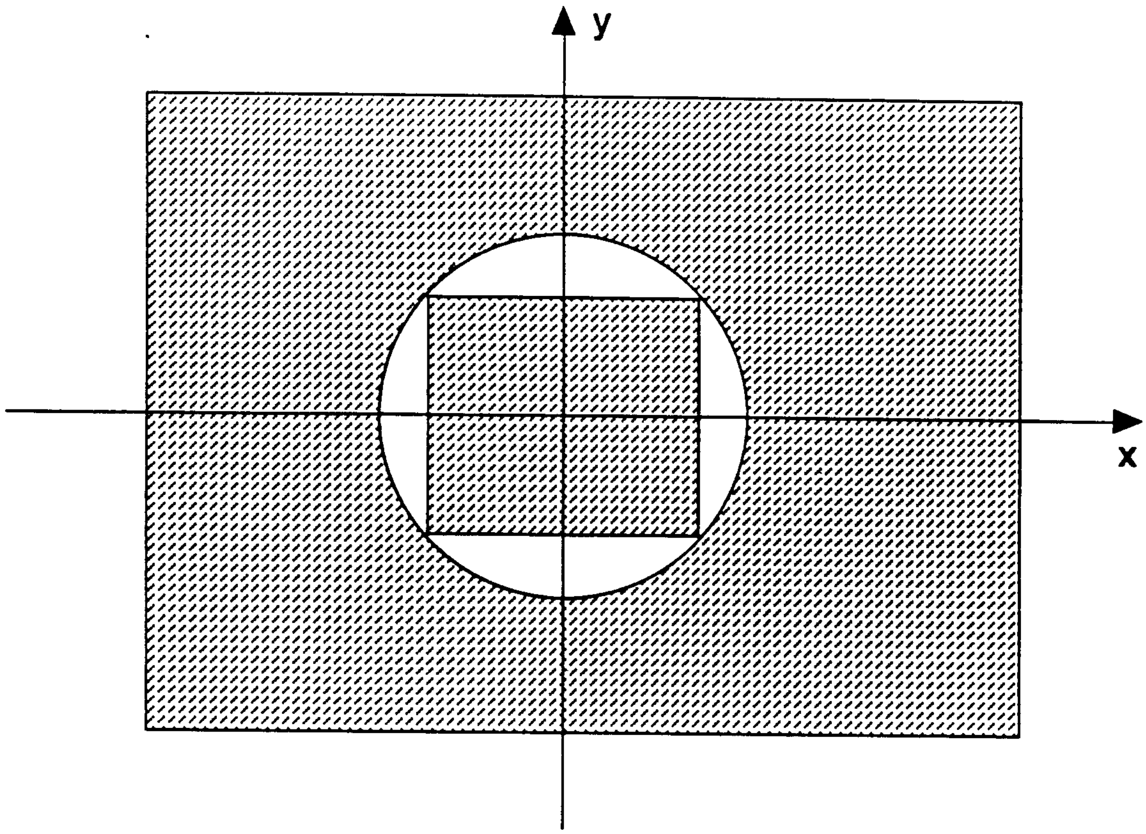
(20%) (a) State the partition law (also sometimes called the 'law of total probability'), carefully defining all statistical assumptions for which the law holds true.

(80%) (b) Suppose that a set of fringes is shown superimposed upon a uniform background intensity level. As the background is made brighter relative to the fringe contrast level, or, as the fringes are made to have lower contrast, they become more difficult to detect.

An observer is shown a viewing screen that is divided into four equal areas. Each area contains the same background intensity level, but only one has a fringe image superimposed upon the background. The observer is asked to identify which of the four areas has the fringes. Even if the observer believes that he does not know where the fringes are, he is required to guess at their location (this is called 'forced choice' detection, and is used to allow for subliminal detection).

This defines a test procedure that is independently repeated a large number of times, under different conditions of background brightness and fringe contrast. The manager of the test procedure randomly changes the location of the fringes among the four areas. The observer is found to correctly identify the fringe locations in fraction p of the tests. However, this includes correct guesses as well as actual detections of the fringes. What is the observer's rate of detection P of the fringe locations, exclusive of guesses? (*Hint:* this is also the test score, corrected for guessing, in a multiple-choice test where each question has four possible answers.) Show all work. A correct guess (subliminal or not) will receive only minimal credit.

An opaque screen, located in the $z = z_1$ plane, has a circular aperture with a square obscuration as shown in the figure. This aperture, which has a diameter of 2 cm, is illuminated with a normally incident, coherent plane wave of $\lambda = 500$ nm and amplitude A, where $|A|^2 = 10$ mW / cm^2, and the diffraction pattern is observed in the $z = z_2$ plane located a distance of $z_{12} = 10$ Km away.



Either: (a) make a normalized sketch of the diffraction pattern irradiance along the x axis, indicating such pertinent features as locations of zeros, relative magnitude of side lobes, etc.;

or: (b) find the value of the irradiance at the point $(0, 0, z_2)$.

WRITTEN PRELIM EXAM – SECOND DAY

SPRING 1998

February 25, 1998
8:30 a.m. to 12:30 p.m.

Please answer any five questions.

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$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

1) A Mach-Zehnder interferometer using a 633 nm light source is used to test a diffraction grating. When the grating is illuminated at normal incidence the first-order diffraction angle is 5 degrees.

   a) (40%) The resulting interference pattern obtained by interfering the first-order diffracted beam with a reference beam shows a peak-valley error of 1/10 fringe. What is the peak-valley error in units of microns in the grating line spacing? State any assumptions being made.

   b) (30%) The same grating is tested using a wavelength of 500 nm. What is the peak-valley error of the resulting interference fringes?

   c) (30%) The interferometer is adjusted to interfere the second-order diffracted beam with the reference beam. What is the peak-valley error in the resulting interference pattern? The wavelength is 633 nm.

1. What is the physical meaning of retarded potentials? (20%)

2. Describe the Hertz vectors and a possible application for them. (30%)

3. What is the finesse of an etalon and what is its value for $r=\sqrt{R}=.9$ and 0.99. (20%)

4. Describe total internal reflection and Brewster angle, plot their dependence on the angle of incidence, explain their polarization dependence, and identify a possible application of each. (30%)

A noncoherent source of (mean) wavelength λ illuminates an x-y plane at distance z. The irradiance distribution on this plane is a constant I_0 and the spatial coherence function, to a good approximation, has the form,

$$\Gamma(x_1 - x_2,\ y_1 - y_2,\ z) = I_0 \exp\left[-\frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{(\lambda z/\alpha)^2}\right],$$

(a) Derive and describe the source characteristics that produced this spatial coherence function and give a physical interpretation of the parameter α with reference to the source.

(b) Obtain the spatial coherence function in a plane $z' > z$ and obtain the interval of spatial coherence in the plane z'.

[Equal weight for parts a and b]

The entire question is concerned with one wavelength only. Our usual conventions for phase shift on reflection apply. If you use a different convention you must state it. Remember that phase shift on reflection is defined at the surface where the reflection is taking place.

(a) Sketch in an admittance diagram the form of the contours of constant reflection for an incident medium of admittance y_0. Actual values are not required but indicate the way in which the contours vary as the reflectance increases. (20%)

(b) Add to the diagram the boundaries of the four quadrants of phase shift on reflection. Label the four quadrants. (20%)

(c) The internal (substrate side) and external reflectances of thick metal films deposited on a substrate are normally different. Let the admittance of a metal be α-iβ. Let the external medium have admittance y_0 and the substrate y_{sub}. Show by any means that the internal and external reflectances are equal if

$$\alpha^2 + \beta^2 = y_0 y_{sub}$$ (40%)

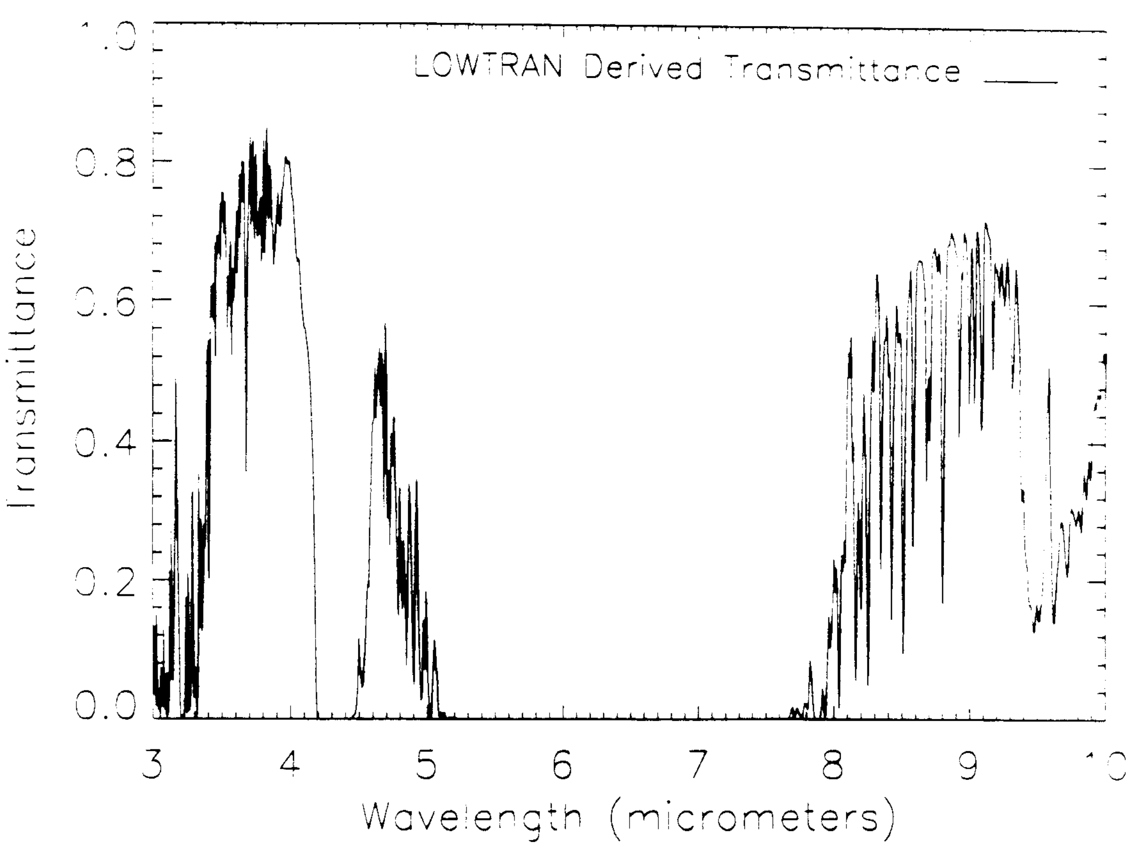
(d) Assume that $y_0 < y_{sub}$. Show by any means that when the internal reflectance is greater than the external then the phase shift associated with the external reflectance must necessarily be in the second quadrant but that this is not necessarily true for the internal reflectance. (20%)

Assume there is a satellite orbiting a planet with no atmosphere. Compute the radiance (integrated over all wavelengths) at the satellite (viewing the dark side of the planet) if the surface temperature of the planet is 600K. Be sure to state all assumptions. (30%)

If an isothermal atmosphere at a temperature of 550K and emissivity of 0.9 is added above the surface, describe what happens to the radiance at the sensor? (20%)

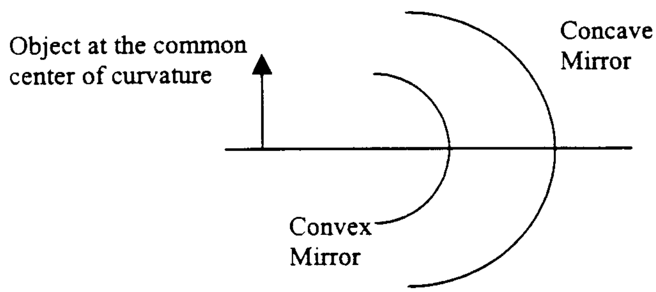
If the atmosphere now has a spectral transmittance as shown in the figure below, what center wavelength would you select for a 100 nm wide spectral band and a detector with flat spectral responsivity if you want to obtain data with a high signal-to-noise ratio and thermal information about the surface? Explain your answer. (20%)

Now assume the temperature of the atmosphere decreases linearly with height from a value of 600K at the surface to a value of 400K at the top of the atmosphere. Sketch a graph of radiance versus view angle from 0 to 60 degrees (where 0 degrees is viewing normal to the surface) for a band centered at 9 μm. Explain the shape of your curve. (30%)



A three-reflection optical system is constructed out of concentric concave and convex spherical mirrors. The larger concave mirror has a radius of curvature of -200 mm, and the smaller convex mirror has a radius of curvature of -100 mm. The mirrors are separated by 100 mm. The light path goes first to the concave mirror, then to the convex mirror, returns to the concave mirror, and finally to the image.

The object plane is located at the common center of curvature and is perpendicular to the optical axis. The system is telecentric in image space, and the stop diameter is 10 mm.



a) (30%) Determine:
- the focal length of this system
- the image location and its magnification
- the stop location required to make the system telecentric.

b) (50%) Calculate the amount of spherical aberration, astigmatism, field curvature, coma and distortion in this system (W040, W222, W220, W131 and W311).

c) (20%) Assuming that there are negligible aberrations, what is the linear resolution of this system when used with HeNe laser illumination.

B-2-3 Spring 1998

Describe what happens to spherical aberration, coma and astigmatism of an optical system when the elements are tilted and/or decentered:

a) (50%) When the unperturbed system is corrected for these aberrations.

b) (50%) When the unperturbed system is not corrected.

In Colorado, auto travelers are protected from snowdrifts across the freeway by snow fences. A curious traveler who was not aware of these fences decided to investigate the structure he noticed in the distance. Typically such a picket fence is made of 4-inch black boards on equal spacing with white snow as the background, and it essentially represents a bar pattern with 100% modulation.

a) What is the contrast of the object?(10%)

b) While viewing such a fence, a positive identification of the fence can be made with a resolution of 5 lp / degree. At what distance can the traveler determine that the distant structure is such a fence if the atmospheric transmission is 100%? (10%)

c) Now introduce an absorbing atmosphere which will affect the contrast. Prove that the contrast at the sensor is equal to the atmospheric transmission. For this case, there is no atmospheric scatter (IR spectrum), and long path radiance of the atmosphere can be modeled by $\varepsilon_p L_{bb}$, where ε_p is the effective path emissivity. (40%)

d) At what distance can an infrared camera see the fence if it has an NEFD at the entrance pupil of 10^{-10} (watt/cm^2). The camera has an IFOV sufficiently small to resolve the fence, is sensitive to all wavelengths (0 to ∞), the fence posts have an emissivity of 0.5 and the snow emissivity is 1.0. The temperature of the environment is 0°C, and the fence pickets are at 5°C. (40%)

Consider beam propagation in a nonlinear self-focusing medium as described by the following equation for the electric field envelope $\mathcal{E}(r, z)$

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{i}{2k}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\mathcal{E} + ik_0 n_2 |\mathcal{E}|^2 \mathcal{E},$$

where z is the propagation direction, $k = n_0 k_0 = n_0\omega/c$, ω being the field frequency, n_0 the background refractive-index, and $n_2 > 0$ is the nonlinear coefficient, $|\mathcal{E}|^2$ being scaled so that it is an intensity. Here we consider a Gaussian input beam

$$\mathcal{E}(r, 0) = \mathcal{E}_0 e^{-r^2/w_0^2},$$

where w_0 is the input spot size.

(a) Ignoring beam diffraction meanwhile for a thin medium of length $L << kw_0^2/2$, show that the field suffers a radially dependent nonlinear phase-shift

$$\phi^{NL}(r, L) = k_0 n_2 I_0 L e^{-2r^2/w_0^2},$$

upon propagating through the medium, where I_0 is the peak input intensity (25%).

(b) The nonlinear phase-shift produces a local radial wavevector

$$\mathbf{K}_T(r) = \hat{\mathbf{r}}\frac{\partial \phi^{NL}}{\partial r},$$

where $\hat{\mathbf{r}}$ is the unit vector in the radial direction. Using this concept give an intuitive explanation of why self-focusing causes the incident beam to come to a focus beyond the medium ($z > L$) for high input powers (25%).

(c) Using the local radial wavevector of largest magnitude, estimate the distance f at which the field comes to focus (30%).

(d) As the nonlinear focal length f approaches the medium length L the effects of diffraction in the medium can no longer be ignored. From this obtain an estimate of the critical power for self-trapping in the medium (20%).

A symmetric, confocal laser resonator operates in the lowest order transverse mode at a wavelength of 1 μm. The end mirrors are 1 cm in diameter and the resonator is 1 m long. If the lasing material with an index of refraction of n=1 fills the resonator and gives a loss of 0.01 cm^{-1} and the fractional losses of the two end mirrors are 0.01 and 0.10, answer the following questions about the characteristics of the laser. Show all steps necessary to justify your answers.

1) What is the minimum beam waist of the laser mode in the cavity? [20%]

2) What is the Rayleigh range of the laser beam? [10%]

3) What is the beam divergence? [10%]

4) What is the frequency separation between the laser operating frequency and the next highest longitudinal mode? [20%]

5) What is the Q of the cavity? [20%]

6) What is the lifetime of a photon in this cavity mode? [10%]

7) What is the frequency width of this cavity mode? [10%]

Consider a gas of two-level atoms illuminated by a monochromatic plane wave $\vec{E} = \frac{1}{2}\vec{\varepsilon}\, E_0\, e^{i(kz-\omega t)} + c.c.$ The light-matter interaction is described by the optical Bloch equations

$$\dot{u} = -\beta u - \Delta v, \qquad \dot{v} = -\beta v + \Delta u + \chi w, \quad \dot{w} = -2\beta(w+1) - \chi v.$$

Here $\chi = -\vec{d}_{21} \cdot \vec{\varepsilon}\, E_0 / \hbar$ is the Rabi frequency, and $\Delta = \omega_0 - \omega$ is the detuning of the driving field from atomic resonance. In the following we assume that the atomic dipole moment $\vec{d} \| \vec{E}$ and drop the vector notation.

(a) Give an expression for the expectation value $\langle d \rangle$ of the atomic dipole moment in terms of the Bloch variables. It may help to remember the definition of the Bloch variables, $u = \rho_{21} + \rho_{12}$, $v = i(\rho_{21} - \rho_{12})$ and $w = \rho_{22} - \rho_{11}$. (10%)

(b) Find the steady state value of $\langle d \rangle$ in terms of the quantities χ, Δ and β. (30%)

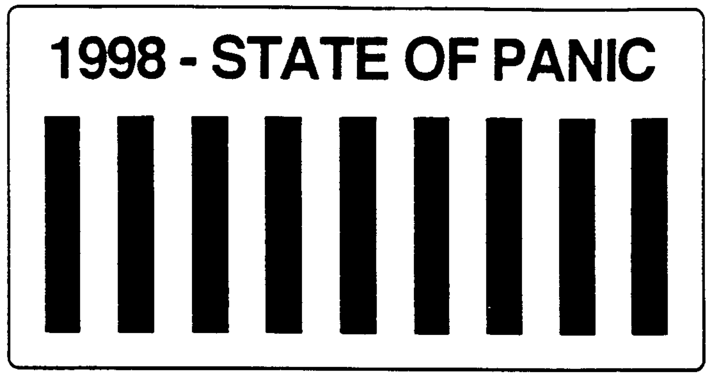
(c) We can define a complex polarizability in the usual way. Show that it has the form

$$\alpha(\omega) = -|d_{12}|^2 \frac{\Delta + i\beta}{\Delta^2 + \beta^2}\left[1 - \frac{\chi^2/2}{\Delta^2 + \beta^2} + \ldots\right].$$

Discuss any qualitatively new features compared to the classical Lorentz atom. (30%)

(d) Consider now the case where the driving field is a Gaussian laser beam. How does the χ^2 term in the *real* index of refraction affect beam propagation for $\Delta > 0$, $\Delta = 0$ and $\Delta < 0$? The phenomenon becomes especially important above a certain critical beam power. Explain in words what happens and why. (30%)

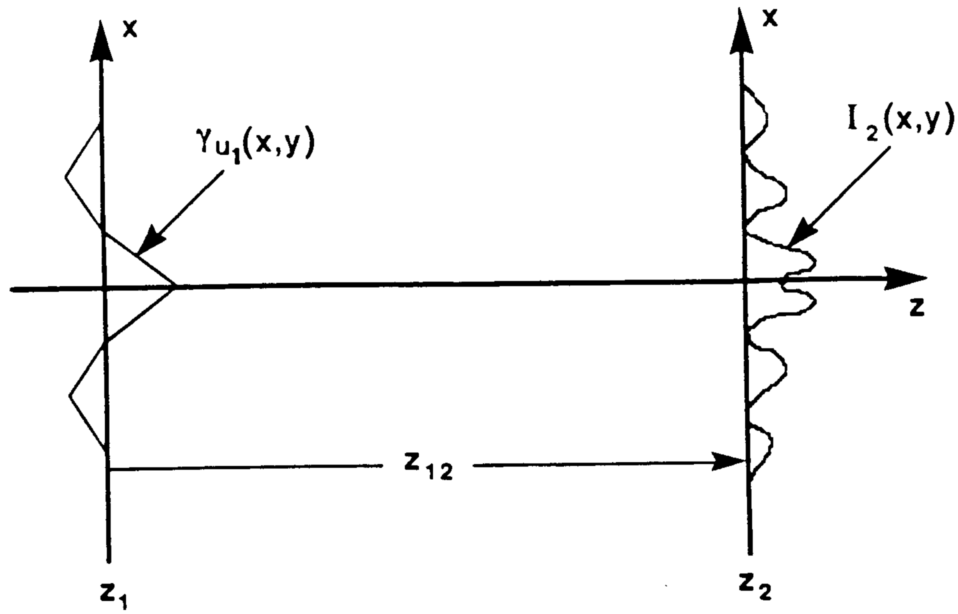
A surveillance camera is used to gather information about movements of automobiles near the White House, and it is necessary to image the license plates of these automobiles. We assume the license plates to be "ideal license plates", displaying a white-and-black bar pattern as shown in the figure, the period of which is 2.5 cm.



For simplicity, assume that the aperture stop of the camera is circular, with diameter $\mathbf{d} = 4$ cm, the entrance and exit pupils are located at the stop, the image plane is located a distance $\mathbf{f}$ from the stop and the camera is located a distance $\mathbf{z_o}$ from the automobiles in question. Further assume that each license plate is characterized by its radiance distribution and that the light is incoherent, but narrowband, with an effective wavelength of $\lambda = 500$nm.

(a) What is the maximum distance $\mathbf{z_o}$ that will allow the stripe pattern of the license plate to be imaged such that the <u>modulation</u> of the fundamental-frequency Fourier component is reduced to no less than 40% of its original modulation as described by the license plate radiance distribution? (50%)

(b) If the license plate images of Part (a) are to be recorded by a 400 x 600-element CCD array, with center-to-center element spacings of 10μm, what is the minimum value of $\mathbf{f}$ (or minimum value of lateral magnification $\mathbf{m}$) required to eliminate aliasing for any orientation of the license plate images on the array? (50%)



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(b) Now if $K = 1\mathrm{W}$, $\eta_0 = 100\ \mathrm{m}^{-1}$ and $z_{12} = 10^5$ m is large enough for z_2 to be in the Fraunhofer region of $u_1(x,y)$, find an expression for the irradiance $I_2(x,y)$, and sketch the profiles $I_2(0,y)$ and $I_2(x,0)$.
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Spectral lines often have Cauchy law (Lorentzian) line shapes. It is required to estimate the true position λ_0 (in wavelength λ space) of the single Cauchy line emanating from a given source. Denote the parameter of the Cauchy law as a.

Data for forming the estimate are taken. These are the independent spectral locations $\lambda_1, \ldots, \lambda_N$ of N photons from the source. To form the estimate $\hat{\lambda}_0$ of λ_0, the arithmetic mean of the data is formed. Denote the mean-square error due to this estimate as e^2.

(20%) (a) What is the usual rule for forming the mean-square error in the sample mean due to N independent data? The answer for e^2 in our case does not follow this rule. Why not?

(80%) (b) Find e^2, showing all work. A correct guess, without any backup analysis, is only worth minimal credit.

The Green's function $G(\mathbf{r}, t)$ for the time-dependent scalar wave equation satisfies

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]G(\mathbf{r}, t) = -4\pi\delta(t)\delta(\mathbf{r}) \,, \tag{1}$$

and it is given explicitly by

$$G(\mathbf{r}, t) = \frac{1}{r}\delta\left(t - \frac{r}{c}\right), \tag{2}$$

where $r = |\mathbf{r}|$.

(*a*) Explain qualitatively what $G(\mathbf{r}, t)$ means. Why is it proportional to $1/r$? Why is there a temporal delta function?

(*b*) If a radiation field $u(\mathbf{r}, t)$ is produced by a specified source $s(\mathbf{r}, t)$, how can it be computed with the aid of the Green's function?

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(*d*) Using (2) and the result of part (*b*), derive the Green's function $G(\mathbf{r})$ for the wave equation found in part (c). Briefly discuss its physical significance.

The enclosed figure illustrates the effect of an externally applied static electric field on a quantum well. The corresponding contribution to the Hamiltonian of the electrons is given by

$$H' \;=\; e\, E_0\, z$$

(a) For the case $E_0 \neq 0$, determine the sign and (approximately) the magnitude (in units of V/cm) of E_0.

(25 %)

(b) For the first two conduction band levels and the first two valence band levels shown in the figure, sketch the corrsponding in-plane dispersion for both cases (i.e., with and without external field). Assume, for simplicity, the bands to be parabolic with equal and positive effective masses for electrons and holes. Make sure the relative energetic positions of the band extrema are correct in your sketch.

(25 %)

(c) Discuss briefly whether the situation shown in the enclosed figure leads to a field-induced red shift or to a field-induced blue shift of the exciton corresponding to the 1-1 transition and of that corresponding to the 2-2 transition. Discuss briefly how the sign of the exciton shift changes if the sign of E_0 is changed.

(25 %)

(d) Discuss the reason for the reduction of exciton oscillator strength with increasing field strength for the exciton corresponding to the 1-1 transition. Furthermore, assume that the quantum well has infinite barriers and, therefore, contains infinitely many bound quantum well states. Discuss briefly the limit of the field-induced reduction of oscillator strength for the exciton of the n-n transition for $n \to \infty$.

(25 %)

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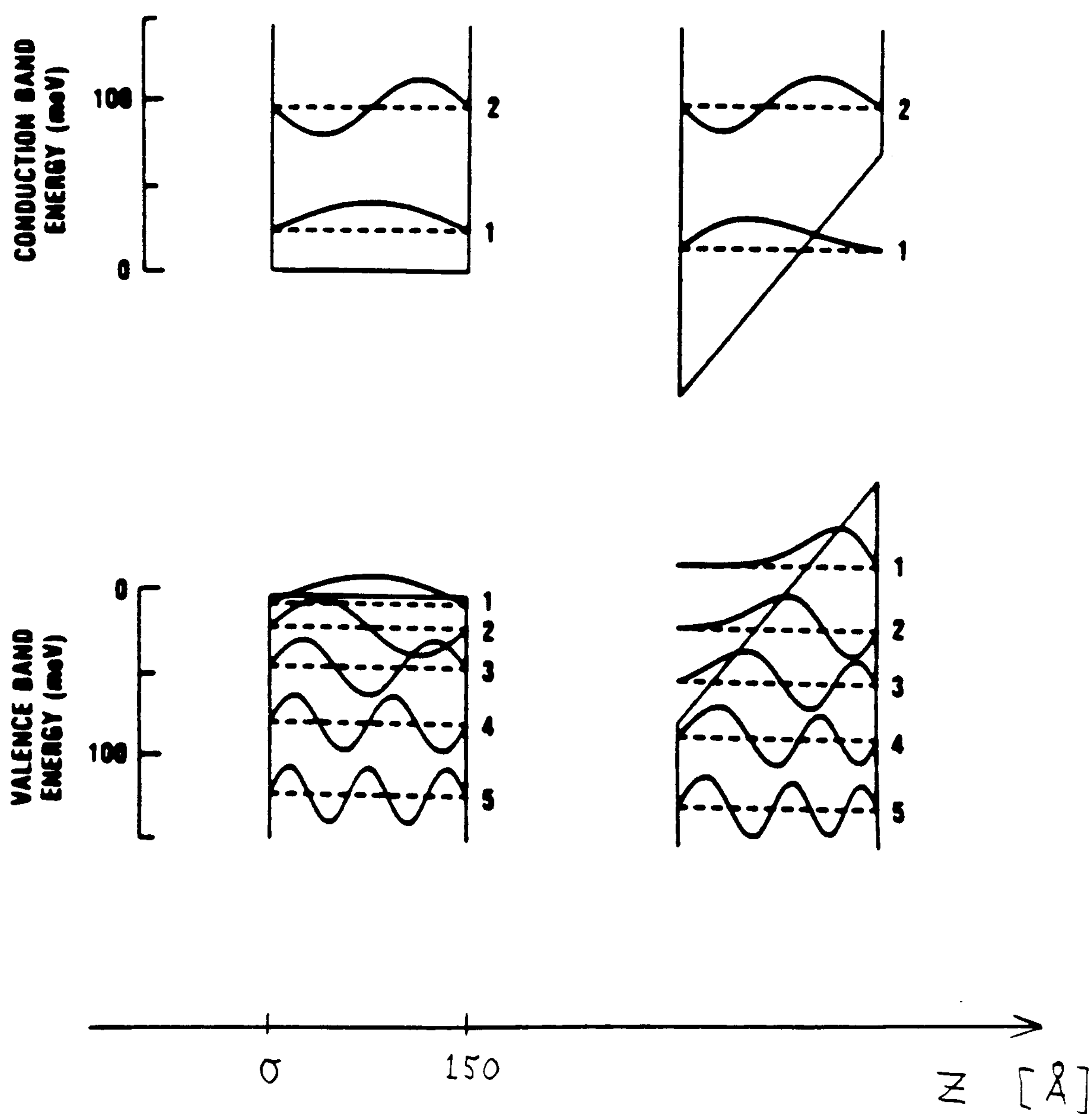

CONDUCTION BAND ENERGY (meV)
100
0
2
1
2
1
VALENCE BAND ENERGY (meV)
0
100
1
2
3
4
5
1
2
3
4
5
0
150
z [Å]

1. Write down Schrödinger's equation for a spherical quantum dot. [20%]
2. Sketch the allowed transitions on an energy-level diagram for electrons and holes in such a dot, both radiative recombination and intersubband transitions. [20%]
3. Sketch the linear absorption spectrum as a function of energy for a single dot for both intersubband and band-to-band transitions. [20%]
4. Sketch the absorption spectrum if you have many dots of slightly different sizes. Mark homogeneous (1 meV) and inhomogeneous (20 meV) distributions. [20%]
5. Explain hole burning experiments in such a system. [20%]

We consider, in one dimension, a two-level atom of mass M interacting with a classical, linearly polarized, monochromatic electromagnetic field of amplitude E_0, frequency ω and wavenumber k,

$$E(X,t) = \hat{\varepsilon}_z E_0 \cos(kX)\cos(\omega t)$$

We denote the *electronic eigenstates* of the atom by $\phi_g(x)$ and $\phi_e(x)$, with corresponding energies E_g, E_e and $E_e - E_g = \hbar\omega_0$. We further assume that the atom is trapped in a square potential well of infinite depth, of width $L >> 2\pi/k$ and centered at $X = 0$.

(a) In the absence of any electric field, write down the Hamiltonian of the atom. Give explicitly its eigenstates and corresponding eigenvalues, introducing an appropriate complete set of quantum numbers to characterize them. [20%]

(b) We assume that the atom-field coupling is adequately described by the electric dipole interaction. Write down this interaction explicitly. [20%]

(c) We now ignore for a while the center-of-mass motion to concentrate on the internal dynamics. Write down the equations describing this dynamics for an atom at position X and initially in its ground state, in the rotating-wave approximation. [20%]

(d) For large absolute values of the atom-field detuning $|\delta|$ the atomic dipole moment can be described by a classical electron oscillator. As a result the atom can be regarded as moving in an effective potential formed by the interaction of the dipole with the electric field. What is the spatial form of this potential? [20%]

(e) Returning then to the original problem where the atomic center-of-mass is included, discuss in words the character of the center-of-mass motion in the two regimes where the kinetic energy is below the maximum atom-field interaction energy, and where it is above this energy. Comment on any qualitative differences that occur depending on whether the motion is treated quantum mechanically or classically. [20%]

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C-2-1 SPRING 1998

Consider beam propagation in a nonlinear self-focusing medium as described by the following equation for the electric field envelope $\mathcal{E}(r, z)$

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{i}{2k}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\mathcal{E} + ik_0 n_2 |\mathcal{E}|^2 \mathcal{E},$$

where z is the propagation direction, $k = n_0 k_0 = n_0\omega/c$, ω being the field frequency, n_0 the background refractive-index, and $n_2 > 0$ is the nonlinear coefficient, $|\mathcal{E}|^2$ being scaled so that it is an intensity. Here we consider a Gaussian input beam

$$\mathcal{E}(r, 0) = \mathcal{E}_0 e^{-r^2/w_0^2},$$

where w_0 is the input spot size.

(a) Ignoring beam diffraction meanwhile for a thin medium of length $L << kw_0^2/2$, show that the field suffers a radially dependent nonlinear phase-shift

$$\phi^{NL}(r, L) = k_0 n_2 I_0 L e^{-2r^2/w_0^2},$$

upon propagating through the medium, where I_0 is the peak input intensity (25%).

(b) The nonlinear phase-shift produces a local radial wavevector

$$\mathbf{K}_T(r) = \hat{\mathbf{r}}\frac{\partial \phi^{NL}}{\partial r},$$

where $\hat{\mathbf{r}}$ is the unit vector in the radial direction. Using this concept give an intuitive explanation of why self-focusing causes the incident beam to come to a focus beyond the medium ($z > L$) for high input powers (25%).

(c) Using the local radial wavevector of largest magnitude, estimate the distance f at which the field comes to focus (30%).

(d) As the nonlinear focal length f approaches the medium length L the effects of diffraction in the medium can no longer be ignored. From this obtain an estimate of the critical power for self-trapping in the medium (20%).

C-1-3

Minor Fall 1998

Consider a first-class dipole-allowed optically created Wannier exciton in a direct-gap bulk semiconductor such GaAs.

(a) Write down the time-independent Schroedinger equation for the center-of-mass motion (not the relative motion!) of the exciton. Give the solution for the center-of-mass wave function $\Phi_{CM}(\vec{R})$ and the center-of-mass energy ε_{CM}, i.e. the energy contribution due to the center-of-mass motion. No formal proof is requested here.

(50 %)

(b) Write down the conservation laws that determine the center-of-mass wavevector in terms of the wavevector of the exciting light field $\vec{q}_i$ inside the crystal and the total exciton energy of the 1s-exciton in terms of the light frequency given as $\hbar\omega$. Neglect any surface and polariton effects. Illustrate the energy conservation law with a sketch of the exciton dispersion and the light dispersion. Indicate in your sketch the bandgap energy E_g, the exciton binding energy E_b, and plot the light dispersion as $\hbar\omega$ vs q_i inside the crystal, assuming that the refractive index of the crystal n is frequency independent.

Assume that you want to make the center-of-mass energy equal to the binding energy by changing the bandgap energy E_g appropriately. Also, assume that you can change the bandgap energy E_g without changing any other parameter. How large would you have to choose E_g? Hint: determine first the appropriate value for q_i and use that value to determine E_g.

If needed, use the following parameter values: m_e=0.05m_0, m_h=0.35m_0 (where m_0 is the electron mass in vacuum), E_b=4 meV, and n=4. You may also use the following *approximate values* for $\hbar^2/m_0 \approx 8 \times 10^{-16}$ eV cm^2 and $\hbar c \approx 2 \times 10^{-5}$ eV cm.
Just in case you don't remember: the light dispersion $\omega = \omega(q_i)$ in the medium is less steep than the light dispersion in vacuum if $n > 1$.

Is the resulting value for E_g typical and realistic for bulk III-V compounds? If not, is it larger or smaller than typical values?

(50 %)

The Green's function $G(\mathbf{r}, t)$ for the time-dependent scalar wave equation satisfies

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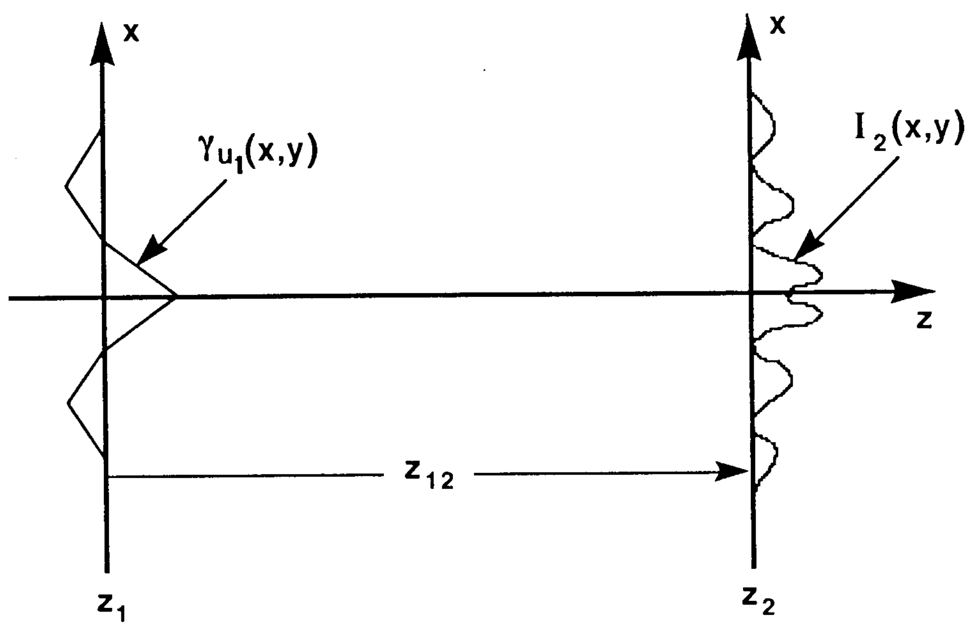

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(a) How large must z_{12} be in order for the observation plane z_2 to be in the Fraunhofer region of the distribution $u_1(x,y)$ when $a = 2 \times 10^{-2}$ m, $b = 4 \times 10^{-2}$ m, $x_0 = 4 \times 10^{-2}$ m and $\lambda = 5 \times 10^{-7}$ m? (25%)

(b) Now if $K = 1$W, $\eta_0 = 100$ m^{-1} and $z_{12} = 10^5$ m is large enough for z_2 to be in the Fraunhofer region of $u_1(x,y)$, find an expression for the irradiance $I_2(x,y)$, and sketch the profiles $I_2(0,y)$ and $I_2(x,0)$. (75%)

A symmetric, confocal laser resonator operates in the lowest order transverse mode at a wavelength of 1 μm. The end mirrors are 1 cm in diameter and the resonator is 1 m long. If the lasing material with an index of refraction of n=1 fills the resonator and gives a loss of 0.01 cm^{-1} and the fractional losses of the two end mirrors are 0.01 and 0.10, answer the following questions about the characteristics of the laser. Show all steps necessary to justify your answers.

1) What is the minimum beam waist of the laser mode in the cavity? [20%]

2) What is the Rayleigh range of the laser beam? [10%]

3) What is the beam divergence? [10%]

4) What is the frequency separation between the laser operating frequency and the next highest longitudinal mode? [20%]

5) What is the Q of the cavity? [20%]

6) What is the lifetime of a photon in this cavity mode? [10%]

7) What is the frequency width of this cavity mode? [10%]

C-2-3 SPRING 1998

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5. Explain hole burning experiments in such a system. [20%]

Consider a gas of two-level atoms illuminated by a monochromatic plane wave $\vec{E} = \frac{1}{2}\vec{\varepsilon}\, E_0\, e^{i(kz-\omega t)} + c.c.$ The light-matter interaction is described by the optical Bloch equations

$$\dot{u} = -\beta u - \Delta v\,, \qquad \dot{v} = -\beta v + \Delta u + \chi w\,, \quad \dot{w} = -2\beta(w+1) - \chi v\,.$$

Here $\chi = -\vec{d}_{21}\cdot\vec{\varepsilon}\, E_0/\hbar$ is the Rabi frequency, and $\Delta = \omega_0 - \omega$ is the detuning of the driving field from atomic resonance. In the following we assume that the atomic dipole moment $\vec{d}\,||\,\vec{E}$ and drop the vector notation.

(a) Give an expression for the expectation value $\langle d\rangle$ of the atomic dipole moment in terms of the Bloch variables. It may help to remember the definition of the Bloch variables, $u = \rho_{21} + \rho_{12}$, $v = i(\rho_{21} - \rho_{12})$ and $w = \rho_{22} - \rho_{11}$. (10%)

(b) Find the steady state value of $\langle d\rangle$ in terms of the quantities χ, Δ and β. (30%)

(c) We can define a complex polarizability in the usual way. Show that it has the form

$$\alpha(\omega) = -|d_{12}|^2 \frac{\Delta + i\beta}{\Delta^2 + \beta^2}\left[1 - \frac{\chi^2/2}{\Delta^2+\beta^2} + \ldots\right].$$

Discuss any qualitatively new features compared to the classical Lorentz atom. (30%)

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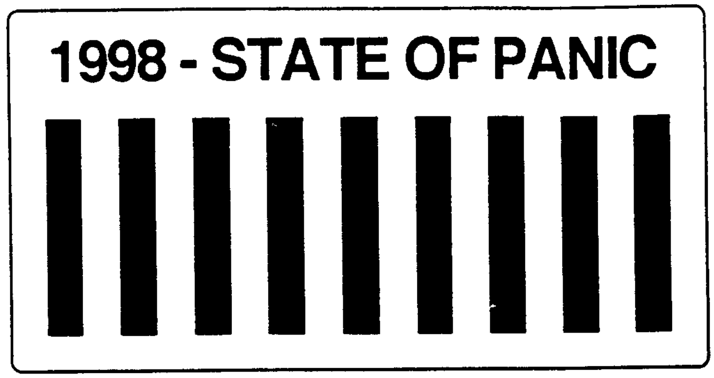

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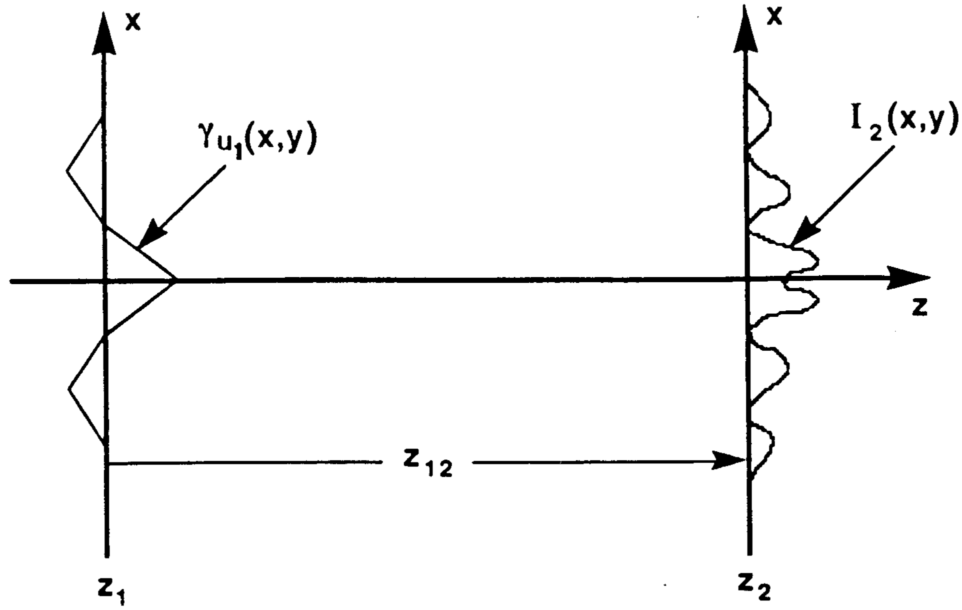

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