WRITTEN PRELIM EXAM – FIRST DAY

SPRING 1999

February 23, 1999
8:30 a.m. to 12:30 p.m.

Please answer two questions from each of the four categories (for a total of 8 questions).

Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J\cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV\cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla\cdot(\mathbf{F}+\mathbf{G}) = \nabla\cdot\mathbf{F} + \nabla\cdot\mathbf{G}$

$\nabla\times(\mathbf{F}+\mathbf{G}) = \nabla\times\mathbf{F} + \nabla\times\mathbf{G}$

$\nabla(\mathbf{F}\cdot\mathbf{G}) = (\mathbf{F}\cdot\nabla)\mathbf{G} + (\mathbf{G}\cdot\nabla)\mathbf{F} + \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F})$

$\nabla\cdot(\phi\mathbf{F}) = \phi(\nabla\cdot\mathbf{F}) + \mathbf{F}\cdot\nabla\phi$

$\nabla\cdot(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot\nabla\times\mathbf{F} - \mathbf{F}\cdot\nabla\times\mathbf{G}$

$\nabla\cdot(\nabla\times\mathbf{F}) = 0$

$\nabla\times(\phi\mathbf{F}) = \phi(\nabla\times\mathbf{F}) + \nabla\phi\times\mathbf{F}$

$\nabla\times(\mathbf{F}\times\mathbf{G}) = \mathbf{F}(\nabla\cdot\mathbf{G}) - \mathbf{G}(\nabla\cdot\mathbf{F}) + (\mathbf{G}\cdot\nabla)\mathbf{F} - (\mathbf{F}\cdot\nabla)\mathbf{G}$

$\nabla\times(\nabla\times\mathbf{F}) = \nabla(\nabla\cdot\mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla\times\nabla\phi = 0$

$\oint_S(\mathbf{F}\cdot\mathbf{n})\,da = \int_V(\nabla\cdot\mathbf{F})\,d^3x$

$\oint_C \mathbf{F}\cdot d\ell = \int_S(\nabla\times\mathbf{F})\cdot\mathbf{n}\,da$

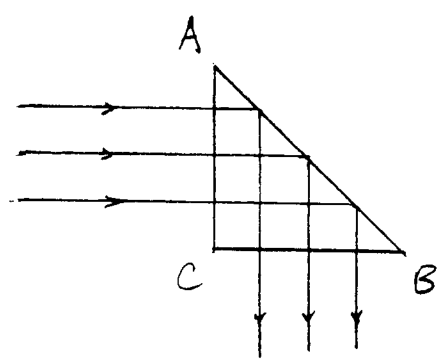
$\oint_S \phi\mathbf{n}\,da = \int_V \nabla\phi\,d^3x$

$\oint_S \mathbf{F}(\mathbf{G}\cdot\mathbf{n})\,da = \int_V[\mathbf{F}(\nabla\cdot\mathbf{G}) + (\mathbf{G}\cdot\nabla)\mathbf{F}]\,d^3x$

$\oint_S(\mathbf{n}\times\mathbf{F})\,da = \int_V(\nabla\times\mathbf{F})\,d^3x$

A parallel beam of light of wavelength 450.0 nm is deflected through 90 degrees by a right-angle prism of index 1.6 (see figure).

- (60%) What is the mathematical form of the fields in the air space beyond the hypotenuse surface AB of the prism? Give a physical interpretation of the form of the field in this region.
- (40%) Use <u>your</u> mathematical expression to compute the numerical value of the distance beyond the hypotenuse surface of the prism at which the mean square value of the E-field is (1/e) the mean square value of E in the air layer immediately adjacent to this surface.



Category A

Day 1

A Fizeau interferometer is used to compare a spherical surface of 100 m radius of curvature with a flat mirror. The flat mirror is 100 mm in diameter and the spherical surface (uncoated glass) is 50 mm in diameter. The source has two wavelengths, 500 nm and 550 nm. Let the two surfaces touch at the center of the spherical surface.

a) (20%) Is the center fringe bright or dark? Explain.

b) (40%) Let there be no tilt between the two surfaces so circular fringes are obtained. If the two surfaces touch in the center of the spherical mirror, how many bright circular fringes for the 500 nm wavelength are obtained before the fringe contrast is a minimum?

c) (40%) What angle will the flat mirror have to be tilted relative to the center of the spherical surface so there are no closed fringes for

   i) For the 500 nm wavelength?

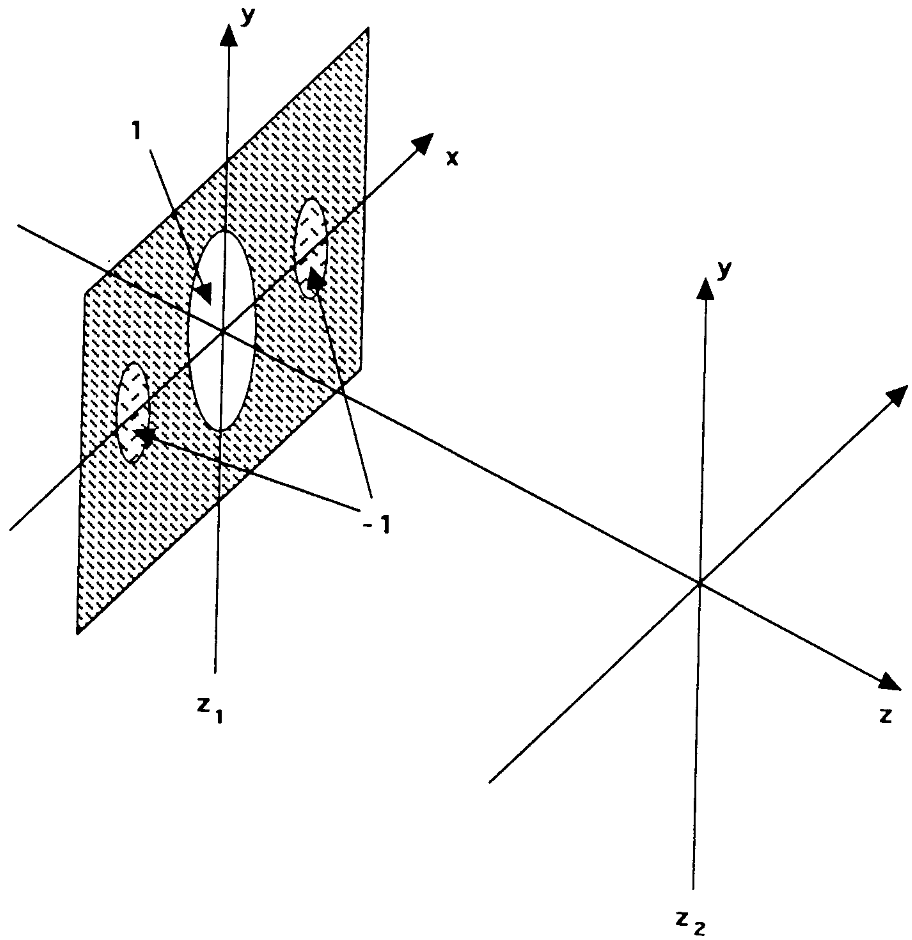
   ii) For the 550 nm wavelength?

Gaskill (Arvind)

A-1-3

Spring 1999

Three circular apertures are located in an opaque screen in the $z = z_1$ plane as shown. The larger aperture has a diameter of 2 mm, is centered at the origin, $(0, 0, z_1)$, and has a transmittance of 1 (it is a clear aperture). The two smaller apertures have diameters of 1 mm, are centered at $(-3\text{ mm}, 0, z_1)$ and $(+3\text{ mm}, 0, z_1)$, respectively, and each has a transmittance of -1 (they are covered with half-wave phase plates). These three apertures are illuminated with a normally incident, coherent plane wave of $\lambda = 500$ nm and amplitude A, where $|A|^2 = 10\text{ mW/cm}^2$.



(a) Appproximate the minimum distance between the z_1 and z_2 planes that will ensure the latter is in the Fraunhofer region of the aperture combination. (30%)

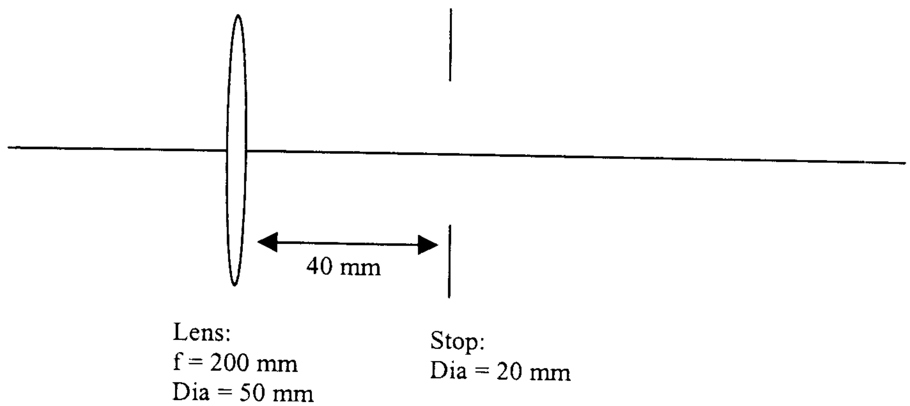
(b) Find an expression for the irradiance in the z_2 plane if the distance between the two planes is 2000 m (assume that the Fraunhofer condition is satisfied). (40%)

(c) **Either:** make a rough, normalized sketch of the irradiance along the x axis, indicating pertinent features such as locations of zeros, etc.; **or**, find the value of the irradiance at the point $(0, 0, z_2)$. (30%)

B-1-1

Spring 1999

The following optical system consists of a thin lens followed by a stop. The object is at infinity. Determine the unvignetted object space field-of-view (in degrees) and the marginal and chief ray heights at the lens.



You have been asked to design a special effects lens that introduces a significant amount of third-order pincushion distortion. It is to be used with a 35 mm film format (24x36 mm), and the distortion at the corner of the image must be 25%. The f/2.8 lens has a focal length of 28 mm. Determine the value of the wavefront aberration coefficient for third-order distortion.

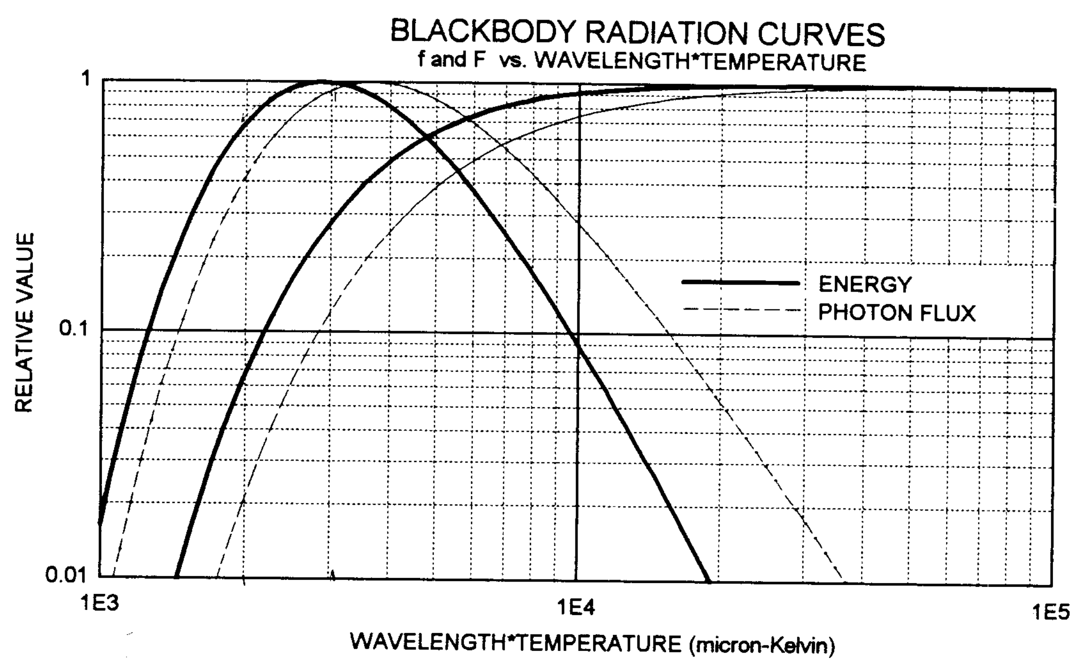
A radiometer consists of a lens (25 mm diameter, 100 mm focal length, lossless) with a 3 mm diameter detector located at the rear focal point. A graybody calibration source ($\varepsilon = 0.9$, 800°C, 15 mm diameter aperture) is available and is temporarily placed at a distance of 250 mm from the radiometer. The source is fitted with a filter that transmits 50% of the radiation between the wavelengths 2 and 3 μm and zero elsewhere.

a) Determine the in-band radiance of the calibration source (including the filter) (20%)

b) How can you easily determine the power on the detector? Is this a viable calibration configuration? Explain your response! (40%)

c) Now you are permitted to move the detector along the optical axis until the source is in focus. What is this detector location and what is the power on the detector? (40%)

The following graph may prove helpful.



Consider a parabolic 2-band semiconductor with bandstructure shown in Fig. 1. Furthermore, assume the semiconductor to be inverted, i.e. it contains electrons in the conduction band and holes in the valence band. The density of electrons shall be the same as the density of holes.

(a) Sketch the momentum dependence of the electron and the hole distribution functions, $f_e(|\vec{k}|)$ and $f_h(|\vec{k}|)$, assuming zero-temperature quasi-thermal Fermi distributions (note that in this case there are no differences between $f_e(|\vec{k}|)$ and $f_h(|\vec{k}|)$).

Assume the quasi-Fermi energy of the electrons ε_F^e to be 40meV above the renormalized bandgap E_g'. Determine the quasi-Fermi energy of the holes (remember that we have very simple zero-temperature quasi-Fermi distributions). Indicate schematically in Fig. 1 the occupied electron and hole states. Specify the interval of optical frequencies $[\hbar\omega_1, \hbar\omega_2]$ (given in units of eV) in which you can use this semiconductor system for lasing.

(80 %)

(b) Assume you need to choose an appropriate laser cavity, and that you have only two cavities to choose from, both of which support only one optical mode. Cavity A has its mode at 1.72eV and cavity B as its mode at 1.54eV. Which cavity would you choose?

Furthermore, assume you need to choose an appropriate optical pumping source (i.e. an external pump laser whose light can be absorbed by the semiconductor so that electron-hole pairs are created), and that you have only two pump lasers to choose from. One pump laser has an output light frequency $\hbar\omega_p$ of 1.72eV, and the other has it at 1.54eV. Which optical pump laser would you choose?

(20 %)

Note: All answers need to include a brief discussion in order to justify the answer.

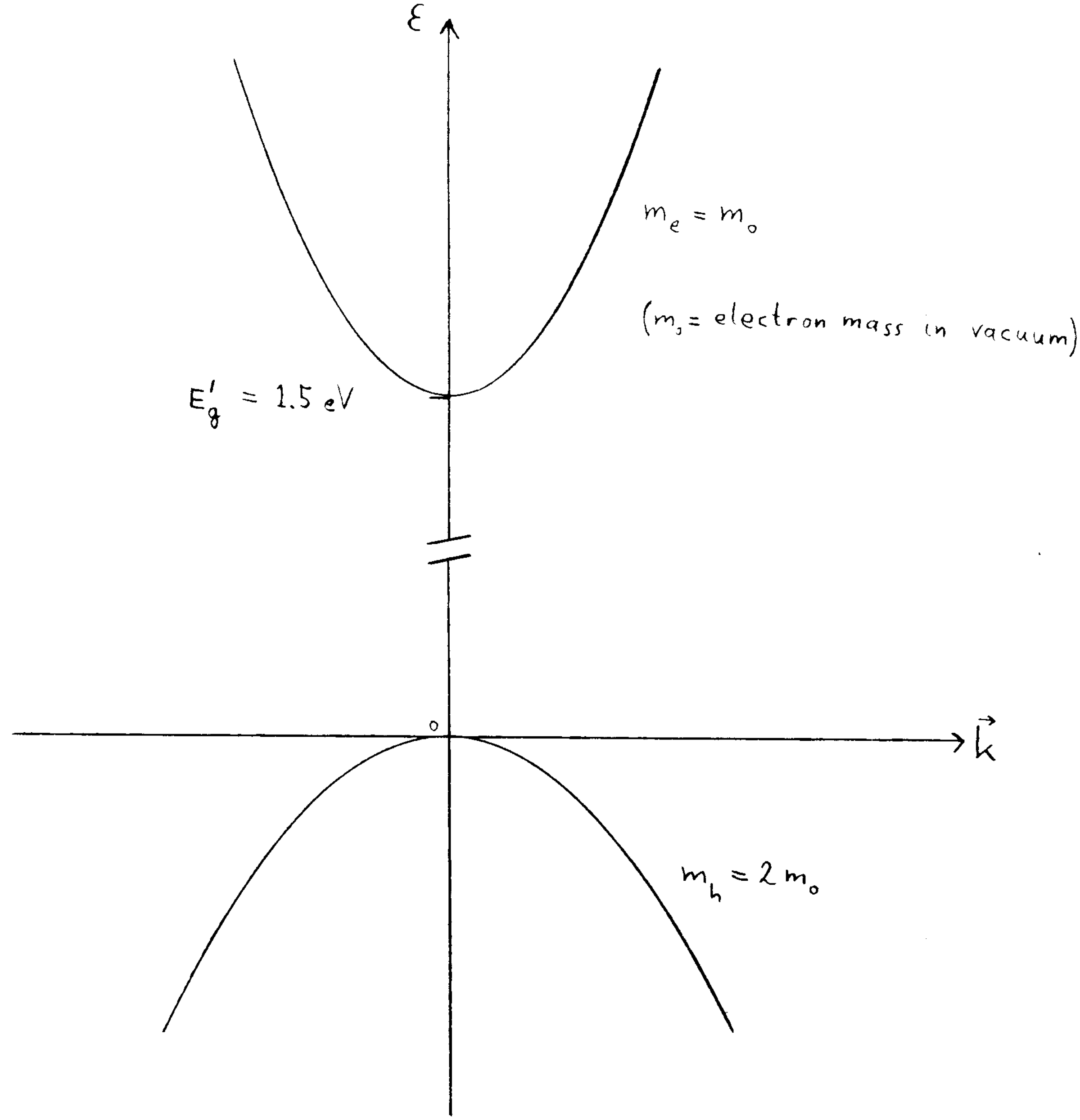


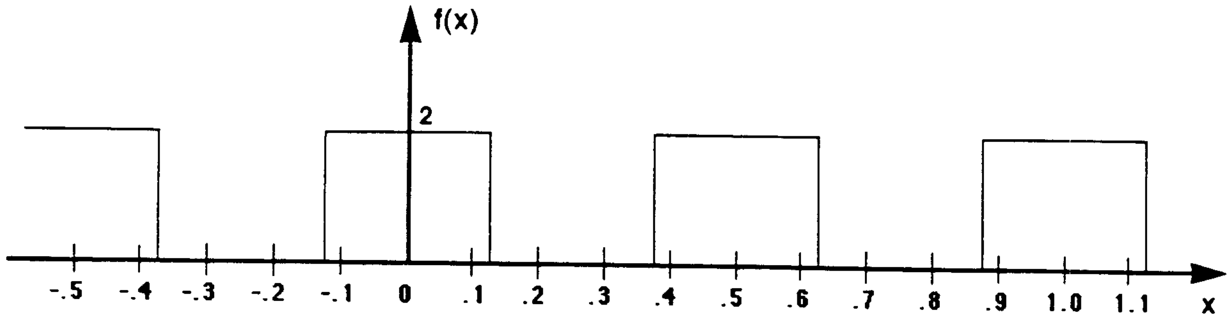
Fig. 1

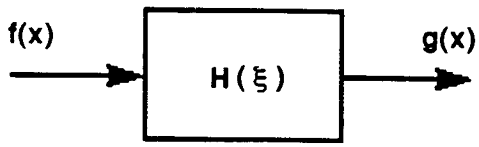
Suppose a strong, monochromatic beam of light at frequency ω passes through an atomic vapor, and suppose that ω is nearly resonant with only one transition in the atoms making up the sample.

Assuming that the atomic energy levels and eigenfunctions are already known, we can start from the Schrödinger equation and obtain an analytic result for the steady-state absorption coefficient α of the vapor in this situation. However, to do this, we must make three important approximations: one in the Hamiltonian, and two in the resulting equations of motion for the probability amplitudes C_i.

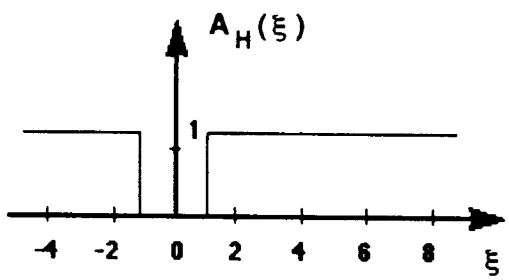
(a) Name and briefly describe each of these three approximations. (60%)

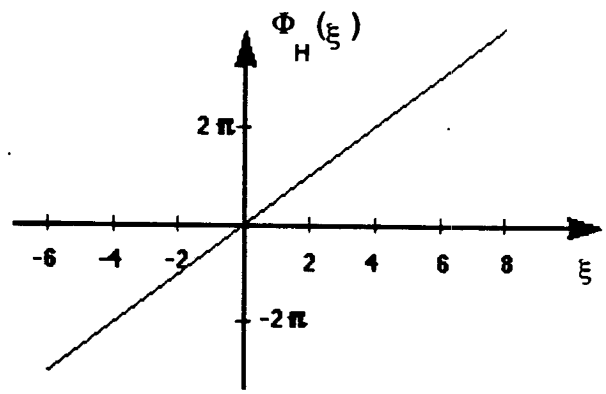
(b) For each of the three approximations you listed in part (a), write out the equation(s) in which the approximation is being made and show which term or terms are being neglected or modified when the approximation is made. (40%)





$$H(\xi) = A_H(\xi)\, e^{-j\Phi_H(\xi)}$$





The input signal to a high-pass filter is a periodic square-wave function of amplitude 2 and period 0.5. The amplitude- and phase-transfer functions of the system are as shown.

(a) Find an expression for the output signal g(x). If you choose, you may include f(x) explicitly as part of your result to keep it as simple as possible. (50%)

(b) Sketch a few periods of g(x), showing important features. (50%)

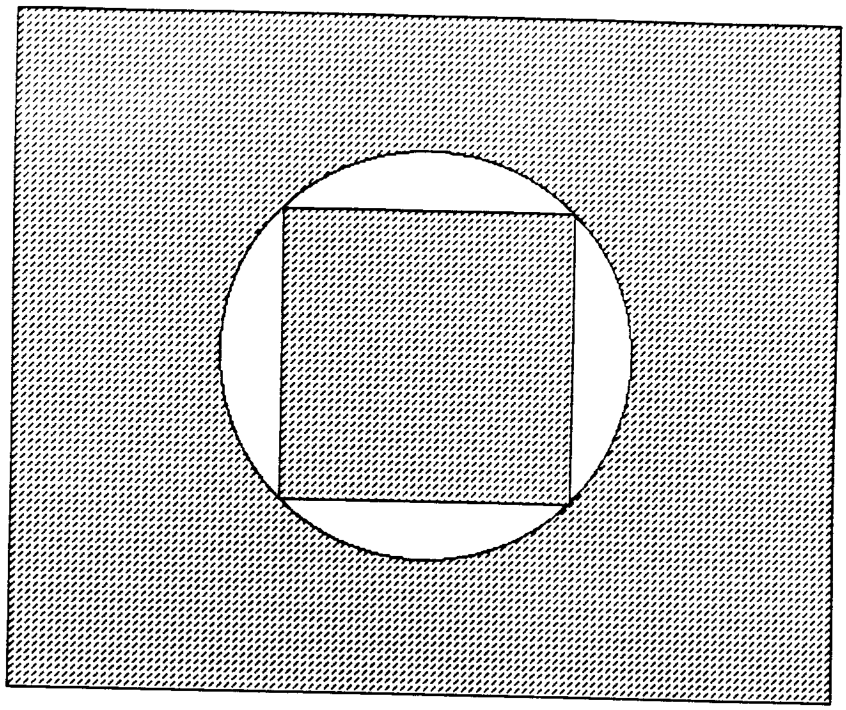
D-1–2

(10%) (a) What is the first-order optics relation for a thin lens that expresses the image distance y in terms of the object distance x? Denote the lens focal length as f.

(80%) (b) Suppose that the object is placed in object space with an uncertainty (standard deviation) in position σ. The probability density function on x is Gaussian. Then by your answer to part (a) distance y must be random as well. Find the probability density function on y.

(10%) (c) Explain the extent to which the model chosen in part (b) is valid, and the conditions under which it no longer is a good model.

0734 kill



Given the aperture shown.

(a) Find an expression for the irradiance of the Point-Spread Function (PSF) of an imaging system for which this aperture represents the pupil function. Assume the light to be narrowband, of center wavelength **λ**, the effective focal length to be **f**, and the circular portion of the aperture to have a diameter of **D**. (50%)

(b) Find the ratio of the central value of this PSF to the PSF of a system having an unobscured circular aperture of diameter **D**. (50%)

WRITTEN PRELIM EXAM – SECOND DAY

SPRING 1999

February 24, 1999
8:30 a.m. to 12:30 p.m.

Please answer any five questions.

Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F}\cdot\mathbf{G}) = (\mathbf{F}\cdot\nabla)\mathbf{G} + (\mathbf{G}\cdot\nabla)\mathbf{F} + \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla\cdot\mathbf{F}) + \mathbf{F}\cdot\nabla\phi$

$\nabla \cdot (\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot\nabla\times\mathbf{F} - \mathbf{F}\cdot\nabla\times\mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla\times\mathbf{F}) + \nabla\phi\times\mathbf{F}$

$\nabla \times (\mathbf{F}\times\mathbf{G}) = \mathbf{F}(\nabla\cdot\mathbf{G}) - \mathbf{G}(\nabla\cdot\mathbf{F}) + (\mathbf{G}\cdot\nabla)\mathbf{F} - (\mathbf{F}\cdot\nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla\cdot\mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F}\cdot\mathbf{n})\, da = \int_V (\nabla\cdot\mathbf{F})\, d^3x$

$\oint_C \mathbf{F}\cdot d\ell = \int_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G}\cdot\mathbf{n})\, da = \int_V [\mathbf{F}(\nabla\cdot\mathbf{G}) + (\mathbf{G}\cdot\nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n}\times\mathbf{F})\, da = \int_V (\nabla\times\mathbf{F})\, d^3x$

Category A

Day 2

Two-wavelength holography is used to test a concave aspheric mirror. The two wavelengths being used are 488 and 514.5 nm.

a) (10%) Sketch a reasonable setup being careful to show which planes are conjugate.

b) (20%) What is the equivalent wavelength?

c) (30%) If the surface of the aspheric mirror differs from a spherical surface by

$$(100 \text{ microns})\ \rho^4,$$

where $0 < \rho \leq 1$, how many interference fringes are obtained in the two-wavelength interferogram if the tilt is minimized, and the slope of the aspheric wavefront relative to a spherical wavefront is minimized?

d) (40%) If the diverger lens has no spherical aberration at 488 nm and 1 wave of spherical aberration (single pass) at 514.5 nm, how much error will result in the measurement of the surface of the aspheric mirror?

A - 2 - 2

Macleod Question Day 2 (OPTI 577)

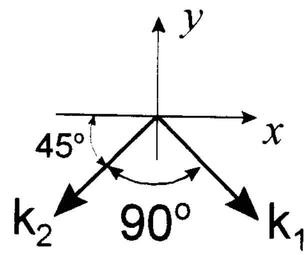
A single quarterwave dielectric layer of characteristic admittance y_f transforms the admittance of a substate, y_{sub}, according to what is known as the *Quarterwave Rule*.

(a) State the Quarterwave Rule (you need not prove it) (3)

A dielectric incident medium is denoted by A with admittance y_a. A dielectric emergent medium (or substrate) is denoted by B with admittance y_b. Film materials P with admittance y_p and Q with admittance y_q are to be used for a two-layer antireflection coating for B in A. We are given that no two admittances are equal and that $y_a<y_p<y_q<y_b$.

(b) Show that if $y_a<y_q^2/y_b<y_p^2/y_a<y_b$ then it is possible to construct a perfect antireflection coating with materials in order $A \mid P \mid Q \mid B$ and that this can be done with each layer thickness less than a quarterwave. (3)

(c) Show further that if the conditions in part (b) are satisfied then is also possible to construct a perfect antireflection coating with layers once again less than a quarterwave in thickness, but with the order of the layer materials reversed, i.e. $A \mid Q \mid P \mid B$. (4)



Two plane waves polarized in the z direction interfere in free space. The propagation vectors for the waves are shown in the diagram above. Both waves have the same wavelength and are coherent with respect to each other.

1.) Derive an expression for the interference pattern created by the two waves. You may normalize the irradiance.

2.) Determine the direction of the time-average Poynting vector for each wave individually.

3.) Determine the direction of the time-average Poynting vector for the combination of the two waves.

4.) How does the direction of the time-average Poynting vector for the combination of the two waves relate to the orientation of the fringe surfaces?

Consider a thin unslanted absorption hologram formed by the interference of two plane waves with a 10° inter-beam angle in silver halide film. The exposing wavelength is 500 nm, the film is 5 μm thick, refractive index n = 1.6. The beams overlap in a square region that is 1 cm on a side.

a) What is the grating period and volume Q parameter for this grating? Is it thin?
b) Write an expression for the resulting transmittance function of the hologram after processing the film. (Use variables not numerical values in this expression.)
c) What is the width of the +1 diffraction order (i.e. separation between 0 intensity points) at a distance of 10 cm from the hologram plane when illuminated with a normally incident plane wave?

As a sensor designer, you are faced with the task of improving the spatial resolution of a satellite sensor from a 20-meter ground instantaneous field of view (IFOV) to a 10-meter ground IFOV. . The sensor uses a whiskbroom scanning mirror to sample in the cross-track direction and the platform's forward motion provides along-track sampling. The orbital altitude of the platform is 800 km.

A) A common method for improving spatial resolution is to decrease the size of the detectors used. If the original detectors are 30 micrometers in size, what is the size of the detectors in the improved system? (10%)

B) Decreasing detector size typically results in poorer signal-to-noise ratio. Describe two ways to alter the sensor design (other than improved detector technology) so that the detector size can be reduced without decreasing the signal-to-noise ratio. (30%)

C) Quantitatively give three additional sensor or platform modifications that can be used to decrease the ground instantaneous field of view to 10 meters. (40%)

D) Of the five methods given in parts B and C, which would you implement? Defend your answer by giving one disadvantage for each of the other four methods. (20%)

Describe the behavior of each of the third-order aberrations when the system is perturbed by tilts and/or decenters.

1) (20%) Write down in wave aberration coefficient form all possible seventh order monochromatic aberrations. Which of these involve new shapes to the wave aberration function that do not exist in the lower order aberrations?

2) (40%) Discuss the connection between the wave aberration function and the wavefront itself. Also discuss the connection between the transverse ray aberration and the wave aberration function.

3) (40%) How are aspherics dealt with in the calculation of aberration coefficients? How do they affect the first-order properties of a system? If a surface aspheric is represented by a rotational power series expansion, how many terms are required to account for the third-order aberrations?

Consider a Schmidt camera having a clear aperture of 200 mm at the correcting plate and a mirror radius of curvature of 1000 mm. The mirror is made from fused silica and the plate from Bk7 glass. The camera works in the visible range of the spectrum.

A) Make a sketch of the Schmidt camera and explain how it works. 10 %

B) What is the unvignetted full field of view when the mirror has a diameter of 250 mm ? 10 %

C) As a function of the radius of curvature of the mirror and the aperture derive an equation for the aspheric profile of the correcting plate. Do not include second order terms in the aperture; only the fourth order term. 40%

D) What should be the focal length of a thin lens (n=1.5) located near the image surface to flatten the field? Comment on the aberrations introduced by this field flattener lens. 40%

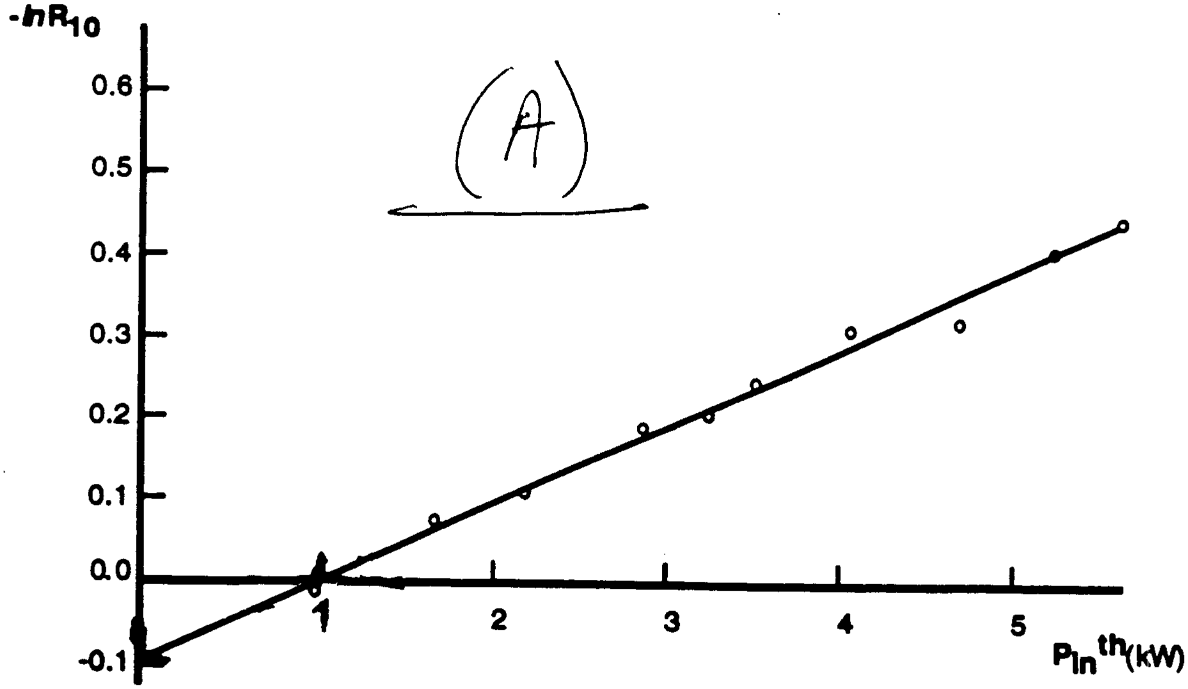
Prelim Exam Question from 541-Lasers
Dick Powell

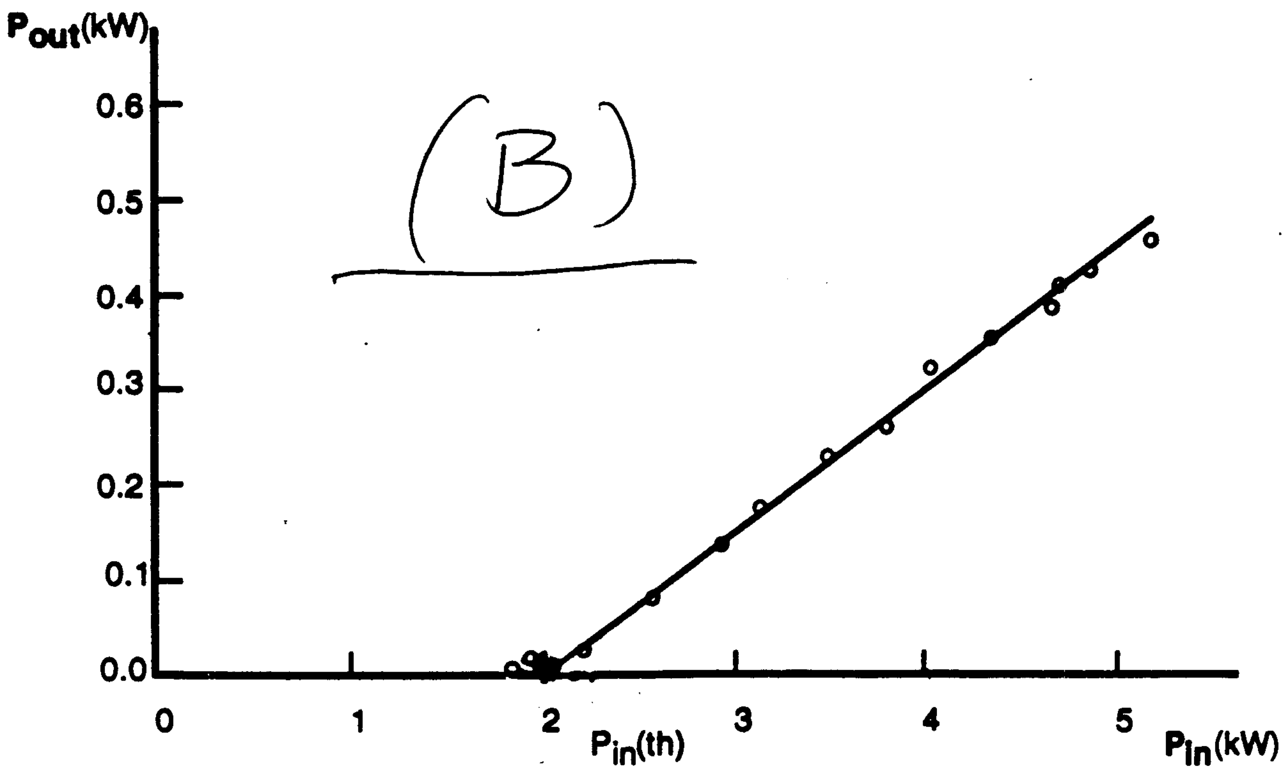
You are given a 4mm-diameter laser rod made from a material with the following properties:

- A 4-level laser transition a 1μm from a metastable state with a fluorescence lifetime of 200 μm and a quantum efficiency of 95%;
- An index of refraction of 1.5;
- An optical damage level of 4 J/cm^2; and
- A pump input power for the laser of 4.4 kW with a pumping efficiency of 14.3%.

Having made a laser out of this rod, you perform experiments to give you a Findley-Clay curve and a slope efficiency curve as shown in plots A and B, respectively. Answer the following questions (each worth the same number of points):

1) What is the reflectivity of the output coupler used in the final laser system?
2) What is the power output of the laser operating at the input power given above?
3) What is the saturation power density of the system?
4) What is the cross section of the laser transition?
5) What is the linewidth of the laser transition given in wavenumbers?


(A)
-lnR10
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
1
2
3
4
5
Pin th(kW)


(B)
Pout(kW)
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
Pin(th)
Pin(kW)

Prelim Question

This question is about radiative coupling between periodic quantum wells at low temperature. The electron/heavy-hole excitonic transition is assumed to be narrow (≤ 1 meV). Each question has equal credit.

1. What is the Bragg condition on the period d of periodic quantum wells in order to have the strongest radiative coupling?

2. What happens to the reflectivity spectrum of periodic quantum wells as the period d is changed through the Bragg condition?

3. How does the Bragg-resonance reflectivity linewidth depend upon the number N of quantum wells?

4. Explain how one can extract the radiative broadening linewidth Γ_0 of a single quantum well from reflectivity measurements on a periodic quantum well sample.

5. Since radiative coupling affects transmission measurements on a periodic quantum well sample, what is the best way to extract the absorption spectrum?

6. Consider a pump-probe experiment on a periodic quantum well sample where d is far from the Bragg condition. In atomic physics the absorption transition is always pushed away from the frequency of an intense off-resonance pump. Is this always the case for an excitonic transition in quantum wells? Why?

7. At Bragg resonance, both the transmission T and reflectivity R have local maxima. Explain how this can be in terms of the true absorption $A \equiv 1 - R - T$ and the spatial dependence of the optical field within the periodic quantum wells.

8. N radiatively coupled quantum wells should have N normal modes. What happens to the oscillator strengths of the N quantum wells exactly at Bragg?

9. What evidence is there for these normal modes in R, T, A, or photoluminescence?

10. How can you modify the dispersion curve of periodic quantum wells?

C-2- 3

The equations describing third-harmonic generation of plane-wave fields propagating along the z-axis in a bulk isotropic medium are

$$\frac{d\mathcal{E}^{\omega}}{dz} = i\kappa_1 \mathcal{E}^{3\omega}[(\mathcal{E}^{\omega})^*]^2 e^{i\Delta k z}, \qquad (1)$$

$$\frac{d\mathcal{E}^{3\omega}}{dz} = i\kappa_2 (\mathcal{E}^{\omega})^3 e^{-i\Delta k z}, \qquad (2)$$

where $\mathcal{E}^{\omega}$ and $\mathcal{E}^{3\omega}$ are the slowly-varying envelopes of the fundamental and third-harmonic fields respectively, $\kappa_{1,2} > 0$ are material constants, and $\Delta k = (k^{3\omega} - 3k^{\omega})$ is the wavevector mismatch for the nonlinear interaction. The fields are normalized so that $|\mathcal{E}|^2$ yields the intensity I.

Consider the case where there is no third-harmonic field present at the input $z = 0$, and a large wavevector mismatch.

(a) For a large wavevector mismatch the fundamental field may be set equal to its input value $\mathcal{E}^{\omega}(0) = \sqrt{I^{\omega}(0)}$ to first-order. Using Eq. (1) obtain an expression for the third-harmonic field in this limit, and plot the evolution of the third-harmonic intensity versus propagation distance z (40%).

(b) Next substitute your first-order solution from part (a) for the fundamental and third-harmonic fields into the right hand side of Eq. (2) and drop any fast oscillating terms. Then show that to second-order the fundamental field evolves as $\mathcal{E}^{\omega}(z) = \mathcal{E}^{\omega}(0)e^{i\Phi_{NL}(z)}$, where the nonlinear phase shift is given by

$$\Phi_{NL}(z) = -\frac{\kappa_1\kappa_2}{\Delta k}[I^{\omega}(0)]^2 z, \qquad (3)$$

so that the fundamental field accumulates an intensity dependent phase-shift (40%).

(c) Assuming normal dispersion does the nonlinear effect in Eq. (3) correspond to a self-focusing nonlinearity or a self-defocusing nonlinearity? Explain (20%).

C-2- 4

Consider a plane-wave optical field polarized in the $x - y$ plane and at frequency ω propagating along the principal axis z of an anisotropic crystal. The equation for the linear optical polarization $P_j(\mathbf{r}, t)$, with $j = x, y$, is given by

$$\left[\frac{d^2}{dt^2} + \Gamma_j \frac{d}{dt} + \omega_j^2\right] P_j = \omega_j^2 \epsilon_0 \chi_s E_j,$$

where ω_j is the electron oscillator resonance frequency along the j direction, Γ_j the corresponding damping rate, and χ_s is the non-resonant value of the linear optical susceptibility.

1. Expressing the field and the polarization in with respect to plane-wave factors

   $$E_j(\mathbf{r}, t) = \frac{1}{2}\mathcal{E}_j(z, t)e^{i(k_j z - \omega t)} + c.c, \qquad P_j(\mathbf{r}, t) = \frac{1}{2}\mathcal{P}_j(z, t)e^{i(k_j z - \omega t)} + c.c,$$

   derive an expression for the frequency-dependent linear susceptibility $\chi_j(\omega)$ (35%).

2. The equations describing the evolution of the field envelopes through the medium are

   $$\frac{d\mathcal{E}_j}{dz} = iC\chi_j(\omega)\mathcal{E}_j,$$

   where C is a real positive constant. Consider the resonant case with $\omega = \omega_x = \omega_y$, but with different damping rates $\Gamma_x > \Gamma_y$. For an initial input field which is right-circular polarized

   $$\mathcal{E}(z = 0) = \frac{\mathcal{E}_0}{\sqrt{2}}(\mathbf{x} + i\mathbf{y}),$$

   derive an expression for the evolution of the left-circular component of the field intensity through the medium, and plot the general form of this solution with propagation distance (50%).

3. For the same conditions as above what will be the polarization state of the field for large propagation distances, large compared to any absorption lengths for the medium that is (15%).

Consider an electronic camera with a diffraction-limited, 50mm F.L., F/2 lens and a CCD detector. The object is a cosinusoidal test pattern with diffuse reflectance given by

$$R(x, y) = \tfrac{1}{2}[1 + \cos(2\pi\nu_0 x)] .$$

This object is located 5m from the lens and illuminated with quasi-monochromatic radiation of wavelength 500nm.

(a) 10% Compute the magnification of the system (an accuracy of 1% is OK).

(b) 30% Compute the maximum *object* spatial frequency ν_0 for which any modulation is visible in the image plane. Call this maximum frequency ν_{max}. What is the corresponding maximum spatial frequency in the image? In both cases, specific numerical answers are required (again to 1% accuracy or so).

(c) 30% Suppose the CCD detector has square elements of width Δ and center-to-center spacing 2Δ in both dimensions. In order to avoid aliasing when $\nu = \nu_{max}$, Δ must be less than or equal to some maximum value Δ_{max}; compute the numerical value of Δ_{max}.

(d) 30% In practice, CCD detectors have Δ greater than the value you calculated above. For each of the following situations, sketch the irradiance distribution on the image plane and draw in the detector elements. Then sketch the detector output for a few cycles of the test pattern.

(i) $\nu_0 = \nu_{max}/2$, $\Delta = \Delta_{max}$

(ii) $\nu_0 = \nu_{max}/20$, $\Delta = 10\Delta_{max}$

(iii) $\nu_0 = \nu_{max}/2$, $\Delta = 10\Delta_{max}$

Two products A and B are competing for sales. They monopolize the market, so that a person can only buy one of A or B on a given purchase. The quality of A exceeds that of B, as defined in the following. If a person buys A, then on his next purchase he will tend to buy A again with the probability 0.8. On the other hand if the person buys B, then on the next purchase he will tend to buy B again with only the probability 0.4.

(10%) (a) What kind of a random process does a typical sequence of purchases

BAABAAABABAAAABAABAAA ....

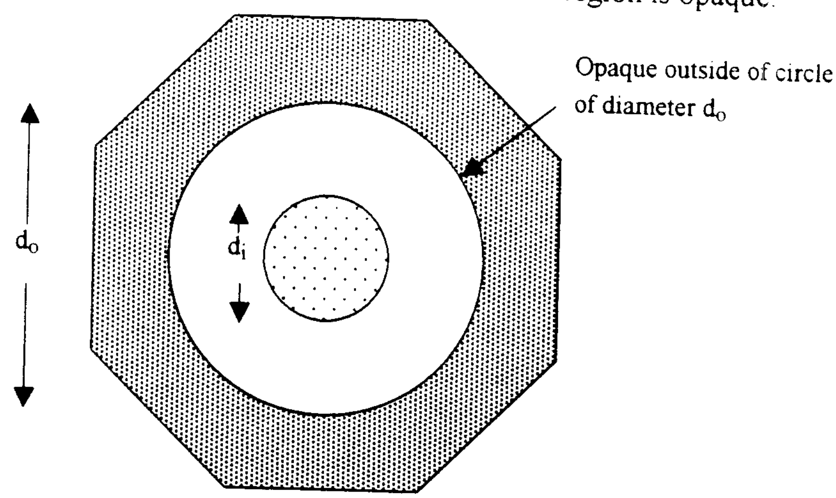
define? (I am looking for a name).

(10%) (b) What is the partition law (or 'law of total probability')?

(80%) (c) In the long run how often is A bought and how often is B bought? That is, if A_n is the event that A is bought on purchase n, and B_n is the event that B is bought on purchase n, what are the probabilities $\lim_{n \to \infty} P(A_n)$ and $\lim_{n \to \infty} P(B_n)$?

Spring 1999

The aperture shown below is illuminated at normal incidence with a unit-amplitude plane-wave of wavelength λ. The Fraunhofer diffraction pattern is observed in the focal plane of a lens having focal length f. The aperture is circular and has a circular central region of diameter d_i. The region outside the diameter d_o circular region is opaque.



Find an expression for the irradiance distribution in the Fraunhofer diffraction pattern of the aperture shown above if

a) (30%) The central region of diameter d_i is opaque.

b) (30%) The central region of diameter d_i transmits 50% of the incident amplitude.

c) (40%) The inner circular aperture of diameter d_i is transparent, but introduces a 45 degree phase change to the light transmitted through it.

D-2-4

Spring 1999

In this problem, you will derive the transfer function of free space and physically interpret your findings.

a) A 3D plane wave has a complex representation given by $A\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$. What relation between the three components of $\mathbf{k}$ (k_x, k_y, k_z) and the angular frequency ω must be satisfied in order for the 3D plane wave to satisfy the scalar wave equation, $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)(A\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]) = 0$?

b) Decompose the field distribution in the $z = 0$ plane, $u(x, y, 0)$, by its Fourier transform, $U(\xi, \eta; 0)$. Assume propagation in the positive-z direction. By using the relationship derived in (a), find the expression for $U(\xi, \eta; z')$ in the plane $z = z'$ in terms of $U(\xi, \eta; 0)$. What is the transfer function of free space?

c) Explain the physical meaning of the transfer function associated with propagation from the $z = 0$ plane to the $z = z'$ plane.

d) Using the result of (b), describe the behavior associated with all spatial frequencies.