WRITTEN PRELIM EXAM – FIRST DAY

FALL 2000

September 25, 2000
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A - B) + \cos(A + B)$

$2\sin A \sin B = \cos(A - B) - \cos(A + B)$

$2\sin A \cos B = \sin(A + B) + \sin(A - B)$

$2\cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

A 2mm x 2mm infrared photodiode detector has a peak spectral specific detectivity, D*, at 10 μm, D*(10μm) = 2.5 (10^{11}) $\left[\frac{cm-\sqrt{Hz}}{watt}\right]$ with a background irradiance (E_q) of 10^{16} photon/sec-cm^2. The detector is BLIP (Background Limited Infrared Photodetector) operated.

a) What is its quantum efficiency?

b) A laser source at 3.39 μm (HeNe laser) is incident on the detector with a irradiance of 1 nano-watt per centimeter squared, ($E^s_e = 10^{-9}$ watt/cm^2).
The electronics used have an electrical bandwidth (Noise equivalent bandwidth -Δf) of 2000Hz. What will be the signal to noise ratio out of this detector?

Certain military activities in Waco Texas in 1993 were documented via video recordings from thermal infrared (TIR) 8-12 μm cameras. Occasionally the tapes show short flashes, which some "experts" interpret as specular sunglint. Other "experts" say these flashes must be gunfire, because sunglint is impossible; the sun is very dim in the infrared.

The TIR camera observes radiance exclusively in the 8-12 μm spectral band. Let's check out the opaque objects in the table. Characteristics given are for the TIR band. All are at 300K.

Earth	emittance $\varepsilon = 0.95$
Water	reflectance $\rho = 0.015$
Glass	reflectance $\rho = 0.04$
Aluminum	reflectance $\rho = 0.9$

The sun is at 6000K, has an angular subtense of 32 arc-min (total angle), and is directly overhead. The transmission of the atmosphere in the TIR is 0.6. The radiance of the sky in the TIR is 10 W/m^2-sr and is uniform from zenith to horizon.

(50%) Determine the radiance in the 8-12 μm TIR spectral band for each of the four objects in the table, assuming that their reflectance is diffuse.

(40%) Repeat for the last three objects, but now assume that their reflectance is specular.

(10%) What is your conclusion about the importance of sunglint in the thermal infrared?

The equations and graph below may prove useful.

$$L = \frac{\sigma}{\pi} T^4 \qquad L_\lambda(\max) = \frac{\sigma'}{\pi} T^5$$

$$\sigma = 5.6704 \times 10^{-8}\ Wm^{-2}K^{-4}$$

$$\sigma' = 1.2867 \times 10^{-11}\ Wm^{-2}K^{-5}\mu m^{-1}$$

$$f(\lambda T) = \frac{L_\lambda(\lambda, T)}{L_\lambda(\lambda_{\max}, T)}$$

$$F(\lambda T) = \frac{\int_0^\lambda L_\lambda(\lambda,T)d\lambda}{\int_0^\infty L_\lambda(\lambda,T)d\lambda} = \frac{\int_0^\lambda L_\lambda(\lambda,T)d\lambda}{(\sigma/\pi)T^4}$$

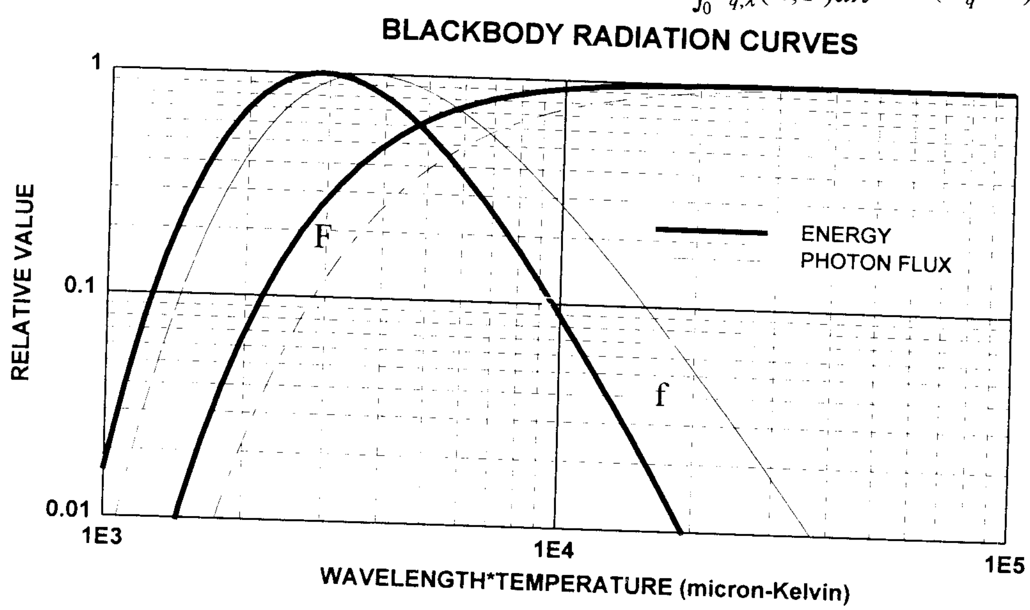
$$L_q = \frac{\sigma_q T^3}{\pi} \qquad L_{q,\lambda}(\max) = \frac{\sigma'_q}{\pi} T^4$$

$$\sigma_q = 1.5205 \times 10^{15}\ \sec^{-1} m^{-2}K^{-3}$$

$$\sigma'_q = 2.1011 \times 10^{11}\ \sec^{-1}\ \mathrm{m}^{-2}\mathrm{K}^{-4}\mu\mathrm{m}^{-1}$$

$$f_q(\lambda T) = \frac{L_{q,\lambda}(\lambda, T)}{L_{q,\lambda}(\lambda_{\max}, T)}$$

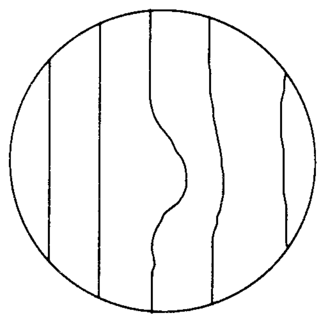
$$F_q(\lambda T) = \frac{\int_0^\lambda L_{q,\lambda}(\lambda,T)d\lambda}{\int_0^\infty L_{q,\lambda}(\lambda,T)d\lambda} = \frac{\int_0^\lambda L_{q,\lambda}(\lambda,T)d\lambda}{(\sigma_q/\pi)T^3}$$



a) The following interferogram was obtained testing a nearly flat mirror in a Twyman-Green interferometer using a helium-neon laser operating at a wavelength of 633 nm. The mirror is tested double pass at a 30-degree angle of incidence. When the mirror being tested is uniformly pushed on so as to shorten the length of the test arm the fringes move to the left in the interferogram.

i) (20 %) What is the peak-valley error, in units of microns, of the mirror surface?

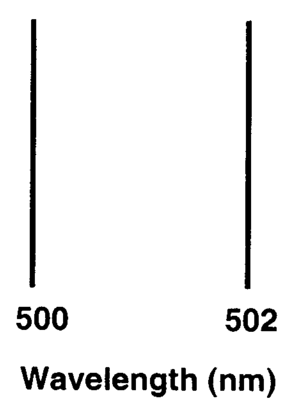
ii) (20 %) Is the center of the mirror a high point or a low point? Explain.



b) (20 %) Newton's fringes are observed with a quasi-monochromatic light of wavelength 500 nm. If the radius of curvature of the lens forming one part of the interfering system is 10 meters, what is the radius of the 20th bright fringe?

c) (20 %) Can holographic interferometry be used to measure deformations of objects whose surfaces are rough compared to the wavelength of the light used? Explain your answer.

d) (20 %) The following two fringes were obtained using Fringes of Equal Chromatic Order (FECO) to test two flat mirrors. What is the separation between the two mirrors?



Fall 2000

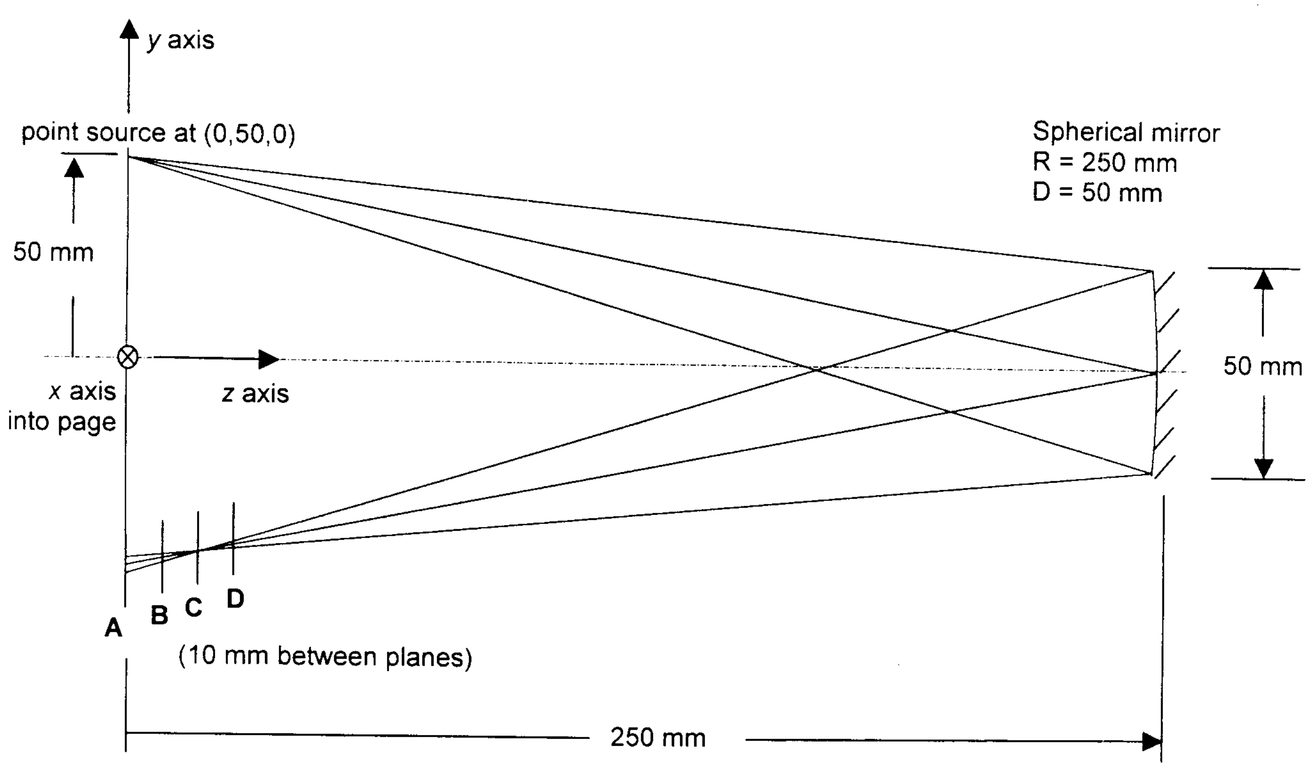
An easy way to think about the source of astigmatism is to use a simple geometrical projection. When the incident light is oblique to the surface, the optical element is foreshortened in one direction causing it to appear more steeply curved. An example is a spherical mirror, with the object and image plane at the center of curvature.

For the case below, consider a spherical mirror with 250 mm radius of curvature, located as shown below at $z = 250$. A point source is placed 50 mm off axis at $y = 50$. The image at plane **A** shows a line 4 mm long.

Draw the image as it goes through focus at planes **A**, **B**, **C**, and **D**. The planes are defined as

A: z = 0
B: z = 10
C: z = 20
D: z = 30

Make sure to show the orientation correctly and show dimensions for the images.



(a) Briefly describe the advantages and/or disadvantages of the following laser media:

i. Two-Level Systems

ii. Three-Level Systems

iii. Four-Level Systems

(b) The gain spectrum for a typical semiconductor laser is on the order of 30 nm wide.

i. Estimate the longitudinal mode separation for a Vertical Cavity Surface Emitting Laser (VCSEL), emitting at 840 nm, with a refractive index of 3.5 and an optical cavity length of 1.2 micron. Hint: The cavity length is 5λ.

ii. Explain why VCSELs lase in a single longitudinal mode.

(c) A Nd:YAG laser has a cylindrical rod with a diameter of 6 mm and a length of 2 cm. The Nd concentration is 1.38×10^{20} active ions per cm^3.

i. Calculate the theoretical maximum for the amount of energy that can be stored in the rod, in excited Nd ions.

ii. Use the result from part i to calculate an upper bound for the peak power in a Q-switched pulse emitted from this laser. Assume the pulse has a Gaussian temporal profile, $P_0 \exp\left[-(t/\tau)^2\right]$, with $\tau = 5$ nsec. (Note: $\int_{-\infty}^{\infty} \exp\left[-(t/\tau)^2\right] dt = \sqrt{\pi}\,\tau$)

(d) The Ti:Sapphire laser is tunable from about 700 nm to about 1100 nm. Use these values to estimate the duration of the shortest pulse which can be produced by modelocking a Ti:Sapphire laser.

Fall 2000

A manufacturer commissions a study of the consumer market for computers. This study finds that, for the typical American family:

(i) *at most one* new computer is independently bought in any given year ;

(ii) the probability p of buying a computer in a given year stays constant over the years; and

(iii) the average number of years *between* successive purchases of a computer is 5 years.

(a) (40%) What is the general form of the probability law governing the total number of computers bought over a 60-year span?

(b) (20%) What are the numerical values for the parameters of the law?

(c) (40%) Consider the event "a computer bought in n consecutive years". What is the general form of the probability law governing this event? (*Note*: The probability law must of course obey normalization.) Use the parameters found in part (b) to get the law in final form.

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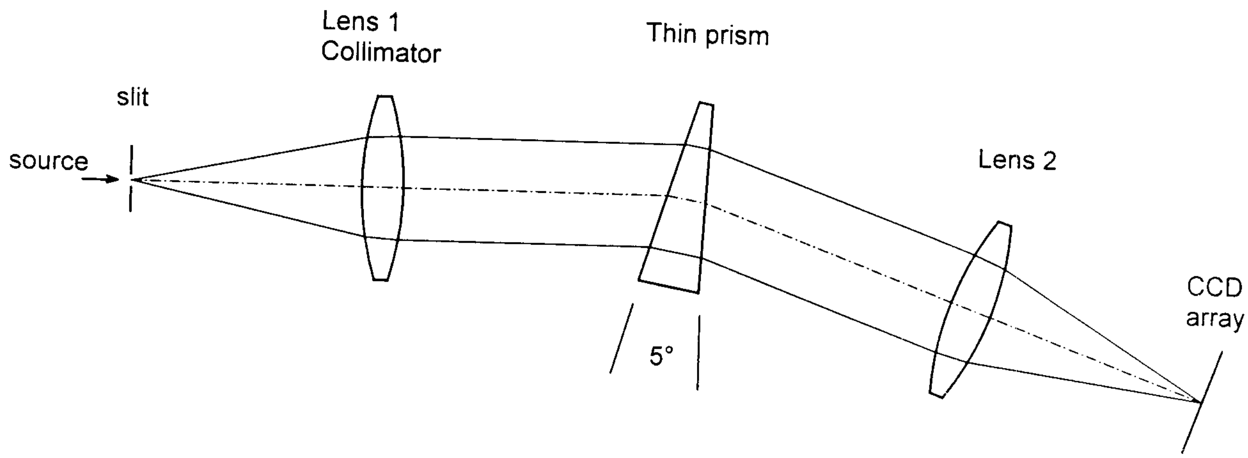
Consider a simple prism spectrograph, shown below:



Given:
Slit size: 10 μm
Lens 1: 200 mm focal length, ideal collimator

SF57 prism with wedge angle = 5°

Refractive index: $n_d = 1.847$

Abbe number: $\upsilon_d = \dfrac{n_d - 1}{n_F - n_C} = 23.8$

n_d is refractive index at the d line, etc.

The F, d, and C lines occur at wavelengths:
F : $\lambda = 486.1$ nm
d : $\lambda = 587.6$ nm
C : $\lambda = 656.3$ nm

CCD detector array with 12 μm pixels

Choose the focal length for lens 2 so that the system provides spectral resolution $R = \dfrac{\lambda}{\Delta\lambda}$ of 100 at a wavelength of 587 nm.

This is equivalent to requiring 5.87 nm (= 587/100) dispersion per 12 μm pixel.

Neglect second order effects.

Consider an electron with mass m, position r, and charge e, having the equation of motion with a damping factor β in presence of an electric field $E = E_0\, e^{i\omega t}$ with $\omega = 10^{15}$ rad/s.

(1) $m\ddot{r} + m\beta\dot{r} = eE$

a. What is the decay, or relaxation time τ in terms of β and what is the harmonic solution to (1) (20%)

b. What is the current density j (N free electrons per unit volume)? (20%)

c. What is the solution for the conductivity σ? (20%)

d. What is the dielectric constant $\hat{\epsilon}$ at optical frequencies using $\hat{\epsilon} \sim 1 + i\,\frac{4\pi\sigma}{\omega}$? (20%)

e. What are the real and imaginary components of the dielectric constant and what is the plasma frequency? (20%)

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Consider a plane wave in a conducting medium given by

$\psi(z,t) = \psi_0 e^{i(kz - \omega t)}$

The wave satisfies the following equation

$$\frac{\partial^2\psi}{\partial z^2} - \mu\sigma\frac{\partial\psi}{\partial t} - \mu\epsilon\frac{\partial^2\psi}{\partial t^2} = 0$$

a. Find the dispersion relation $k = k(\omega)$. (25%)

b. For a real ω, use $k = \pm(\alpha + i\beta)$ with α and β real. Find the expressions for α and β in terms of $Q = \omega\epsilon/\sigma$. (25%)

c. Write $k = (\alpha + i\beta) = |k|\,e^{i\Omega} = |k|\,(\cos\Omega + i\sin\Omega)$

Find α and β in terms of $|k|$ and Ω, and $|k|$ in terms of Q. (25%)

d. Find the wave velocity $v = \omega/\alpha$ and wavelength $\lambda = 2\pi/\alpha$ and the skin depth $\delta = 1/\beta$ in terms of Q. (25%)

In treating the interaction of light with matter, one usually writes the wave function for an atom interacting with the optical field as an expansion in terms of the unperturbed (i.e., no optical field present) atomic wave functions. Assuming a plane wave optical field, this results in a set of equations that can be written as:

$$\frac{dC_j}{dt} = -\frac{i}{\hbar}\sum_k C_k e^{i\omega_{jk}t} V_{jk} \quad \text{where} \quad V_{jk} = -\wp_{jk} E_0 \cos \omega t \tag{1}$$

(a) Identify by name the quantities C_k, $\wp_{jk}$, and ω_{jk} that appear in equation (1) and also write out the defining mathematical equation for each of these three quantities. (15%)

(b) What approximation(s) are made when this set of equations is solved by perturbation theory? Show explicitly how making the approximation(s) makes approximate analytic solutions of equation (1) possible. (30%)

(c) Give an example of a situation in which second-order perturbation theory is the appropriate approximation to use in treating the interaction of light with matter. Next, give a second example of a situation in which it is not appropriate to use perturbation theory to any order. (15%)

(d) The solution of (1) to first order in perturbation theory is

$$\begin{aligned} C_n^{(1)}(t) &= \frac{\wp_{ni} E_0}{\hbar}\left[\frac{\sin[(\omega_{ni}-\omega)t/2]}{(\omega_{ni}-\omega)}\right], \qquad n \neq i \\ C_i^{(1)}(t) &= 1 \end{aligned} \tag{2}$$

Use this result to derive Fermi's Golden Rule for the case where an appropriate sort of optical field acts on a sample of an atomic vapor. You may find it helpful to recall that $\int_{-\infty}^{\infty}(\sin^2 x/x^2)\,dx$ is just π. What did you assume about the optical field in your derivation? (40%)

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Consider the bandstructure ε_k of a one-dimensional crystal, and in particular the simple model of a band that is parabolic across the whole Brillouin zone.

(a)
Give an expression of the energy difference $\Delta\varepsilon$ between the zone boundary and the zone center.
Let the lattice constant at room temperature be $a = 5\mathring{A}$ and the effective mass $m = 0.2m_0$ (where m_0 is the electron mass in vacuum). Give the corresponding energy difference in units of eV. (Use $\hbar^2/m_0 = 7.62 \times 10^{-16} eVcm^2$.)

(40 %)

(b)
Are there any basic physical principles a negative effective mass would violate? If so, give a brief discussion. If not, calculate $\Delta\varepsilon$ for $m = -0.2m_0$.
Are there any basic physical principles an effective mass larger than m_0 would violate? If so, give a brief discussion. If not, calculate $\Delta\varepsilon$ for $m = 2m_0$.

(20 %)

(c)
Close to room temperature T_R, a typical temperature dependence of the effective mass is $m(T) = m(T_R) - m' \cdot \Delta T$ (here, ΔT is the deviation from room temperature, $\Delta T = T - T_R$). Similarly, the linear expansion of the lattice constant is given by $a(T) = a(T_R) + a' \cdot \Delta T$. Here, m' and a' are positive constants. Using the expansion formulas given below, determine the change of $\Delta\varepsilon$ linear in ΔT, i.e. neglect contributions to $\Delta\varepsilon$ that are beyond first order in ΔT.

Within this approximation, determine whether a temperature increase will lead to an increase or a decrease of $\Delta\varepsilon$ if $m' = 2 \times 10^{-5} m_0/K$ and $a' = 5 \times 10^{-5}\mathring{A}/K$ and the values for the room temperature parameters are those given in part (a).

Expansion formals: (i) $(1-x)^{-1} \approx 1 + x$, (ii) $(1+x)^{-2} \approx 1 - 2x$

(40 %)

Consider an optical Gaussian beam having wavelength λ, 1/e (amplitude) radius r_0, and radius of curvature R_c in the xy plane at $z = 0$. Find the location of the waist of the beam along the z-axis in terms of r_0, R_c and λ. Your answer must be valid for both cases $R_c > 0$ and $R_c < 0$.