WRITTEN PRELIM EXAM – FIRST DAY

FALL 2001

September 25, 2001
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$= 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

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$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

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$\sinh x = \frac{1}{2}(e^x - e^{-x})$

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$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F}\cdot\mathbf{G}) = (\mathbf{F}\cdot\nabla)\mathbf{G} + (\mathbf{G}\cdot\nabla)\mathbf{F} + \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F})$

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$\nabla \cdot (\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot\nabla\times\mathbf{F} - \mathbf{F}\cdot\nabla\times\mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla\times\mathbf{F}) + \nabla\phi\times\mathbf{F}$

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$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla\cdot\mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F}\cdot\mathbf{n})\, da = \int_V (\nabla\cdot\mathbf{F})\, d^3x$

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$\oint_S (\mathbf{n}\times\mathbf{F})\, da = \int_V (\nabla\times\mathbf{F})\, d^3x$

Fall 2001

The phase factor, τ, of a transmitted plane wave on total internal reflection is given by

$$e^{-i\tau} = e^{i\omega\left(t - \frac{x \sin\theta_i}{n v_2}\right)} e^{\mp \frac{\omega z}{v_2}\sqrt{\frac{\sin^2\theta_i}{n^2} - 1}}$$

Here, $i = \sqrt{-1}$, n $= n_2/n_1$ is the relative refractive index, v_2 is the velocity in medium 2, ω is the angular frequency, x and z are parallel and perpendicular to the interface, respectively, and θ_i is the angle of incidence. Also, the beam is incident on medium 1 and refracted into medium 2.

1. (25%) What are the conditions for obtaining total internal reflection ?

2. (25%) Explain the physical meaning of each of the two factors in the equation.

3. (25%) What happens to the transmitted field as θ_i increases ?

4. (25%) Describe a potential application for total internal reflection phenomena.

Answer the following as generally as possible, defining all terms you use.

(a) (12.5%) What law of statistics allows the abstract concept of a probability to be a physical observable?

(b) (12.5%) In a city where it rains, on average, 40% of the days, a reader of the daily weather forecast in the newspaper finds that, if rain is predicted, it does rain 80% of the time. Then how much Shannon information, in bits, is contained in a given forecast of rain?

(c) (12.5%) What is the probability density function obeyed by laser speckle intensity at a field point?

(d) (12.5%) If an instrument has the probability p of working on a given trial, what is the probability law on n successive workings of the instrument? (Note: the law should obey normalization)

(e) (12.5%) If a random variable obeys a binomial probability law with parameters N and p, what does it obey in two well-known extreme cases of the sizes of N and p?

(f) (12.5%) If a random variable x obeys a log-normal probability law, what probability law does x itself obey?

(g) (12.5%) If a variable y is formed as $y = \frac{1}{N}\sum_{n=1}^{N} x_n$ where N is a large but finite constant and the x_n are identically and independently random numbers from a Cauchy probability law, what probability law does y obey?

(h) (12.5%) Describe briefly one way of generating a normal random variable on a computer, without use of a normal random number generator.

Suppose that a stochastic process $n(y)$ is filtered by a deterministic processing kernel $f(x)$ to produce an output function

$$g(x) \equiv \int_{-\infty}^{\infty} dy\, n(y) f(x-y).$$

(a) (10%) Is $g(x)$ also a stochastic process?

(b) (20%) How does the power spectrum $S_g(\omega)$ for $g(x)$ relate to $S_n(\omega)$, the power spectrum for $n(y)$?

(c) (10%) You are now given that $n(y)$ is a zero-mean *random noise* process with variance profile $\sigma_n^2(y)$. Define such a process.

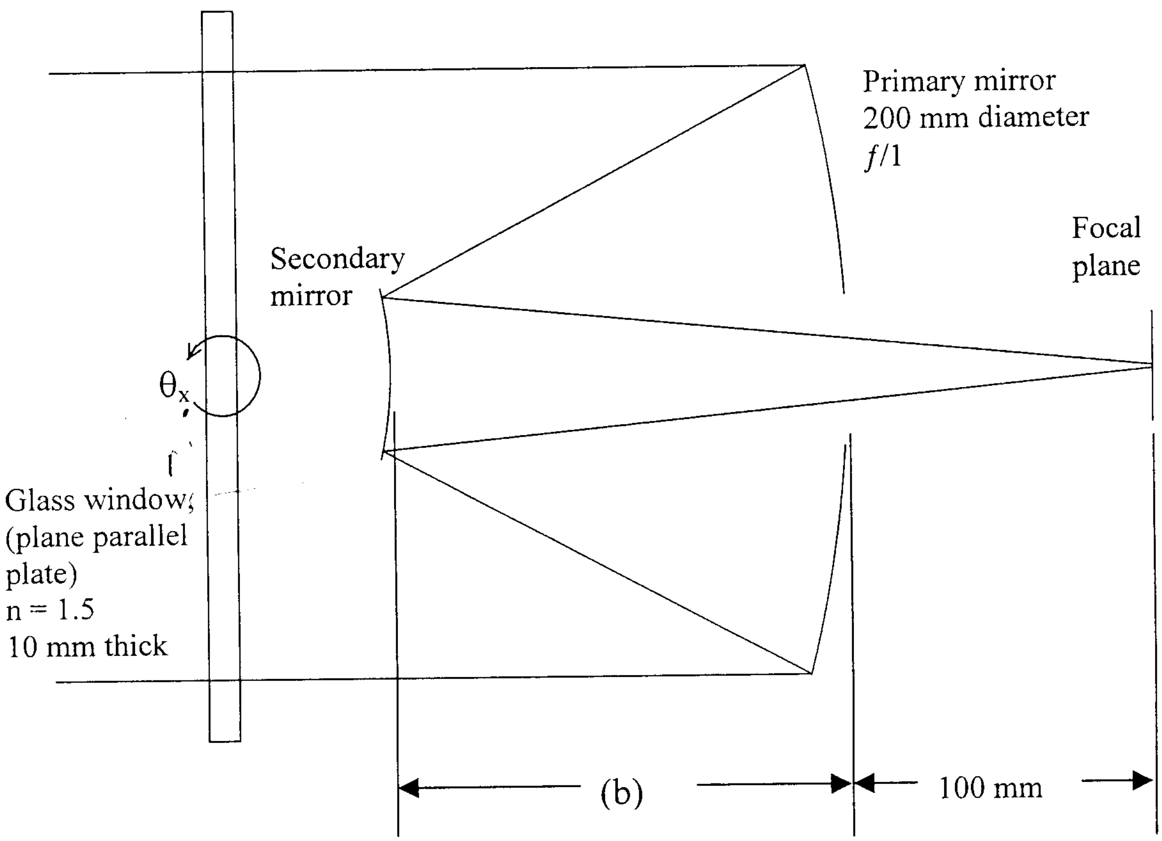
(d) (40%) Derive an expression that relates $\sigma_g^2(x)$, the variance in g at the point x, to the variance $\sigma_n^2(y)$ of the random noise and other given functions of the problem.

(e) (20%) What happens to the answer to part (d) if $n(y)$ also obeys wide-sense stationarity?

IV

Design a Cassegrain telescope using paraxial relationships

Use a 200 mm diameter primary mirror with focal ratio $f/1$
Put the aperture stop at the primary mirror
Set the distance from the primary vertex to the image to be 100 mm
Set the system focal ratio to $f/5$
The telescope is looking at distant objects, through a glass window 10 mm thick



1. (50%) Determine the prescription for the telescope, including
   a) Primary mirror radius of curvature
   b) Spacing from secondary mirror to primary mirror
   c) Secondary mirror radius of curvature
   d) Give the names of appropriate aspheric shapes for the primary and secondary mirrors
   e) Effective focal length for the system

2. (30%) Determine the location and size for the entrance pupil and exit pupil

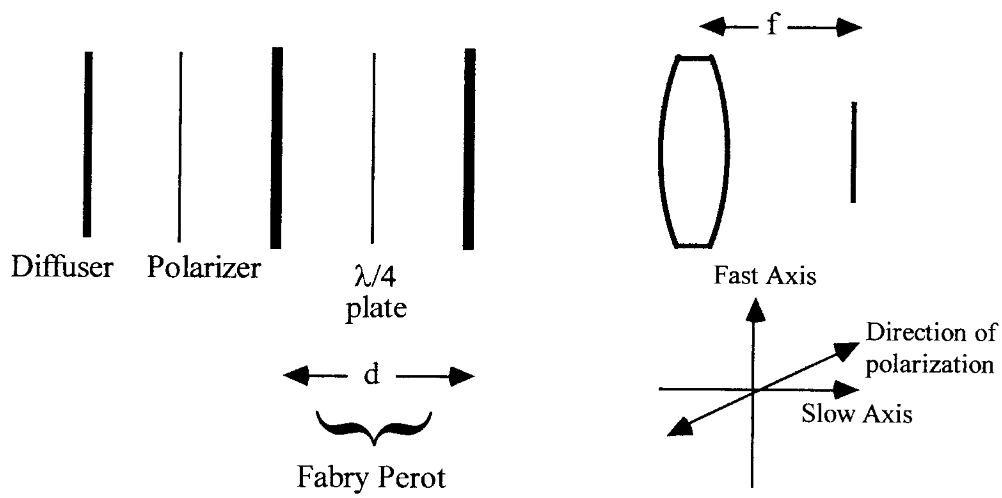
3. (20%) Looking in the image plane, evaluate the following two cases:
   a) Calculate the image size for an object that subtends 10 milliradians
   b) Calculate the shift in image position if the window is tilted 1 milliradian θ_x

An f/10 optical system has 5μm of optical path difference at the edge of the pupil due to third-order spherical aberration.

a) (30%) What is the geometrical spot size at paraxial focus?

b) (70%) What is the geometrical spot size at marginal focus?

A quarter-wave plate is placed inside a Fabry Perot cavity as shown below. The mirror separation is d. Both mirrors have an intensity reflectance of R and they introduce no phase change upon reflection and no absorption. A laser of wavelength λ is used to illuminate a diffuser placed before the Fabry Perot. Between the diffuser and Fabry Perot we place a polarizer such that the light entering the Fabry Perot is plane polarized at an angle θ with respect to the slow axis of the quarter-wave plate. Find the intensity distribution in the focal plane of the lens following the Fabry Perot. You can assume that the quarter-wave plate transmits 100% of all incident energy. Also, for simplicity assume that over the angles of interest the effective optical thickness of the quarter-wave plate is a constant.



WRITTEN PRELIM EXAM – SECOND DAY

FALL 2001

September 26, 2001
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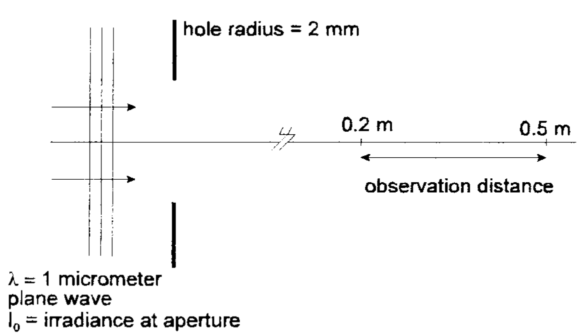
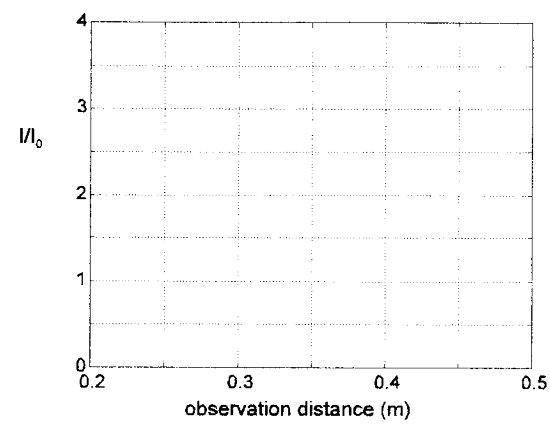
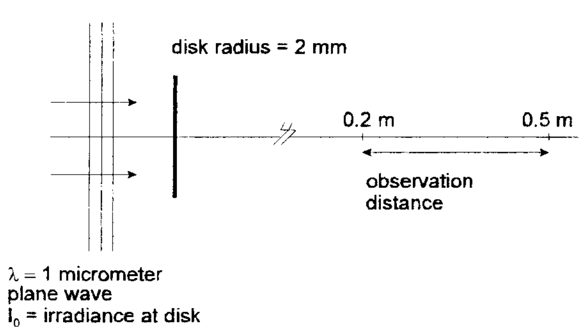
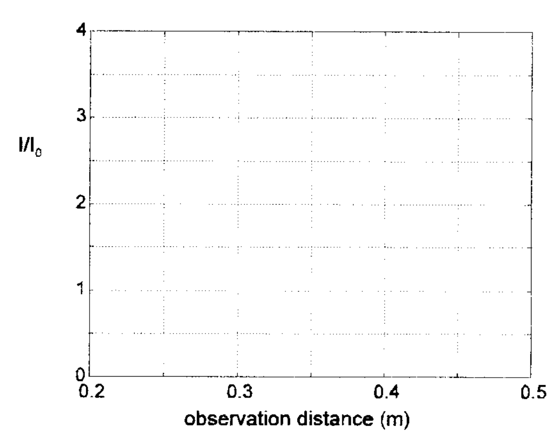
a) (50%) A 2mm diameter aperture in an otherwise opaque screen is illuminated with an on-axis plane wave as shown below. Sketch the on-axis irradiance as a function of the observation distance between 0.2 m and 0.5 mm from the aperture in the space provided. Normalize the vertical axis by I_0, which is the irradiance illuminating the aperture. Indicate on your sketch the location and value of any minima and maxima in the observation range.





b) (50%) A 2mm diameter opaque disk is illuminated with an on-axis plane wave as shown below. Sketch the on-axis irradiance as a function of the observation distance between 0.2 m and 0.5 mm from the disk in the space provided. Normalize the vertical axis by I_0, which is the irradiance illuminating the disk. Indicate on your sketch the location and value of any minima and maxima in the observation range.





Consider a gas laser with a 30 cm long confocal resonator operating continuously with a constant output power of 100 mW at $\lambda = 500$ nm. The gain bandwidth of the lasing transition arises from spontaneous emission (with a decay constant $\gamma = 1 \times 10^{-8}$ sec^{-1}) and from the motion of the atoms, which have a most probable speed of $\overline{u} = 3 \times 10^4$ cm/sec. In your calculations, assume that the gain bandwidth over which lasing is possible is equal to the full width of the unsaturated lasing transition.

(a) Will this laser produce a single output frequency or more than one frequency? If more than one frequency, how many frequencies will be present? (20%)

(b) Make a sketch of the gain coefficient α_g vs. frequency for this laser as it exists during CW operation. Also indicate the laser's output frequency (or frequencies) on your sketch. (35%)

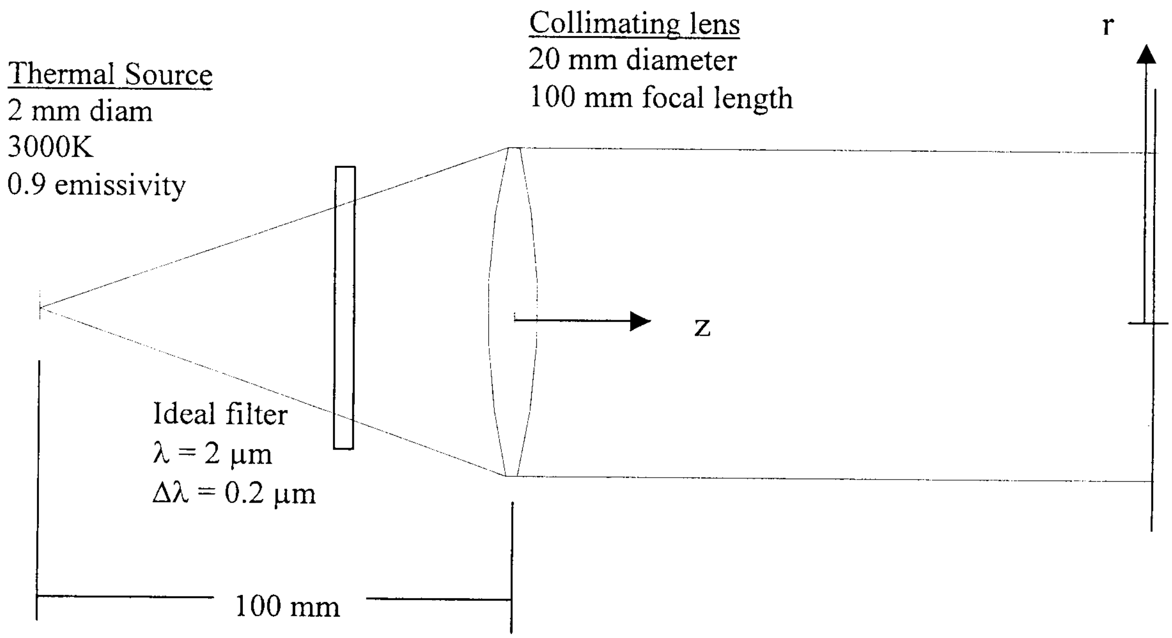
(c) If the resonator length is slowly increased in such a way that an axial mode of the resonator is tuned through the center of the gain bandwidth curve, what, if anything, will happen to the output power of the laser? Explain your answer. (20%)

(d) What is a confocal resonator? Name at least two unique characteristics that this type of resonator has. (25%)

IX **Fall 2001**

Consider the following collimator with a thermal source

Circular source, 2 mm diameter
Lambertian graybody at 3000K with emissivity of 0.9
20 mm diameter collimating lens with 100 mm focal length



For each of the four locations listed :

a) z = 0 (at the lens)
b) z = 500 mm
c) z = 1000 mm
d) z = 2000 mm

Calculate the on-axis photon flux density (photon irradiance) in the wavelength band from λ = 1.9 to 2.1 μm. Give the value in photons/sec/cm^2. You can ignore cosine effects.

X

Fall 2001

A photodiode has recently been fabricated from the newly-discovered Dereniakium, which has a direct bandgap of 0.85 eV. The detector has a responsive quantum efficiency of 0.85 and an active area of 5 mm^2.

40% Sketch the absolute spectral responsivity (A/W) as a function of wavelength, and point out relevant features of the curve.

30% Three different lasers are **simultaneously** directed at this detector. It receives 1 mW from a He-Ne laser at 632.8 nm, 1 mW from a Nd-YAG laser at 1.06 μm, and 2 mW from an Ir-Ne laser at 1.532 μm. The $1/e^2$ beam diameter of each laser is 2 mm. Calculate the output current of the detector.

30% Assuming that the only noise present is shot noise due to the photons incident from the three lasers, calculate the S/N ratio for a 100 Hz effective noise bandwidth.

20% 1. Sketch the absorption of a bulk semiconductor in the vicinity of the band edge without and with the Coulomb interaction.

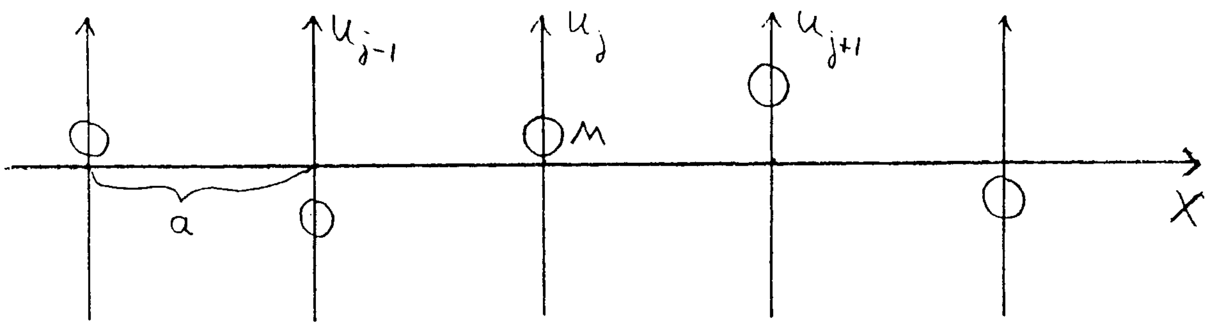
20% 2. Sketch the absorption/gain of a quantum well in the vicinity of the band edge as a function of the carrier density. What effects contribute to this carrier density dependence?

20% 3. Optical Stark effect in a quantum well. Write down a general expression from atomic physics for the Stark shift (generalized Rabi frequency) as a function of optical field amplitude and detuning from the resonance. How does this depend upon the field amplitude when the Rabi frequency is large compared to the detuning, and when it is small compared to the detuning?

20% 4. Sketch the optical Stark shift as a function of time for a subpicosecond pulse detuned several linewidths below the heavy-hole exciton. Assume the pump and probe have the same circular polarization.

20% 5. What is the origin of the coherent oscillations observed when the probe precedes the pump?

Consider transverse lattice vibrations in an infinitely long, one-dimensional monatomic lattice with lattice constant a. Assume that the motion of the atoms is governed by Newton's equation. Denote the displacement of atom #j by u_j. Let the mass of the atom be M, and assume that there are only harmonic nearest-neighbor interatomic forces. For example, the force on atom #j due to the neighbor $j+1$ is $F_{j,j+1} = k \cdot (u_{j+1} - u_j)$ (where k is the force constant), and there is a similar force on atom #j due to the other nearest neighbor.



Specify Newton's equation for the displacement of atom $\#j$ and solve the equation using the ansatz

$$u_j(t) = A\, e^{-i(\Omega t - q x_j)}$$

where $x_j = j \cdot a$ is the position of atom #j along x. Derive the dispersion relation $\Omega = \Omega(q)$ and express it in terms of M, k and a. Use the identity $\frac{1}{2}(1 - \cos(qa)) = \sin^2(qa/2)$ to cast the result into a standard form. Clearly indicate if there is any difference in the solution for positive and negative q values. Sketch the result in the q-interval $[-\pi/a, \pi/a]$.