WRITTEN PRELIM EXAM – FIRST DAY

SPRING 2001

February 20, 2001
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2 \cos^2 A - 1$

$\cos 2A = 1 - 2 \sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

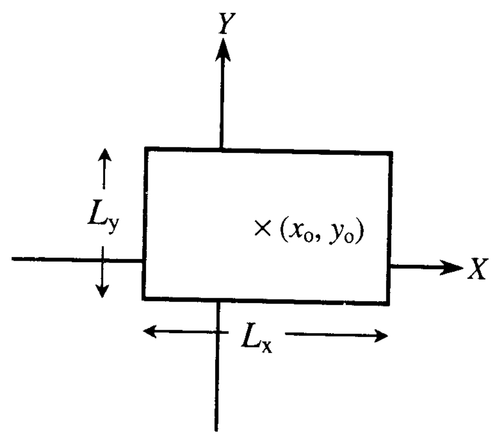
$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Spring 2001

Fourier Optics

Consider a rectangular aperture of dimensions $L_x \times L_y$ centered at (x_o, y_o) in the XY-plane, as shown in the figure. A uniform, monochromatic plane wave (wavelength = λ) propagates along the Z-axis. This beam is incident at $Z = 0$ on an opaque screen containing the rectangular aperture; only that portion of the beam which falls within the aperture is transmitted.



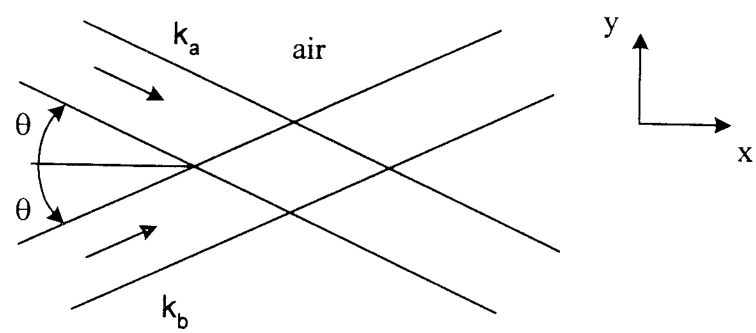
a) (40%) Assuming $x_o = y_o = 0$, find the complex light amplitude distribution in the far field (i.e., Fraunhofer pattern).

b) (40%) Using the shift theorem of Fourier transform theory together with the result obtained in part (a), determine the far field amplitude distribution in the general case where $x_o \neq 0$ and $y_o \neq 0$.

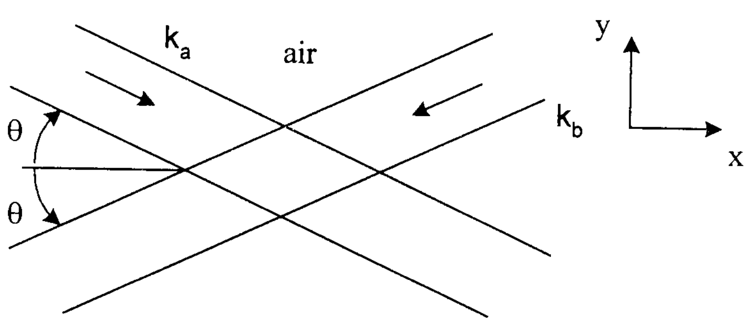
c) (20%) Physically, one expects that when the aperture is shifted from (0, 0) to (x_o, y_o) the far field distribution will be similarly displaced. With regard to the result obtained in (b), is this expectation fulfilled? If not, explain the reason.

Assume z-polarized waves in the problems below.

a) (30%) Two collimated laser beams overlap as shown below. Given that $\lambda_a = \lambda_b = 0.5\ \mu m$ and $\theta = 1^\circ$, show the orientation and spacing of the fringes by sketching a series of straight, equally spaced lines in the xy plane.



b) (30%) same as (a), only the propagation direction for the second beam is changed as shown below.



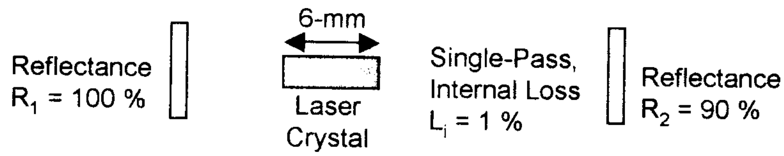
c) (15%) Assume $\lambda_b = \lambda_a + \Delta\lambda$, where $\Delta\lambda = 0.0125$Å. Describe the change in the behavior of the fringes if the beams are traveling as in part (a).

d) (15%) Assume $\lambda_b = \lambda_a + \Delta\lambda$, where $\Delta\lambda = 0.0125$Å. Describe the change in the behavior of the fringes if the beams are traveling as in part (b).

e) (10%) Describe a method to measure $\Delta\lambda$, using a single silicon detector and one of the two geometrics shown above.

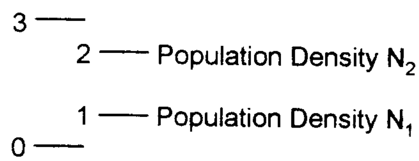
III

A solid-state laser is illustrated below. SPRING, 2001



a. (40 %) The optical loss inside the laser cavity is 1 % per pass due to light scattering in the laser crystal. Find the gain coefficient for the crystal when the laser is above threshold.

b. (30 %) The laser crystal can be accurately modeled using a four-level-system illustrated below. The (non-degenerate) levels 1 and 2 are separated by 1.17 eV. Calculate the ratio N_2/N_1 when the crystal is in thermal equilibrium at room temperature.



c. (30 %) The effective emission cross-section for the lasing transition from level 2 to level 1 is $\sigma_e = 2.8 \times 10^{-19}$ cm^2. Find N_2 - N_1 (the "inversion") when the laser is above threshold.

Consider a compound microscope that has the following:

25X objective with numerical aperture NA of 0.5
Standard 160 mm tube
10X eyepiece
Telecentric (so focal shifts blur, but do not translate the images)

Assume the microscope is focused on an object that has a diameter of 100 microns.

1.) What is the effective focal length for the objective, and for the eyepiece? (10%)

2.) Sketch the layout of this instrument including: (50%)
Positions of the objective and eyepiece.
Positions and sizes of internal and final images.
Positions and sizes of aperture stop, entrance pupil, and exit pupil.

3) Assume diffraction limited performance in visible light (40%)
(a) What is the limiting resolution for the system, in units of microns at the object? How do you define "diffraction limited resolution"?
(b) Convert this to an apparent angle as seen by the viewer.
(c) How does this compare to typical resolution for the eye?
(d) What is the diffraction limited depth of focus for the object? (How far could the object move out of focus while the system performance remains diffraction limited?)

A fixed frequency CW optical field having an irradiance I passes through a sample of atoms and is partially absorbed. Assume that only a single homogeneously broadened transition is near resonance with the field.

(a) Sketch the behavior of the absorption coefficient versus I as the irradiance of the field is slowly increased. (10%)

(b) State Beer's Law in a form that is appropriate for all irradiances I. (10%)

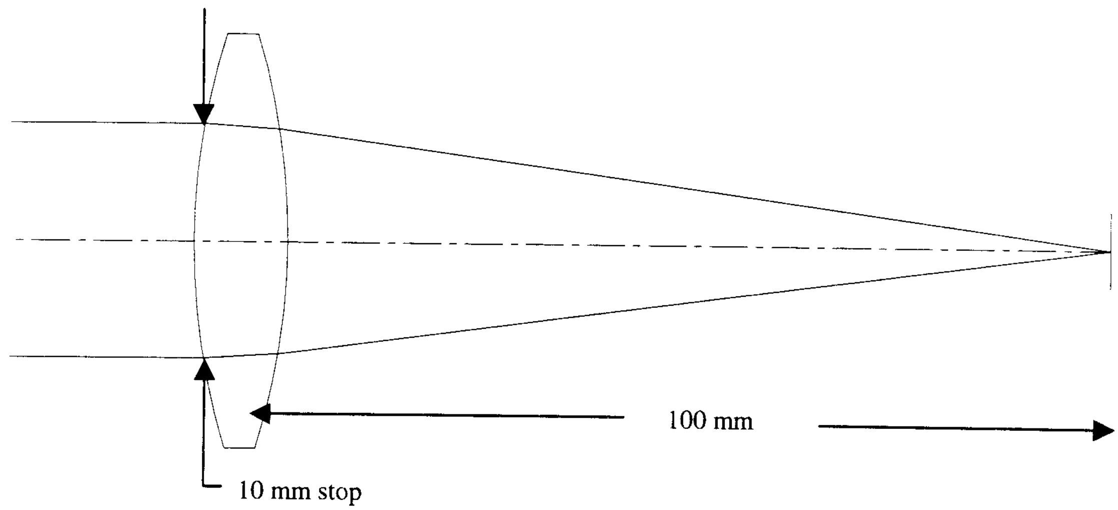
(c) Explain physically what is happening to the atoms in the sample as I is varied, causing the absorption coefficient to change with increasing I. (20%)

(d) How, if at all, does the explanation you gave in part (c) change if the transition is inhomogeneously broadened? (15%)

(e) Write out an appropriate set of rate equations for this problem. (20%)

(f) Could the rate equations of part (e) be used to provide a quantitative description of how the absorption coefficient varies with I? If so, how? (25%)

Consider a single thin lens made of BK7. The focal length of the lens for yellow (*d* – line) light is 100 mm. Assume an object point at infinity and a 10 mm stop at the lens. Parameters for the glass and definitions of the wavelengths are given in the tables below.



1. Draw a layout of the lens indicating where along the axis the yellow (*d*), blue (*F*), and red (*C*) light comes to focus. What is the spacing between the red and blue foci (10%)

2. Assume an image plane 100 mm from the lens, as shown. Draw the following plots for *F*, *d*, and *C* light for the case of on-axis imaging. Be sure to label your axes, including values.
   a) Ray fan diagram (transverse ray aberration *vs.* pupil coordinate) (20%)
   b) OPD diagram (wavefront aberration *vs.* pupil coordinate) (20%)

3. Draw the geometric point spread function for white light for two cases: (20%)
   a) on axis
   b) for a field point 0.01 radians off axis

4. The chromatic effect can be balanced by combining two thin lenses, one made of BK7, and the other made of a different glass, SF57. Calculate the power for each lens to obtain a system focal length of 100 mm and correction of the primary chromatic aberration. (30%)

	BK7	SF57
refractive index n_d	1.517	1.847
Abbe number v_d	64.2	23.8

n_d is refractive index at the *d* line, etc.

Abbe number is defined as $v_d = \dfrac{n_d - 1}{n_F - n_C}$

The *F*, *d*, and *C* lines occur at wavelengths:
$F : \lambda = 486.1$ nm
$d : \lambda = 587.6$ nm
$C : \lambda = 656.3$ nm

Prelim question : 501

Consider an electromagnetic wave incident on a metal surface whose conductivity and dielectric constant are σ and ϵ.

1. Calculate from Maxwell's equations the relaxation time τ of the charge density ρ inside the metal. (25%)
2. Derive the wave equation of the electric vector $\mathbf{E}$ inside the metal. (25%)
3. For a purely monochromatic wave, find an expression for the real and for the imaginary components of the refractive index of the metal. (25%)
4. What is the distance d at which the wave decays inside the metal to 1/e of its initial value. (25%)

WRITTEN PRELIM EXAM – SECOND DAY

SPRING 2001

February 21, 2001
8:30 a.m. to 12:30 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_{\mathrm{S}} (\mathbf{F} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_{\mathrm{C}} \mathbf{F} \cdot d\ell = \int_{\mathrm{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_{\mathrm{S}} (\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}} (\nabla \times \mathbf{F})\, d^3x$

Consider a light beam propagating in a medium characterized by the susceptibility χ. The propagation equation is

$$\left\{\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right\} E(z,t) = -\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}P(z,t)$$

Specifically, consider only solutions of the form

$$\begin{aligned} E(z,t) &= E_0\, e^{-i(\omega t - kz)} \\ P(z,t) &= \chi E_0\, e^{-i(\omega t - kz)} \end{aligned}$$

where the frequency ω is real and the propagation constant k is allowed to be complex, $k = k' + ik''$. The complex susceptibility $\chi = \chi' + i\chi''$ is assumed to be appropriate for the frequency ω under consideration.

(a) Based on the information given above, derive a complex equation that relates k to ω (dispersion relation). It should be formulated in terms of the complex dielectric function, $\varepsilon = \varepsilon' + i\varepsilon''$, which you have to define appropriately. Furthermore, derive two real equations that are equivalent to the complex equation.

(50 %)

(b) Consider the specific case of a medium described in terms of Lorentz oscillators with eigenfrequency ω_0. Consider a light frequency ω above the eigenfrequency ω_0 such that the dielectric function is essentially real and negative ($\varepsilon' < 0$ and $\varepsilon'' = 0$). Using the dispersion relation formulated in terms of two real equations in part (a) of this problem, determine whether k' or k'' is zero in this case. Finally, express the inverse penetration depth α, defined by

$$|E(z,t)|^2 = E_0^2\, e^{-\alpha z}$$

in terms of the light frequency and the dielectric function.

(50 %)

a) (40 %) A quasi-monochromatic slit source of wavelength 500 nm is used in a Young's two-pinhole experiment. The source is 2 meters from the two pinholes, and the pinholes are separated by 2 mm. Fringes are observed 0.5 m from the pinholes. What is the minimum width of the slit source such that zero contrast fringes are obtained?

b) (30 %) Newton's rings are observed using light having two wavelengths of 500 nm and 500+Δ nm. What is Δ if the fringe visibility drops to zero for the 21st dark fringe for the 500 nm wavelength?

c) (30 %) Sketch a setup of the Hanbury-Brown Twiss system for resolving binary stars. Give two advantages and two disadvantages of the Hanbury-Brown Twiss system versus the Michelson Stellar interferometer

Your supervisor asks you to choose an appropriate detector for a laser communications experiment. There is 1550 nm optical radiation exiting a fiber that is rated at NA 0.4. The radiation is digitally modulated at frequencies up to 1 MHz and the peak power is 250 pW. The receiver (detector + interface electronics) output must be captured by a digital oscilloscope.

a. From the table of detectors below, select the one that gives the highest SNR. All data is at 300K. Show your calculations. (60%)

b. Describe appropriate interface electronics. (20%)

c. Describe the effect of a voltage bias on your chosen detector. (20%)

DETECTOR	MODE	RQE	Io (amps)	Eg (eV)
GaP	PV	0.8	1E-12	2.25
GaAs	PV	0.7	1E-11	1.4
Si	PV	0.6	1E-10	1.12
InGaAs type 1	PV	0.7	1E-9	0.75
Ge	PV	0.6	1E-7	0.68
InGaAs type 2	PV	0.6	1E-6	0.60
InGaAs type 3	PV	0.5	1E-5	0.48
PbS	PC	0.4	R_d=1E8 ohms	0.48
InAs	PV	0.5	1E-3	0.41
PbSe	PC	0.4	R_d=1E8 ohms	0.30
InSb	PV	0.5	1E-2	0.22

Let us consider a particle of mass m confined in an infinite square well potential $V(x)$. The wavefunction $\psi(x)$ of this particle obeys the time independent Schrödinger equation. The Hamiltonian for the system is given by:

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \quad \text{with}$$

$$V(x) = 0 \quad \text{for} \quad 0 < x < L$$
$$V(x) = \infty \quad \text{for} \quad x \leq 0,\ x \geq L$$

a) What is the value of the wavefunction of the particle outside the square well? (10%)

b) Calculate the wavefunction of the particle inside the square well. For this problem, seek for solutions of the general form $\psi(x) = A\sin kx + B\cos kx$, where A and B are constants. Invoke the continuity of the wavefunction at the boundaries $x = 0$ and $x = L$. Use the fact that the wavefunction is normalized. (40 %)

c) Calculate the energies that the particle can have in the square well. Show that the energy states are quantized. (20%)

d) What is the lowest energy that the particle can have in the square well? (20%)

e) Describe how the energy difference between the stationary states of the particle characterized by the quantum numbers $n = 1$ and $n = 2$ changes when the value of the width of the square well potential is doubled. (10%)