WRITTEN PRELIM EXAM – FIRST DAY

Fall 2002

September 24, 2002
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

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$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

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$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

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$\sin 2A = 2\sin A \cos A$

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$\cos 2A = 1 - 2\sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

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$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

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$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

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$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Consider a CW laser beam of frequency ω passing through a transparent Lanthanum Fluoride crystal doped with a small percentage of Praseodymium atoms (Pr^{3+}: LaF_3). The Praseodymium atoms in this crystal have a single isolated transition out of their ground state in the visible region of the spectrum (592.52 nm). We want to examine what happens when the frequency of the laser is slowly tuned through resonance with this transition.

If we had a perfect single crystal of this material, the polarization induced in the Praseodymium atoms by the optical field would be given by

$$P = \frac{\wp^2 E_0}{2\hbar}(N_b^0 - N_a^0)\left[\frac{\omega_0 - \omega}{\mathcal{R}^2+\gamma^2} + i\,\frac{\gamma}{\mathcal{R}^2+\gamma^2}\right] e^{-i(\omega t - kz)} \; + \; \text{c.c.}$$

Here $\mathcal{R} = \sqrt{(\omega_0-\omega)^2 + (\wp E_0/\hbar)^2}$ is the generalized Rabi oscillation frequency, $\wp$ is the transition dipole matrix element, γ is the linewidth (half-width at half-maximum), and N_i^0 is the population of state i when no light is present.

(a) Find an expression for the absorption coefficient α produced by this polarization when the laser frequency is near resonance with the transition. Don't worry if you don't have every constant quite right. (25%)

(b) In actuality, perfect crystals of this material cannot be obtained, and the imperfections cause the transition frequencies ω_0 of the Praseodymium atoms to be Gaussian distributed with a width $\Delta\omega_0$ (half-width to the $1/e$ point). At low temperatures, this distribution of transition frequencies becomes much larger than the intrinsic linewidth γ. Find an expression for the absorption coefficient α in this situation. (35%)

(c) For the imperfect crystal at low temperature described in part (b), sketch the behavior of the absorption coefficient α that would be observed as a function of the laser frequency ω as the laser is tuned through the absorption linewidth. Assume that the irradiance of the laser beam is such that $\wp E_0/\hbar > \gamma$. (20%)

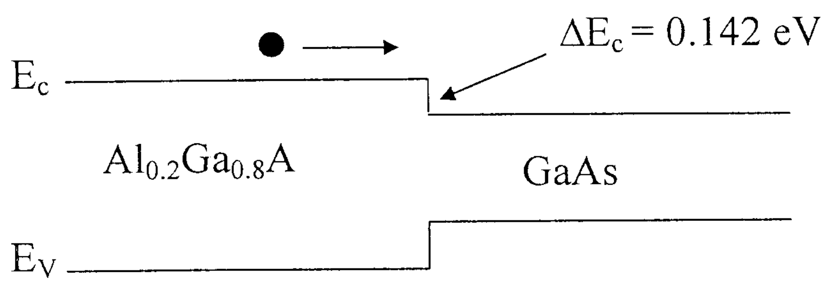
(d) If we again have the situation of part (b), with the irradiance of the laser beam again such that $\wp E_0/\hbar > \gamma$, sketch the behavior of the actual Praseodymium atom population difference $N_b - N_a$ in the crystal as a function of frequency when the laser beam is present with its frequency <u>held fixed</u> at some value near resonance. (20%)

A Keplerian telescope has the following specifications:

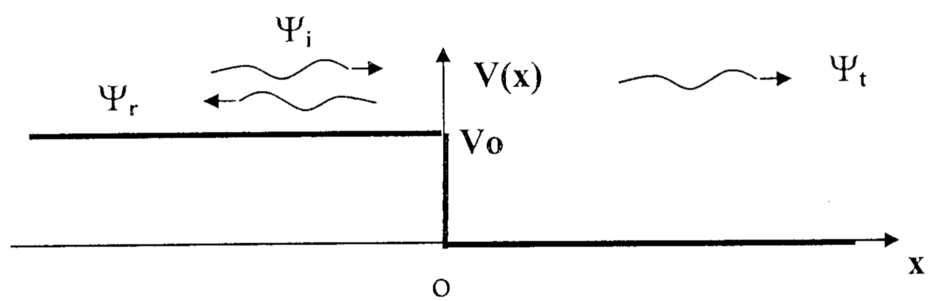
Magnifying Power = 12 X
Length = 260 mm (objective to eye lens)
Unvignetted Field of View = +/- 2 degrees
Eye Relief = 15 mm
Entrance Pupil Diameter = 40 mm
Stop located at the Objective
All elements are thin lenses

Design the telescope. Provide element focal lengths, diameters and spacings. Be sure to verify that all of the above specifications are met.

Using molecular beam epitaxy (MBE) it is possible to form a semiconductor heterojunction, that changes abruptly over one atomic spacing. Consider an electron of energy E and effective mass m* in the AlGaAs region that is incident on the heterojuction of height ΔE_c.



Classically, we would expect the electron to pass over the conduction band discontinuity and continue into the GaAs. However, quantum mechanically the situation can be different. Lets investigate the conduction band using the following model.



Each part is worth (25%)

a) Set up the time independent Schrodinger equation for $x<0$ and $x>0$
b) Solve for the wavefunctions for each region
c) State the boundary conditions at $x=0$
d) Using intuitive physical arguments (like F=ma, etc) explain what happens to the velocity of the electron as it passes from AlGaAs to GaAs. Does the velocity change instantaneously?

A scientific grade CCD array has a quantum efficiency of 0.75 at 700 nm, a dark current of 50 pA/cm^2 at 20°C, and square pixels that are 24 μm on a side. The full-well capacity (maximum that a pixel can store) is 100,000 electrons. The array is placed in the focal plane of a camera (F/2 optics, no losses).

20% At 20°C, how long does it take for dark current to fill a well?

30% You wish to integrate a dim scene with an in-band radiance of 0.01 mW/m^2-sr at 700 nm. Ignoring for a moment the dark current, how long does it take to reach 50% of the full-well capacity?

30% To what temperature must you cool the array such that the generated charge from the scene is 20 times larger than that from the dark current? The functional form of the dark current vs. temperature is

$$i_d = 3 \times 10^{-10} e^{T/10}$$

20% If the CCD array is 1024 x 1024 pixels, the CTE (charge transfer efficiency) is 0.99995 and it takes three transfers to move a well of stored charge the distance of one pixel (i.e., 3ϕ clocks), the resulting image of a uniform scene will be flawed. Describe the appearance of the image. Is there a viable scheme to reduce this flaw?

Question #5 FALL 2002

a). Obtain general expressions for the complex current density j and specific conductivity σ of a metal using the equation of motion of an electron (20%).

b-e). A plane wave with frequency of 1 MHz is incident normally on a good conductor. Let the conductor have specific conductivity $\sigma = 5.8 \times 10^7$ siemens/meter, permittivity $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}$ F/m, and magnetic permeability $\mu = \mu_0 = 4\pi \times 10^{-7}$ N/m. . Define, write the expressions, and calculate the following properties that take place inside the conductor in SI units:

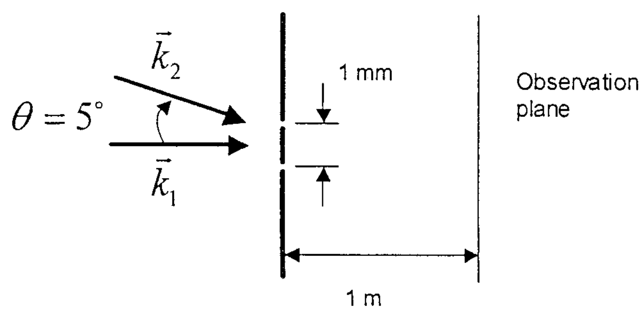
b. Skin depth (20%)

c. Phase velocity (20%)

d. Group velocity (20%)

e. Average Poynting vector (20%)

Two plane waves illuminate a Young's double pinhole interferometer as shown. $\lambda_1 = 532$ nm and $\lambda_2 = 541.4$ nm. The angle between the plane waves is 5 degrees. $\vec{k}_1$ is normally incident onto the aperture plane. The pinhole spacing is 1 mm, and the distance from the aperture plane to the observation plane is 1 m. Ignore the finite size of the pinholes, and assume that the observation region is limited to a small area around the axis. Sketch the observation-plane irradiance pattern , showing any important features. It may not be possible to show both fine and macroscopic features in the same sketch, so show fine features in one sketch and macroscopic features (features > 10 times larger than the smallest feature) in a second sketch.



WRITTEN PRELIM EXAM – SECOND DAY

Fall 2002

September 25, 2002
8:30 a.m. to 12:00 p.m.

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$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Consider a photon-counting detector illuminated by a steady (not modulated, not noisy) light source. Let the mean rate of detection be denoted by λ.

(a) Give a simple expression for the probability of detecting N photons in a measurement time T.

(b) Now suppose you make 10 measurements in 10 nonoverlapping time intervals, each of duration T. Give an expression for the probability of detecting N_1 photons in the first interval, N_2 in the second interval, etc.

(c) Use the expression derived in part (b) to find an expression for the maximum-likelihood estimate of the rate λ.

(d) Compute the variance of this maximum-likelihood estimate.

Two plane waves with the same wavelength, but different amplitudes, interfere in space.

a.) (40%) If $\vec{k}_1 = k\hat{k}_1$ and $\vec{k}_2 = k\hat{k}_2$, where $\hat{k}_1 = 0.1\hat{x} + 0.866\hat{z}$ and $\hat{k}_2 = \hat{z}$, what is the orientation of fringe planes?

b.) (30%) What is the minimum spacing between fringe planes if $l_1 = l_2 = 632.8\,\text{nm}$?

c.) (30%) Does visibility depend on orientation of the observation screen?

You are given an ideal telecentric, "4F", imaging system. It consists of two lenses of focal-length *f* separated by a distance *2f*. The input plane to the system is *f* upstream of the first lens and the output plane is *f* downstream of the second lens. Midway between the lenses is an aperture with complex-amplitude transmittance $a(x, y)$. There is a transparency, with complex amplitude transmittance $t(x, y)$ in the input plane.

Case 1:

The transparency is illuminated by coherent light in the form of a normally-incident plane wave.

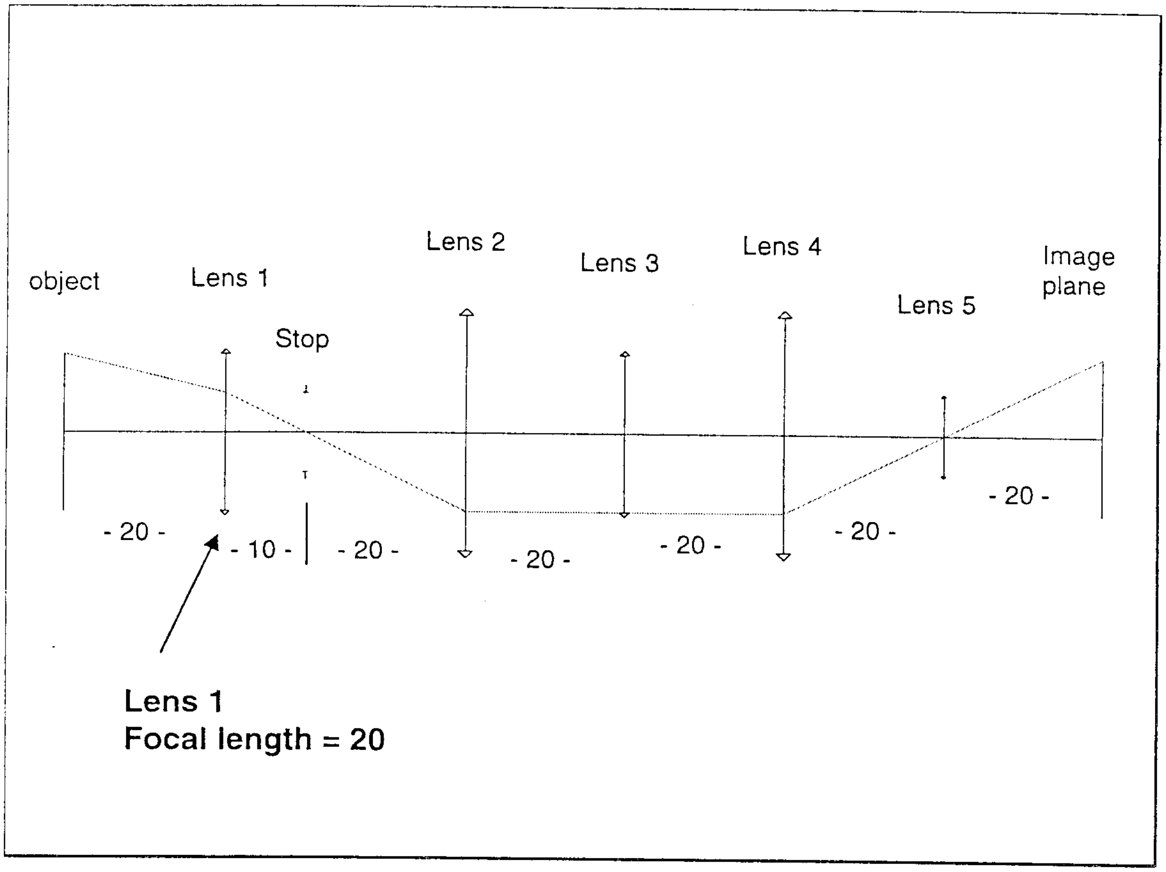
- What is the coherent transfer function of the system? Show the conversion to reduced coordinates for the spatial frequencies.
- What is the coherent point spread function of the system?

Case 2:

The same transparency is illuminated with quasi-monchromatic incoherent light.

- What is the incoherent transfer function of the system? How is it related to the coherent transfer function?
- What is the modulation transfer function (MTF) of the system?
- What is the incoherent point spread function of the system?
- A cosine grating, $a(x, y) = \frac{1}{2}\left[1 + \cos(2\pi\xi_0 x)\right]$, replaces the aperture. What is the resulting incoherent point spread function?
- What is the output of the system with the grating in place?

Consider the following optical system that uses ideal, paraxial elements to relay an image from the object plane to the image plane.
The **chief ray** is drawn.
The spacings between elements are given, and the drawing is to scale.



Answer the following questions (equal weighting for each)

1. Determine the positions of the entrance pupil and the exit pupil
2. Use the graphical information above to determine the focal lengths for Lenses 2 - 5
3. a. Locate the position of the internal image (a real image formed somewhere other than at the image plane).
   b. Determine the system magnification.
4. Change the power of only one lens to make this system telecentric in image space, yet maintain the image position and magnification.

You are given the task of helping to construct a laser. You have the following items available:

- a 0.75-meter long argon-ion tube, sealed with Brewster windows
- a planar high reflector
- a concave output coupler with a radius of curvature of 2.0 meters

(a) In constructing the laser, you need to make a stable optical cavity with the two available mirrors, separated by a distance L. What range of distances can you choose for L in order to ensure that the optical cavity is stable?

(b) You decide to build the cavity with a mirror separation of 1.0 meter. What is the axial cavity mode frequency spacing in units of Hertz? How does this result depend upon the wavelength of the light in the cavity?

(c) You turn on the pumping mechanism for the argon-ion gain medium, and align the cavity so that laser light is generated, and is transmitted through the concave output coupler. What primary measurable effect will the Brewster windows have on the laser light?

(d) Argon-ion lasers may lase at several different center wavelengths, corresponding to different atomic transitions. If you were to isolate the light from a single atomic line and look at the frequency spectrum of the light, you would find a comb of frequencies within a Gaussian-shaped envelope. The linewidth of this envelope is about 3.5 GHz. What is the most significant broadening mechanism that contributes to this linewidth? Be sure to write down the name of this mechanism, and include a brief description of the physical origin of this mechanism (i.e., what is happening within the laser?).

(e) Is this broadening mechanism an example of homogeneous or inhomogeneous broadening?

(f) Where in the cavity does the optical field reach its minimum beam waist? Ignoring the focusing properties of the output coupler, is the output laser beam converging, diverging, or collimated?

(g) Sketch beam profiles of three non-degenerate Hermite-Gaussian transverse modes. What can you add within the cavity to ensure that the laser operates only in the fundamental transverse mode?

All parts equally weighted.

Question #12 FALL 2002

a. Describe physically and supply a complete mathematical representation of a right-circularly polarized electromagnetic plane wave of circular frequency ω propagating in free space along positive z direction of a right-handed coordinate system. The Poynting vector has a value of 2 [Wm^{-2}]. (State the convention of handedness that you have used.)

b. The wave of item (a) enters a dielectric medium of refractive index n. What is the Poynting vector in this medium?

c. The wave of item (a) propagating in free space encounters a counter-propagating right-circularly polarized plane wave of the same circular frequency and the same irradiance. Describe the superposition of these two waves along the z axis and the state or states of polarization (orientation of the electric field) along the z axis.

All parts equally weighted.

MINOR WRITTEN PRELIM EXAM

FALL 2002

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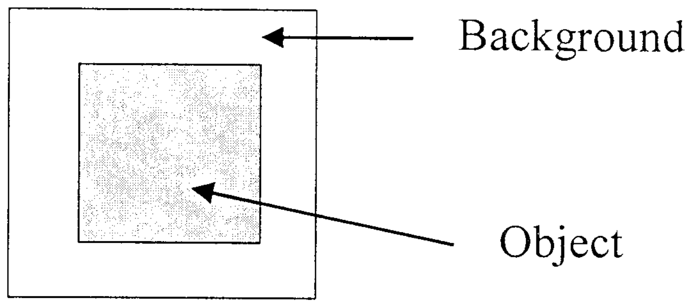
$\oint_{\mathrm{C}} \mathbf{F} \cdot d\ell = \int_{\mathrm{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

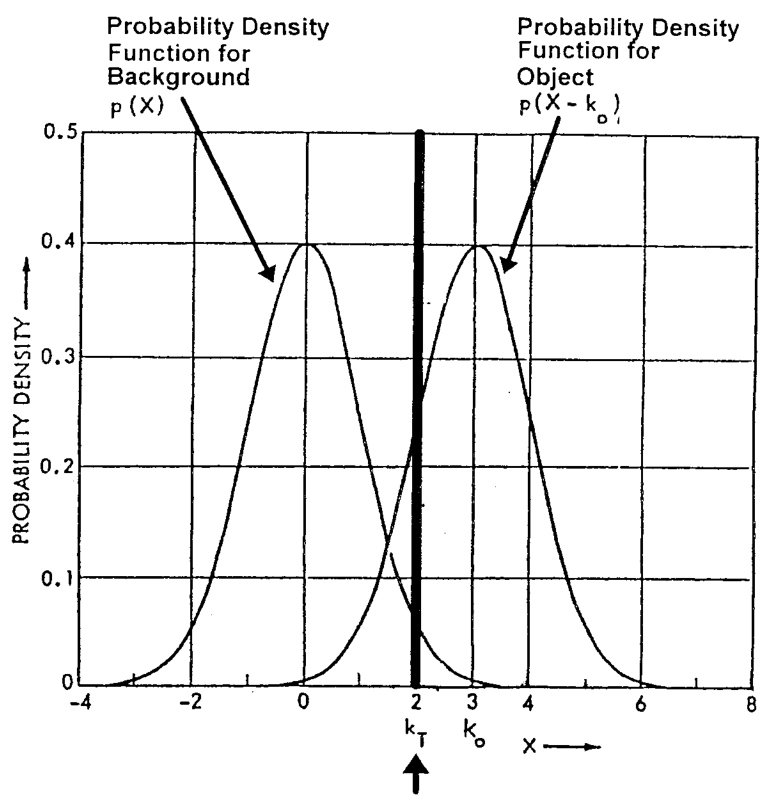
$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_{\mathrm{S}} (\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}} (\nabla \times \mathbf{F})\, d^3x$

A human observer views a scene consisting of a uniform object on a uniform background as shown in the schematic below. The difference in the mean light photon fluence emitted by object and background are large enough to permit detection by the observer, and the viewing angle falls in the range of viewing angles for which the Rose Model of Vision holds.



The Rose-Model of Vision postulates that the probability for detection for quantum noise limited viewing of such a scene follows a "Normal (Gaussian)" law. Notice that the distributions are plotted in units of standard deviations.



Normalized probability densities. Note that kT and ko are threshold and output signal-to-noise ratios respectively

The enclosed sketch displays normalized probability densities for background and for object as well as a decision line at position k_T.

a. What is the importance of the decision line?

b. Define in terms of the decision line k_T and output signal-to-noise ratio k_0 the

Probability for detection ("True Positives")
Probability for false alarm ("False Positives")
Probability for True Negatives
Probability for False Negatives.

c. What is the relation between the quantities True Positives, False Positives, True Negatives and False Negatives?

d. Assuming the observer can select the position of the decision line:
(1) Where would he place it in order to maximize the probability of detection?
(2) Where would he place it in order to minimize the probability of false alarm?

e. What is the probability for detection for the specific values of $k_T = 3$ and $k_0 = 3$ in the above probability densities ?

FALL 2002

OPTI 533

- Let $x(n_1, n_2)$ be a 2-D sequence defined for all (n_1, n_2), but known to be zero outside the rectangle $\{0, 1, \ldots, N_1 -1\} \times \{0, 1, \ldots, N_2-1\}$.
- Let $\tilde{x}(n_1, n_2)$ be the periodic extension of $x(n_1, n_2)$ with period $N_1 \times N_2$.
- Let $\hat{x}(n_1, n_2)$ be the periodic extension of $x(n_1, n_2)$ with period $2N_1 \times 2N_2$.
- Note that then one period of $\hat{x}$ is a zero padded version of one period of $\tilde{x}$.

a) Show that the DFT of $\tilde{x}$ gives $N_1 \times N_2$ evenly spaced samples from the DSFT (discrete-space Fourier transform) of x.

b) Show that the DFT of $\hat{x}$ gives $2N_1 \times 2N_2$ evenly spaced samples from the DSFT of x.

Consider a photon-counting detector illuminated by a steady (not modulated, not noisy) light source. Let the mean rate of detection be denoted by λ.

(a) Give a simple expression for the probability of detecting N photons in a measurement time T.

(b) Now suppose you make 10 measurements in 10 nonoverlapping time intervals, each of duration T. Give an expression for the probability of detecting N_1 photons in the first interval, N_2 in the second interval, etc.

(c) Use the expression derived in part (b) to find an expression for the maximum-likelihood estimate of the rate λ.

(d) Compute the variance of this maximum-likelihood estimate.

You are given an ideal telecentric, "4F", imaging system. It consists of two lenses of focal-length *f* separated by a distance *2f*. The input plane to the system is *f* upstream of the first lens and the output plane is *f* downstream of the second lens. Midway between the lenses is an aperture with complex-amplitude transmittance $a(x, y)$. There is a transparency, with complex amplitude transmittance $t(x, y)$ in the input plane.

Case 1:

The transparency is illuminated by coherent light in the form of a normally-incident plane wave.

- What is the coherent transfer function of the system? Show the conversion to reduced coordinates for the spatial frequencies.
- What is the coherent point spread function of the system?

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The same transparency is illuminated with quasi-monchromatic incoherent light.

- What is the incoherent transfer function of the system? How is it related to the coherent transfer function?
- What is the modulation transfer function (MTF) of the system?
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- A cosine grating, $a(x, y) = \frac{1}{2}\left[1 + \cos(2\pi\xi_0 x)\right]$, replaces the aperture. What is the resulting incoherent point spread function?
- What is the output of the system with the grating in place?

MINOR WRITTEN PRELIM EXAM

FALL 2002

September 24, 2002
8:30 a.m. to 12:00 p.m.

Please answer all questions.

Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your name and the problem number. Staple together all sheets for a given problem.

Insert your answers and this exam into the manila envelope supplied. The exam questions will be returned to you along with your answers after they have been graded.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A - B) + \cos(A + B)$

$2\sin A \sin B = \cos(A - B) - \cos(A + B)$

$2\sin A \cos B = \sin(A + B) + \sin(A - B)$

$2\cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Consider a CW laser beam of frequency ω passing through a transparent Lanthanum Fluoride crystal doped with a small percentage of Praseodymium atoms (Pr^{3+}: LaF_3). The Praseodymium atoms in this crystal have a single isolated transition out of their ground state in the visible region of the spectrum (592.52 nm). We want to examine what happens when the frequency of the laser is slowly tuned through resonance with this transition.

If we had a perfect single crystal of this material, the polarization induced in the Praseodymium atoms by the optical field would be given by

$$P = \frac{\wp^2 E_0}{2\hbar}(N_b^0 - N_a^0)\left[\frac{\omega_0 - \omega}{\mathcal{R}^2 + \gamma^2} + i\,\frac{\gamma}{\mathcal{R}^2 + \gamma^2}\right] e^{-i(\omega t - kz)} \;+\; \text{c.c.}$$

Here $\mathcal{R} = \sqrt{(\omega_0 - \omega)^2 + (\wp E_0/\hbar)^2}$ is the generalized Rabi oscillation frequency, $\wp$ is the transition dipole matrix element, γ is the linewidth (half-width at half-maximum), and N_i^0 is the population of state i when no light is present.

(a) Find an expression for the absorption coefficient α produced by this polarization when the laser frequency is near resonance with the transition. Don't worry if you don't have every constant quite right. (25%)

(b) In actuality, perfect crystals of this material cannot be obtained, and the imperfections cause the transition frequencies ω_0 of the Praseodymium atoms to be Gaussian distributed with a width $\Delta\omega_0$ (half-width to the $1/e$ point). At low temperatures, this distribution of transition frequencies becomes much larger than the intrinsic linewidth γ. Find an expression for the absorption coefficient α in this situation. (35%)

(c) For the imperfect crystal at low temperature described in part (b), sketch the behavior of the absorption coefficient α that would be observed as a function of the laser frequency ω as the laser is tuned through the absorption linewidth. Assume that the irradiance of the laser beam is such that $\wp E_0/\hbar > \gamma$. (20%)

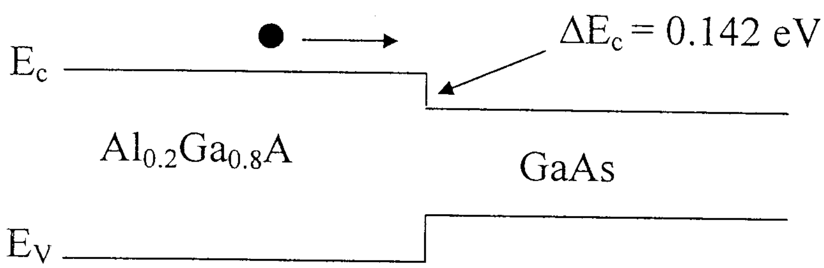
(d) If we again have the situation of part (b), with the irradiance of the laser beam again such that $\wp E_0/\hbar > \gamma$, sketch the behavior of the actual Praseodymium atom population difference $N_b - N_a$ in the crystal as a function of frequency when the laser beam is present with its frequency held fixed at some value near resonance. (20%)

A Keplerian telescope has the following specifications:

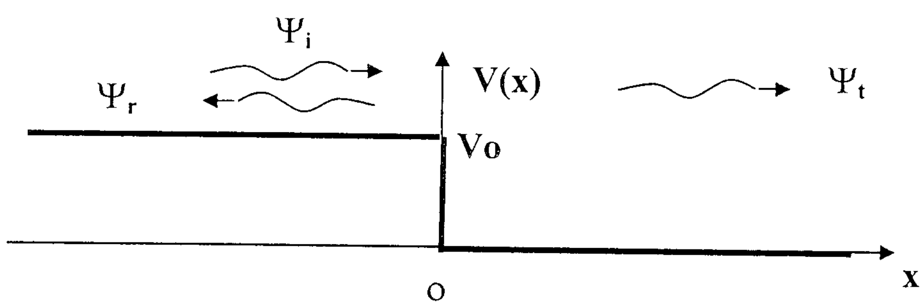
Magnifying Power = 12 X
Length = 260 mm (objective to eye lens)
Unvignetted Field of View = +/- 2 degrees
Eye Relief = 15 mm
Entrance Pupil Diameter = 40 mm
Stop located at the Objective
All elements are thin lenses

Design the telescope. Provide element focal lengths, diameters and spacings. Be sure to verify that all of the above specifications are met.

Using molecular beam epitaxy (MBE) it is possible to form a semiconductor heterojunction, that changes abruptly over one atomic spacing. Consider an electron of energy E and effective mass m* in the AlGaAs region that is incident on the heterojuction of height ΔE_c.



Classically, we would expect the electron to pass over the conduction band discontinuity and continue into the GaAs. However, quantum mechanically the situation can be different. Lets investigate the conduction band using the following model.



Each part is worth (25%)

a) Set up the time independent Schrodinger equation for $x<0$ and $x>0$
b) Solve for the wavefunctions for each region
c) State the boundary conditions at $x=0$
d) Using intuitive physical arguments (like F=ma, etc) explain what happens to the velocity of the electron as it passes from AlGaAs to GaAs. Does the velocity change instantaneously?

A scientific grade CCD array has a quantum efficiency of 0.75 at 700 nm, a dark current of 50 pA/cm^2 at 20°C, and square pixels that are 24 μm on a side. The full-well capacity (maximum that a pixel can store) is 100,000 electrons. The array is placed in the focal plane of a camera (F/2 optics, no losses).

20% At 20°C, how long does it take for dark current to fill a well?

30% You wish to integrate a dim scene with an in-band radiance of 0.01 mW/m^2-sr at 700 nm. Ignoring for a moment the dark current, how long does it take to reach 50% of the full-well capacity?

30% To what temperature must you cool the array such that the generated charge from the scene is 20 times larger than that from the dark current? The functional form of the dark current vs. temperature is

$$i_d = 3 \times 10^{-10} e^{T/10}$$

20% If the CCD array is 1024 x 1024 pixels, the CTE (charge transfer efficiency) is 0.99995 and it takes three transfers to move a well of stored charge the distance of one pixel (i.e., 3ϕ clocks), the resulting image of a uniform scene will be flawed. Describe the appearance of the image. Is there a viable scheme to reduce this flaw?

Question #5 FALL 2002

a). Obtain general expressions for the complex current density j and specific conductivity σ of a metal using the equation of motion of an electron (20%).

b-e). A plane wave with frequency of 1 MHz is incident normally on a good conductor. Let the conductor have specific conductivity $\sigma = 5.8 \times 10^7$ siemens/meter, permittivity $\varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}$ F/m, and magnetic permeability $\mu = \mu_0 = 4\pi \times 10^{-7}$ N/m. . Define, write the expressions, and calculate the following properties that take place inside the conductor in SI units:

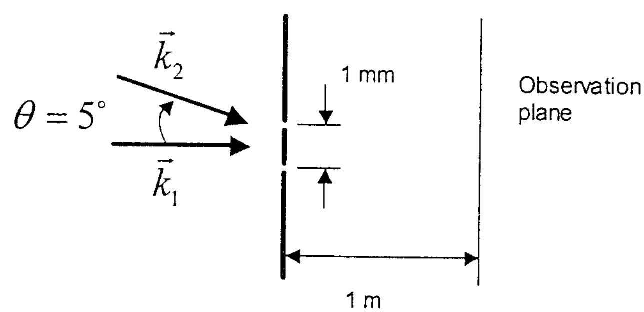
b. Skin depth (20%)

c. Phase velocity (20%)

d. Group velocity (20%)

e. Average Poynting vector (20%)

Two plane waves illuminate a Young's double pinhole interferometer as shown. $\lambda_1 = 532$ nm and $\lambda_2 = 541.4$ nm. The angle between the plane waves is 5 degrees. $\vec{k}_1$ is normally incident onto the aperture plane. The pinhole spacing is 1 mm, and the distance from the aperture plane to the observation plane is 1 m. Ignore the finite size of the pinholes, and assume that the observation region is limited to a small area around the axis. Sketch the observation-plane irradiance pattern , showing any important features. It may not be possible to show both fine and macroscopic features in the same sketch, so show fine features in one sketch and macroscopic features (features > 10 times larger than the smallest feature) in a second sketch.



Question #7 FALL 2002

Consider a photon-counting detector illuminated by a steady (not modulated, not noisy) light source. Let the mean rate of detection be denoted by λ.

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(c) Use the expression derived in part (b) to find an expression for the maximum-likelihood estimate of the rate λ.

(d) Compute the variance of this maximum-likelihood estimate.

Question #8

FALL 2002

Two plane waves with the same wavelength, but different amplitudes, interfere in space.

a.) (40%) If $\vec{k}_1 = k\hat{k}_1$ and $\vec{k}_2 = k\hat{k}_2$, where $\hat{k}_1 = 0.1\hat{x} + 0.866\hat{z}$ and $\hat{k}_2 = \hat{z}$, what is the orientation of fringe planes?

b.) (30%) What is the minimum spacing between fringe planes if $l_1 = l_2 = 632.8\,\text{nm}$?

c.) (30%) Does visibility depend on orientation of the observation screen?

You are given an ideal telecentric, "4F", imaging system. It consists of two lenses of focal-length *f* separated by a distance *2f*. The input plane to the system is *f* upstream of the first lens and the output plane is *f* downstream of the second lens. Midway between the lenses is an aperture with complex-amplitude transmittance $a(x, y)$. There is a transparency, with complex amplitude transmittance $t(x, y)$ in the input plane.

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The transparency is illuminated by coherent light in the form of a normally-incident plane wave.

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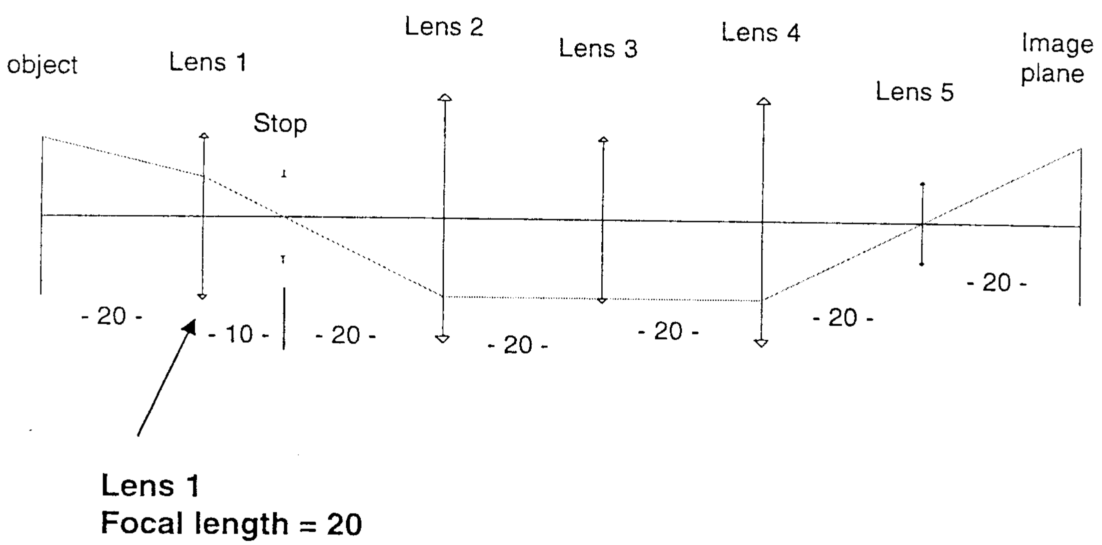
- A cosine grating, $a(x, y) = \frac{1}{2}\left[1 + \cos(2\pi\xi_0 x)\right]$, replaces the aperture. What is the resulting incoherent point spread function?
- What is the output of the system with the grating in place?

Consider the following optical system that uses ideal, paraxial elements to relay an image from the object plane to the image plane.
The **chief ray** is drawn.
The spacings between elements are given, and the drawing is to scale.



Answer the following questions (equal weighting for each)

1. Determine the positions of the entrance pupil and the exit pupil
2. Use the graphical information above to determine the focal lengths for Lenses 2 - 5
3. a. Locate the position of the internal image (a real image formed somewhere other than at the image plane).
   b. Determine the system magnification.
4. Change the power of only one lens to make this system telecentric in image space, yet maintain the image position and magnification.

Question #11

FALL 2002

You are given the task of helping to construct a laser. You have the following items available:

- a 0.75-meter long argon-ion tube, sealed with Brewster windows
- a planar high reflector
- a concave output coupler with a radius of curvature of 2.0 meters

(a) In constructing the laser, you need to make a stable optical cavity with the two available mirrors, separated by a distance L. What range of distances can you choose for L in order to ensure that the optical cavity is stable?

(b) You decide to build the cavity with a mirror separation of 1.0 meter. What is the axial cavity mode frequency spacing in units of Hertz? How does this result depend upon the wavelength of the light in the cavity?

(c) You turn on the pumping mechanism for the argon-ion gain medium, and align the cavity so that laser light is generated, and is transmitted through the concave output coupler. What primary measurable effect will the Brewster windows have on the laser light?

(d) Argon-ion lasers may lase at several different center wavelengths, corresponding to different atomic transitions. If you were to isolate the light from a single atomic line and look at the frequency spectrum of the light, you would find a comb of frequencies within a Gaussian-shaped envelope. The linewidth of this envelope is about 3.5 GHz. What is the most significant broadening mechanism that contributes to this linewidth? Be sure to write down the name of this mechanism, and include a brief description of the physical origin of this mechanism (i.e., what is happening within the laser?).

(e) Is this broadening mechanism an example of homogeneous or inhomogeneous broadening?

(f) Where in the cavity does the optical field reach its minimum beam waist? Ignoring the focusing properties of the output coupler, is the output laser beam converging, diverging, or collimated?

(g) Sketch beam profiles of three non-degenerate Hermite-Gaussian transverse modes. What can you add within the cavity to ensure that the laser operates only in the fundamental transverse mode?

All parts equally weighted.

a. Describe physically and supply a complete mathematical representation of a right-circularly polarized electromagnetic plane wave of circular frequency ω propagating in free space along positive z direction of a right-handed coordinate system. The Poynting vector has a value of 2 [Wm^{-2}]. (State the convention of handedness that you have used.)

b. The wave of item (a) enters a dielectric medium of refractive index n. What is the Poynting vector in this medium?

c. The wave of item (a) propagating in free space encounters a counter-propagating right-circularly polarized plane wave of the same circular frequency and the same irradiance. Describe the superposition of these two waves along the z axis and the state or states of polarization (orientation of the electric field) along the z axis.

All parts equally weighted.

Question 12
Day 2 /Track 1

(

Suppose that a 2-level atom is described by the following time-dependent superposition state when in the presence of a laser field:

$$|\Psi(t)\rangle = c_g(t)|\psi_g\rangle e^{-iE_g t/\hbar} + c_e(t)|\psi_e\rangle e^{-iE_e t/\hbar}.$$

Here, $|\psi_g\rangle$ and $|\psi_e\rangle$ are the atom's ground and excited energy eigenstates (respectively) when the field is not present. For this problem, make use of the terms and parameters given above and in the following list:

Ω_0 (the on-resonance Rabi frequency),
Δ (the detuning of the field from atomic resonance),
Γ (the natural lifetime of the excited state).

If you use any other terms in this problem, be sure to define them!

(a) Evaluate $\langle\psi_g|\psi_e\rangle$. *[1 point]*

(b) Make a plot of $|c_g(t)|^2$ for the range of times $0 \leq t \leq t_1$, where $t_1 = 6\pi/\Omega_0$, $c_g(0) = 0$, and $\Delta = 0$. Label your plot axes, and on each axis, label the range of values associated with this plot (ie, maximum and minimum plotted values for $|c_g(t)|^2$ and t). *[2 points]*

(c) Make a new plot of $|c_g(t)|^2$ for times $0 \leq t \leq t_1$, where again $t_1 = 6\pi/\Omega_0$ and $c_g(0) = 0$, but now $\Delta = \sqrt{3}\Omega_0$. What is the generalized Rabi frequency for this case? Again, label the range of values for the plotted points. *[2 points]*

(d) If $c_g(t_2) = -1/2$ for some time t_2, what is the probability of finding the atom in the *excited* state at time $t = t_2$? Provide a number for your answer. *[1 point]*

(e) For the case with $c_g(t_2) = -1/2$, write an expression for $\langle E\rangle$, the energy expectation value for the state $|\Psi(t_2)\rangle$ at time $t = t_2$. Write your answer in terms of E_g and E_e only. *[1 point]*

(f) If the laser turns off at time t_3, and $c_e(t_3) = 1$, write an expression for $|c_e(t > t_3)|^2$ that accounts for spontaneous emission. (Note that this is c_e, *not* c_g.) Check that your answer makes sense in the limits $t = t_3$ and $t = \infty$. *[1 point]*

(g) If a laser beam propagates through a gas of these two-level atoms, what is the maximum possible steady-state value of the ratio N_e/N_{tot}, where N_e is the steady-state population density of the excited state, and N_{tot} is the total population density? *[1 point]*

(h) Can a gas of these two-level atoms provide steady-state gain and amplify a laser beam propagating through it? *[1 point]*