Track I

WRITTEN PRELIM EXAM – FIRST DAY

Fall 2006

September 19, 2006
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

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$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

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$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

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$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

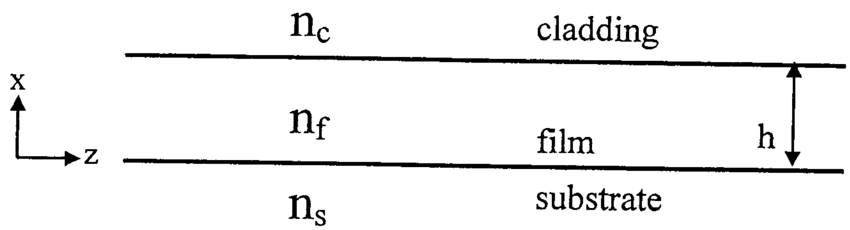
$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Fall 2006 Comprehensive Exam, Track I Question 1



$n_c < n_s < n_f$

Consider an asymmetric 3-layer slab waveguide shown above (y-invariant). Starting from the transverse portions of the electric field amplitudes in the three regions (see below), **derive the *characteristic equation* for TE (Transverse Electric) modes** (70%). **When applying the boundary conditions include very short justifications** (30%). Note: it is not necessary to simplify the equation to its final form.

$E_y(x) = Ae^{-\gamma_c(x-h)}$	$h \leq x$	(cladding)
$E_y(x) = B\cos\kappa_f x + C\sin\kappa_f x$	$0 < x < h$	(film)
$E_y(x) = De^{\gamma_s x}$	$x \leq 0$	(substrate)

Here κ_f is the transverse wavevector, and γ_s and γ_c are the decay (attenuation) coefficients in substrate and cladding, respectively. *A*, *B*, *C* and *D* are amplitude coefficients.

Hint: Equations for the different vector components may help:

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -i\omega\mu_0 H_z \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\varepsilon E_z$$

$$\frac{\partial E_z}{\partial y} + i\beta E_y = -i\omega\mu_0 H_x \qquad \frac{\partial H_z}{\partial y} + i\beta H_y = i\omega\varepsilon E_x$$

$$\frac{\partial H_z}{\partial x} + i\beta H_x = -i\omega\varepsilon E_y \qquad \frac{\partial E_z}{\partial x} + i\beta E_x = i\omega\mu_0 H_y$$

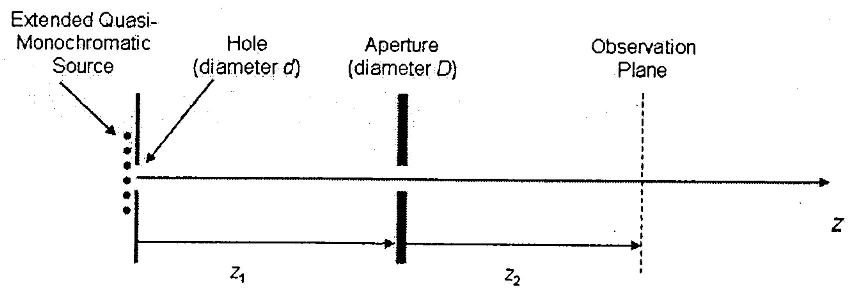
Fall 2006 Comprehensive Exam, Track I Question 2

A ball lens is used as a 100X objective in a microscope with an optical tube length of 200 mm. The optical tube length is the distance from the rear focal point of the objective to the intermediate image presented to the eyepiece of the microscope. The ball lens is a sphere of radius R with an index n = 1.500.

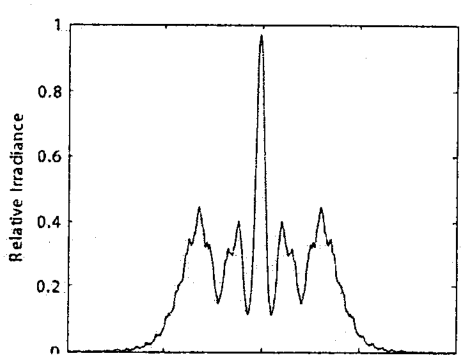
Determine the required ball radius R and the object-side working distance provided by this objective.

Fall 2006 Comprehensive Exam, Track I Question 3

A.) (5 pts.) A hole with diameter $d = 0.5\text{x}10^{-3}$ m is placed over an extended quasimonochromatic source as shown below. Estimate the largest diameter over which good spatial coherence is observed at the plane of the aperture at a distance of $z_1 = 0.5$ m from the source. The source has an average wavelength of $\lambda = 1\text{x}10^{-6}$ m. State any assumptions that you make.



B.) (5 pts.) The irradiance pattern profile shown below is observed at some distance z_2 from a $D = 0.5\text{x}10^{-3}$ m aperture. Calculate z_2. State any assumptions that you make.



Fall 2006 Comprehensive Exam, Track I Question 4

(a. 2 points) What is the origin of electronic conduction in inorganic semiconductors? What is the origin of electronic conduction in organic compounds and how does it differ from that in semiconductors?

(b. 2 points) To describe light emission and absorption in organic molecules it is customary to use the configuration coordinate diagram. Draw schematically such a diagram and include absorption and emission processes on the diagram. Also use this diagram and draw schematically absorption and emission lines as a function of photon energy.

(c. 2 points) What is the Bloch wavefunction in an inorganic solid? Does the concept of Bloch wavefunction work for amorphous organic solids and why?

(d. 2 points) What are the differences between Wannier and Frenkel excitions and in what types of solids do they apply?

(e. 2 points) Is it possible to make glass active and use it for amplifiers and lasers? Describe why and how.

Fall 2006 Comprehensive Exam, Track I
Question 5

A ground-based sensor is being used to characterize the radiance output of the moon. The sensor has two operating modes, lunar and stellar. Lunar mode consists of the system operating with a silicon detector that is square with an area of 0.0625 mm^2 at the bottom of a tube that is 0.707 m in length and has a diameter of 0.354 m. Stellar mode consists of placing a lens with a focal length of 0.707 m on the front of the tube. A 10-nm wide spectral filter centered at 600 nm is placed directly in front of the detector.

The system operates with a 10-bit analog-to-digital converter and is used to view a star that is 1000light years away as well as the full moon and empty space. The outputs from the sensor in the various modes while viewing the moon, star, and deep space are:

Sensor output for stellar and lunar modes while viewing star, moon, and deep space			
Mode	Star	Moon	Deep space
Stellar	219 counts	1023 counts	60 counts
Lunar	60 counts	405 counts	60 counts

7 points
A) Compute an average spectral radiance from the lunar disk based on the above data. Other information that may be of use is:

- Irradiance from the star is measured independently to be 1.62×10^{-11} W/m^2
- Radius of the star is 7.00×10^{8} m
- Earth-moon distance is 384,400 km
- Lunar radius is 1,738 km

3 points
B) Describe a method that you could use to determine an average reflectance for the lunar surface. Can your method be used with the current sensor system? Explain quantitatively.

Trade I

WRITTEN PRELIM EXAM – SECOND DAY

Fall 2006

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Fall 2006 Comprehensive Exam, Track I Question 7

Consider an optical imaging system with 10 cm aperture operating at 0.5 μm wavelength. With the stop wide open at F/2, the system suffers from primary (3rd order) spherical aberration. The magnitude of the aberration is determined by adjusting to paraxial focus where the image of an on-axis point is 200 μm in diameter and has the characteristic appearance from spherical aberration.

At paraxial focus for this system, the marginal rays have not yet come to focus for this system. (Hint: Use this information to determine the sign of the aberration.)

Draw graphs and sketches as indicated below. Label your axes and show scale. The plots should have approximately the correct shape.

1. At paraxial focus, for an on-axis field point, draw the following (50%)
   a) Draw the ray fan diagram.
   b) Sketch the image from a point object and show a cross section of the irradiance distribution.
   c) Draw the wave fan (or OPD) diagram.

2. Longitudinal spherical aberration (20%)
   Draw a plot of the longitudinal spherical aberration.
   The longitudinal aberration is defined as zonal focus shift as function of pupil coordinate.

3. Stop the system down to F/10 (30%)
   a) Draw the new OPD fan diagram.
   b) Draw the new ray fan diagram.
   c) Sketch the new image from a point object and show a cross section of the irradiance distribution. (Make sure that your answer makes physical sense.)

Fall 2006 Comprehensive Exam, Track I Question 7

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   a) Draw the new OPD fan diagram.
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   c) Sketch the new image from a point object and show a cross section of the irradiance distribution. (Make sure that your answer makes physical sense.)

Fall 2006 Comprehensive Exam
Track I
Question 8

The impulse-response of a Linear Shift-Invariant (LSI) system is $h(x) = 3\,\text{sinc}(x)$. Let the input to this system be $f(x) = 2\sin\left(\frac{2}{3}\pi x\right) + 4\cos(2\pi x)$. Determine the output of the system.

Note: $\text{sinc}(x) = \sin(\pi x)/(\pi x)$.

The wave equation for propagation of an electric field $\vec{E}(\vec{r}, t)$ in a metal occupying the half-space $z > 0$ is

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial \vec{J}}{\partial t},$$

which is coupled to the Drude model for the current density

$$\tau \frac{\partial \vec{J}}{\partial t} + \vec{J} = \sigma_s \vec{E},$$

$\sigma_s > 0$ being the static (zero frequency) conductivity, and τ the average time between electron collisions.

(a) By considering monochromatic plane-wave fields in the metal for $z > 0$

$$\begin{aligned} \vec{E}(\vec{r}, t) &= \frac{\hat{\mathbf{e}}_x}{2}\left[E_t(\omega)e^{i(K(\omega)z - \omega t)} + \text{c.c.}\right], \\ \vec{J}(\vec{r}, t) &= \frac{\hat{\mathbf{e}}_x}{2}\left[J_t(\omega)e^{i(K(\omega)z - \omega t)} + \text{c.c.}\right], \end{aligned}$$

where c.c means complex conjugate, show that the complex propagation wavenumber $K(\omega)$ obeys $K^2(\omega) = \omega^2/c^2 + i\omega\mu_0\sigma(\omega)$, with the frequency-dependent conductivity given by $\sigma(\omega) = \sigma_s/(1 - i\omega\tau)$. (4)

(b) Next consider that a field is incident from vacuum onto the metal from the half-space $z < 0$ and that some is also reflected

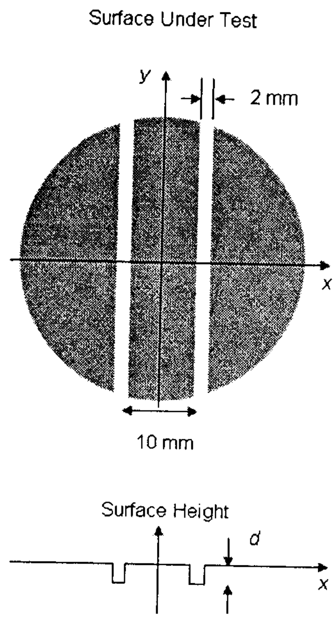
$$\vec{E}(\vec{r}, t) = \frac{\hat{\mathbf{e}}_x}{2}\left[(E_i(\omega)e^{i\omega z/c} + E_r(\omega)e^{-i\omega z/c})e^{-i\omega t} + \text{c.c.}\right], \quad z < 0.$$

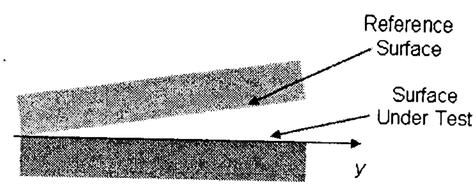
By demanding that the field and its first z-derivative be continuous at the interface at $z = 0$ derive an expression for the field reflectivity coefficient $r(\omega) = E_r(\omega)/E_i(\omega)$. (4)

(c) Using the results from parts (a) and (b), find an approximate value for the field reflectivity coefficient $r(\omega)$ in the high-frequency limit $\omega \longrightarrow \infty$, and comment on how this relates to your expectations for the metal reflectivity for frequencies much larger than the plasma frequency. (2)

Fall 2006 Comprehensive Exam, Track I Question 10

A surface is tested in the Fizeau geometry shown below with $\lambda = 546$ nm. Two shallow trenches are oriented in the test surface, where $d = 136.5$ nm. Draw an accurate representation of the fringes observed if the maximum wedge between the perfectly flat reference surface and the surface under test is 2.5λ. Notice that the relative orientation of the wedge is in the y direction. State any assumptions that you make.



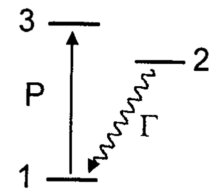


Fall 2006 Comprehensive Exam
Track I
Question 11

Sketch a diagram of a long distance, point-to-point DWDM fiber optic system. Assume that the system uses Raman amplification in addition to EDFAs. Briefly explain the functionality of the key building blocks of the system.

Fall 2006 Comprehensive
Exam, Track I
Question 12

Consider a single mode, single frequency, homogeneously broadened CW laser that has a standard 3-level atomic gain medium, as shown in the figure to the right. The population densities $\mathbf{N_1}$, $\mathbf{N_2}$, and $\mathbf{N_3}$ correspond to atomic levels of increasing energy, respectively. A pump source promotes level 1 atoms to level 3 at a rate **P**, upon which the excited atoms **instantaneously** decay to level 2. Spontaneous emission of light from level 2 to level 1 occurs at a rate Γ. Absorption and stimulated emission of light also occur in the usual way, with an intensity-dependent rate **R(I)** (not indicated in figure). For the following questions, assume that the laser operates at the atomic resonance.



(a) *(1 pt)* Write the usual rate equation for $\mathbf{N_1}$, using the parameters given. Include terms corresponding to absorption and stimulated emission of light between levels 1 and 2.

(b) *(2 pts)* Let the total population density be given by $\mathbf{N_T = N_1 + N_2}$, which is approximately correct. Assume that there is a closed shutter in the cavity so that laser light can not build up (thus **R(I) = 0**), but the pump source is still on and spontaneous emission still occurs. For these conditions, use your rate equation to solve for the small-signal steady-state population density difference $\mathbf{N_2\text{-}N_1}$, writing your answer in term of $\mathbf{N_T}$ and the ratio $\mathbf{P/\Gamma}$ **only**. <u>No other parameters should appear in your answer.</u>

<u>GRAPHING.</u> For each of the following questions, make a separate quantitatively accurate graph of the indicated laser parameter versus $\mathbf{P/\Gamma}$ over the range $\mathbf{P/\Gamma = 0}$ **to 5**. On the horizontal axis of each graph, make tick marks at the integer values of $\mathbf{P/\Gamma}$, and label these values.

(c) *(2 pts)* Still assuming that the shutter in the cavity is closed, graph $\mathbf{N_2\text{-}N_1}$ versus $\mathbf{P/\Gamma}$, corresponding to your answer to part (b). On the vertical axis of your graph, make labeled tick marks for the values $\mathbf{-N_T}$ and $\mathbf{+N_T}$.

(d) *(2 pts)* Graph the small-signal gain coefficient γ_0 versus $\mathbf{P/\Gamma}$. Your answer should be consistent with your answer to part (c), and is independent of whether the cavity shutter is open or closed. Assume that $\gamma_0 = \gamma_T$ at the value $\mathbf{P/\Gamma = 3}$, where γ_T is the threshold gain coefficient. On the vertical axis of your graph, make a labeled tick mark for the value γ_T.

(e) *(2 pts)* For this and the last question, assume that the cavity shutter is open so that laser light can build in the cavity, and assume steady-state operation of the CW laser. Graph the **steady-state** gain coefficient γ versus $\mathbf{P/\Gamma}$. On the vertical axis of your graph, make a labeled tick mark for the value γ_T.

(f) *(1 pt)* Graph the steady-state **laser output power** versus $\mathbf{P/\Gamma}$. Assume that the laser output power is **100 mW** when $\mathbf{P/\Gamma = 5}$.