WRITTEN PRELIM EXAM – FIRST DAY

Spring 2007 2006

February 13, 2007

8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2 \cos^2 A - 1$

$\cos 2A = 1 - 2 \sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Spring 2007 Written Comprehensive Exam
Applied Track
Day 1 Question #1

Design an object-space telecentric imaging system consisting of a thin lens and a stop. The system has an object-to-image distance of 450 mm, and it images a 20 mm diameter object onto a 10 mm square detector. The diameter of the image fills the width of the detector. The system operates at a working f-number of 4 ($f/\#_W = 4$ or NA = 0.125) and is unvignetted for this object.

Provide the focal length and diameter of the lens, the stop diameter, and the required spacings.

Spring 2007 Written Comprehensive Exam
Applied Track
Day 1 Question #2

Consider the quantum state of an electromagnetic field occupying a single mode of an optical cavity. The mode's resonance frequency is ω. For the following questions, you are given that the electric field operator for the light in this mode takes the form

$$\hat{\mathcal{E}} = \mathcal{E}_p(\hat{a} + \hat{a}^\dagger),$$

where $\mathcal{E}_p$ can be treated as a constant for this problem. The operators $\hat{a}$ and $\hat{a}^\dagger$ act on a photon number state $|n\rangle$ in the following way:

$$\begin{aligned} \hat{a}|n\rangle &= \sqrt{n}\cdot|n-1\rangle, \\ \hat{a}^\dagger|n\rangle &= \sqrt{n+1}\cdot|n+1\rangle. \end{aligned}$$

(a) [*1 pt.*] Are the photon number states eigenstates of $\hat{\mathcal{E}}$?

(b) [*1 pt.*] Evaluate $\langle n|\hat{a}\hat{a}^\dagger|n\rangle$.

(c) [*1 pt.*] Evaluate $\langle n|\hat{a}^\dagger\hat{a}|n\rangle$.

(d) [*1 pt.*] Write the electromagnetic field Hamiltonian $\hat{H}$ in terms of $\hat{a}$ and $\hat{a}^\dagger$, or in terms of a photon number operator $\hat{N}$. Write $\hat{H}$ in a form that explicitly includes zero point energy.

(e) [*1 pt.*] Evaluate $\langle\hat{\mathcal{E}}\rangle$ for the state $|n\rangle$. (Remember that $\langle\hat{\mathcal{E}}\rangle = \langle n|\hat{\mathcal{E}}|n\rangle$.)

(f) [*2 pts.*] Evaluate $\langle\hat{\mathcal{E}}^2\rangle$ for the state $|n\rangle$.

(g) [*2 pts.*] The variance $\sigma_\mathcal{E}^2$ of $\hat{\mathcal{E}}$ is the square of the standard deviation $\sigma_\mathcal{E}$, and is used to characterize the fluctuations or uncertainty in the electric field amplitude. Determine the variance of the electric field amplitude for the photon number state $|n\rangle$.

(h) [*1 pt.*] Evaluate $\sigma_\mathcal{E}^2$ for the state $|0\rangle$. What is the significance of the (correct) answer to this calculation?

10 pts. total.

Spring 2007 Written Comprehensive Exam
Applied Track
Day 1 Question #3

A periodic function $f(x)$ with period "Δx" is represented by the Fourier series $\sum_{n=-\infty}^{\infty} F_n \exp\left(2\pi i \frac{nx}{\Delta x}\right)$.

a. What is the 1-D Fourier transform of $f(x)$?

b. What is the 1-D Fourier transform of $\mathrm{rect}(x) f(x)$?

c. What is the 2-D Fourier transform of $f(x)$?

Spring 2007 Written Comprehensive Exam
Applied Track
Day 1 Question #4

Consider a polarizable particle with dipole moment **p**(**R**,t) and center of mass position **R** in interaction with an electromagnetic field, and described by the Lorentz equation

$$\left(\frac{d^2}{dt^2}+\omega_0^2\right)\mathrm{p}(\mathrm{R},\mathrm{t})=\frac{e}{m}\mathrm{E}(\mathrm{R},\mathrm{t})$$

with e the electron charge, ω_0 the natural frequency of the dipole of mass m, and **E**(**R**,t) the electric field evaluated at the center of mass position **R**=(X,Y,Z). (note: boldface is used to denote a vector quantity.)

a) By considering a time harmonic field of frequency $\omega\neq\omega_0$, $\mathbf{E}(\mathbf{R},t) = [\mathbf{E}(\mathbf{R},\omega)\exp(-i\omega t)+ \mathbf{E}^*(\mathbf{R},\omega)\exp(i\omega t)]/2$, obtain the corresponding time harmonic variation of the dipole moment $\mathbf{p}(\mathbf{R},t) = [\mathbf{p}(\mathbf{R},\omega)\exp(-i\omega t)+ \mathbf{p}^*(\mathbf{R},\omega)\exp(i\omega t)]/2$. (3)

b) Find an expression for the optical potential, or time-averaged dipolar interaction energy, $U(\mathbf{R})=\langle -\mathbf{p}(\mathbf{R},t)\cdot\mathbf{E}(\mathbf{R},t) \rangle$ expressed in terms of $\mathbf{E}(\mathbf{R},\omega)$, where $\langle\ldots\rangle$ signifies a time average over an optical period $2\pi/\omega$. (hint: the time average allows one to neglect fast oscillating terms). (3)

c) Consider an electric field composed of two counter-propagating fields of wavevectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and identical polarization. Show that this yields an optical potential U(**R**) that is modulated in space, and determine the direction of the modulation. (2)

d) Next consider the case of a Gaussian beam of focused spot size W at Z=0, $\mathbf{E}(X,Y,Z=0,\omega)=\mathbf{E}_0\exp[-(X^2+Y^2)/W^2]$. Obtain the condition on the field frequency ω relative to the natural frequency ω_0 of the dipole for the Gaussian beam to provide a confining optical potential for the dipole in the (X,Y) plane. (2)

Spring 2007 Written Comprehensive Exam
Applied Track
Day 1 Question #5

Problem

A1) Sketch the attenuation in dB/km for a **typical** silica optical fiber (used in modern Telecommunications) as a function of wavelength within the wavelength range of 800 nm - 1800 nm. The sketch should show the distinct features of the attenuation spectrum and give an approximate value for the minimum attenuation.

(4 points)

A2) Very briefly explain how it, in principle, could be possible to further reduce the minimum attenuation if:

(1 point)

i) you are limited to using a silica based fiber.
ii) you can use fibers made using other materials.

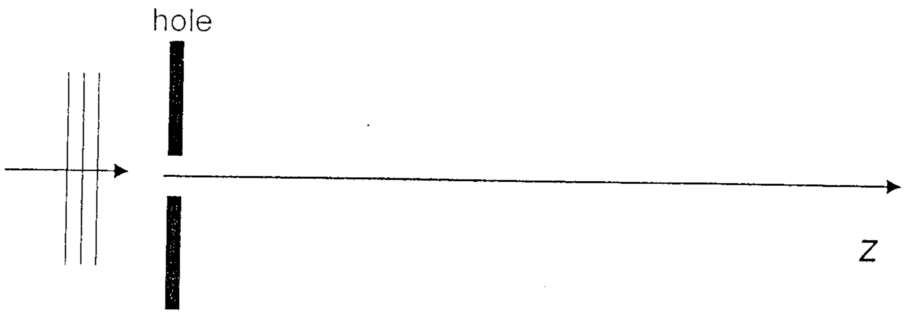
B1) Describe a double-heterostructure for laser diodes (e.g. draw a simplified "flat band configuration" under strong forward bias).

(2 points)

B2) mention the three key advantages that a double heterostructure offers in laser diodes compared to a simple homojunction.

(3 points)

A round hole of diameter 2 mm is illuminated with an on-axis $\lambda = 0.667\ \mu m$ plane wave as shown below.



a.) (10 pts.) Sketch the on-axis irradiance from $z = 0.15$ m to $z = 0.3$ m, indicating the positions of odd and even Fresnel numbers.

b.) (5 pts.) Sketch the transverse irradiance profiles at each integer Fresnel number in part (a). Indicate the geometrical boundary in your sketch.

WRITTEN PRELIM EXAM – SECOND DAY

Spring 2007

February 14, 2007
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$\oint_{\mathrm{C}} \mathbf{F} \cdot d\ell = \int_{\mathrm{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_{\mathrm{S}} (\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}} (\nabla \times \mathbf{F})\, d^3x$

Spring 2007 Written Comprehensive Exam
Applied Track
Day 2, Question #7

Consider an imaging system with 100 mm focal length and 25 mm pupil, focused at infinity with ±2° field of view. Due to an offense against the sine condition, this system suffers from coma, but no other aberrations. The amount of coma in the wavefront is given as

$$\Delta W = W_{131} H \rho^3 \cos\phi$$

where

W_{131} = 20 μm (Seidel coefficient)
H = normalized field coordinate
ρ, ϕ = normalized pupil coordinates

H is normalized to equal 0 on axis and equal 1 at the edge of the field at 2°.
ρ is normalized to equal 0 on axis and equal 1 at the edge of the pupil at 12.5 mm

1.) For the case of a point source 1° off axis:

a) (15%) Sketch a plot of the OPD (wave fan)
(All plots should be quantitative. Label the axes and give units. Axis crossings and extreme points should be correct.)

b) (15%) Sketch a plot of the transverse ray aberration (ray fan)

c) (20%) Sketch the image.
(Include the shape and size of the image as well as the orientation and position with respect to the center of the field)

2.) Repeat the sketches from 1) assuming the sensor is 1 mm *inside* of focus.

a) (15%) Wave fan

b) (15%) Ray fan

c) (20%) Sketch the image.

Spring 2007 Written Comprehensive Exam
Applied Track
Day 2, Question #8

Statistics and stochastic processes

Part I:

Consider a radio antenna that measures a function of a known frequency (let's say 1) but with random amplitude a and random phase ϕ. The measurement $f(t; a, \phi)$ can be written as,

$$f(t; a, \phi) = a \cos(t + \phi).$$

Let us further assume that a and ϕ are independent random variables. The amplitude a has a mean of μ_a and a variance of σ^2, and the phase ϕ is uniformly distributed between $-\pi$ and π.

1. (20%) What is the mean of this stochastic process?
2. (30%) What is the autocorrelation of this process? Is $f(t; a, \phi)$ wide-sense stationary? (Useful trig identity: $\cos(A)\cos(B) = \frac{1}{2}(\cos(A - B) + \cos(A + B)))$

Part II:

Consider a general stochastic process $g(t)$ which is known to be wide-sense stationary with mean m and autocorrelation $R_g(\Delta t)$. Now consider the stochastic process $G(\xi)$, the Fourier transform of $g(t)$.

3. (20%) What is the mean of the stochastic process $G(\xi)$?
4. (30%) What is the autocorrelation of $G(\xi)$? Is $G(\xi)$ wide-sense stationary?

Spring 2007 WRitten Comprehensive Exam
Applied Track
Day 2, Question #9

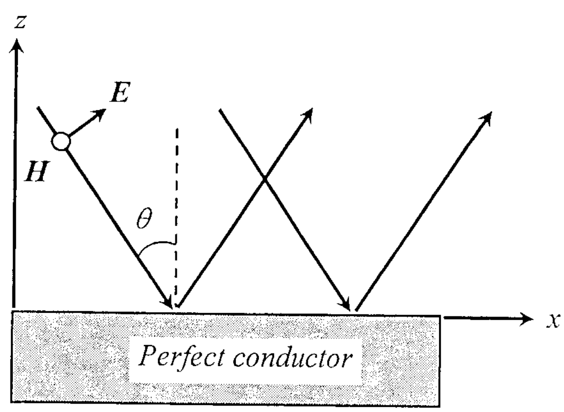
Consider an optical absorption process in a semiconductor. For both direct-gap and indirect-gap semiconductors sketch the electronic bandstructure and indicate the transition process (from valence to conduction band). Clearly distinguish energy and momentum transfer due to photons and phonons, respectively, and do not neglect the photon momentum. In addition to the sketch, write down the equations for energy and momentum conservation in the two cases (i.e. direct and indirect gap), clearly distinguishing scalar from vectorial quantities.

For the case of the indirect-gap semiconductor, and assuming the conduction-band minimum to be at the Brillouin zone boundary, give a quantitative estimate for the momentum transfer due to phonons vs that due to photons (for this, assume the photon wavelength to be in the visible part of the spectrum, use a reasonable value for a typical lattice constant, use $\hbar \approx 10^{-34}$ Js and express your results in units of kg m/s). Can you neglect one with respect to the other?

(10 points)

Spring 2007 Written Comprehensive Exam
Applied Track
Day 2, Question #10

A p-polarized, monochromatic, plane electromagnetic wave is incident from free space onto the flat surface of a perfect electrical conductor. The incident beam has frequency $f_o = \omega_o/2\pi$, electric-field amplitude E_o, magnetic field amplitude H_o, and angle of incidence θ.



a) Write expressions for the incident beam's E- and H-fields that describe their space and time dependence in terms of f_o, θ, E_o, the speed of light in vacuum c, and the impedance Z_o of the free-space. (Note: $c = 1/\sqrt{\mu_o \varepsilon_o}$, $Z_o = \sqrt{\mu_o/\varepsilon_o}$, and $H_o = E_o/Z_o$.) (3)

b) Write expressions similar to those in (a) for the E- and H-fields of the reflected beam. (3)

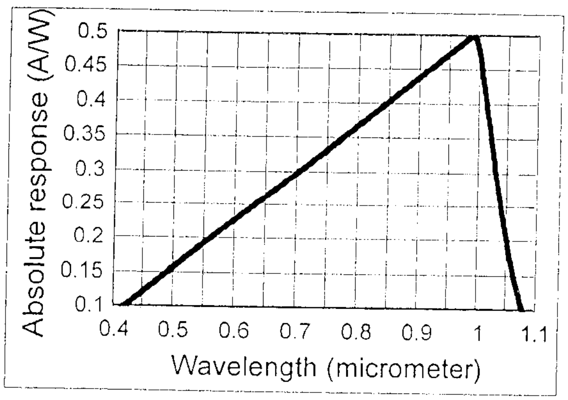
c) Find the distribution of surface current density $J_s(x, t)$ at the front facet of the mirror. (2)

d) Find the distribution of surface charge density $\sigma_s(x, t)$ at the front facet of the mirror. (2)

Note: Surface current density is given by the H-field discontinuity at the surface; surface charge density is related to the E-field discontinuity at the surface.

Spring 2007 Written Comprehensive Exam
Applied Track
Day 2, Question #11

Assume the sun has a radius of 6.96×10^8 m, is at a distance of 1.496×10^{11} m, and at a temperature of 5600 K. A simple tube radiometer is built to operate over the 0.80 to 0.90 μm spectral interval with a field of view that is much larger than the solid angle subtended by the sun. The detector has an area of 10 mm^2. Furthermore, the output from the sun at 0.85 μm can be assumed to have a value of 4.4624×10^7 W/(m^2 μm) and the detector response is as shown in the accompanying figure.



a) Determine the output from the radiometer.

b) What happens to the radiant flux through the detector if the aperture area is doubled?

c) What is the noise equivalent power (NEP) if the SNR is 150?

d) What is the output if a lens of area 10 cm^2 and appropriate focal length is added to the radiometer?

e) The tube radiometer now views a lamp in the laboratory. What temperature is needed for the lamp to give a radiant flux equal to that computed when viewing the sun. Assume the lamp is spherical with a radius of 0.4652 cm and is viewed at a distance of 100 cm. Also assume that the output of the lamp as a function of temperature at a wavelength of 0.85 μm can be approximated by

$$M_\lambda(T) = (2.48 \times 10^{-8}) T^{4.07} \text{ W/(m}^2\ \mu\text{m)}.$$

Spring 2007 Written Comprehensive Exam
Applied Track
Day 2, Question #12

PART I. Suppose that a laser is to be constructed from a homogeneously-broadened gain medium. The optical resonator will be 0.3 m long, and use two mirrors: a flat mirror, and a concave mirror with radius-of-curvature $\mathcal{R}$. The resonator axis will lie along the $\hat{z}$ direction.

(a) [*1 pt.*] Specify a possible numerical value for $\mathcal{R}$ that will permit the construction of a cavity that is well within the region of cavity stability.

(b) [*1 pt.*] What is the free spectral range of this cavity?

(c) [*1 pt.*] Assume that the laser is now assembled. It emits a beam of wavelength λ, and has a transverse profile that is Gaussian. The beam propagates in the $\hat{z}$ direction, and a lens brings the beam to a focus at position $z = 0$. At the focus, the radius of the beam (ie, the beam waist) is w_0. Write an expression for the Rayleigh range z_0 of the beam.

(d) [*1 pt.*] For the conditions of part (c), write an expression for the beam radius $w(z)$, valid for $z > 0$. (If you do not know the answer, you may instead make a qualitatively accurate plot of $w(z)$ for $z > 0$ for half-credit).

(e) [*2 pts.*] For the conditions of part (c), write an expression for the beam's wavefront radius-of-curvature $R(z)$, valid for $z > 0$. **ALSO**, qualitatively plot $R(z)$ for $z > 0$.

PART II. Now suppose that the laser beam described above is collimated by a lens, rather than coming to a focus. At position $z = 0$, the beam enters a very long glass cell containing an atomic gas. The gas has a resonance near the laser beam's frequency, and is thus able to absorb light from the beam. The absorption coefficient for the light in the gas is α, and the input intensity I_0 of the light is much less than the saturation intensity of the atomic transition.

(f) [*2 pts.*] Give an expression for $I(z)$, the laser beam intensity within the cell, in terms of α.

(g) [*2 pts.*] Suppose that an external energy source can pump enough energy into the gas such that the gas can now *amplify* the laser light. Plot $I(z)$, the intensity of the beam within this gain medium, and ensure that your plot qualitatively shows all important trends in $I(z)$.

10 pts. total.