MINOR WRITTEN PRELIM EXAM

Fall 2009

September 23, 2009
8:30 a.m. to 12:00 p.m.

Please answer all questions.

Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your name and the problem number. Staple together all sheets for a given problem.

Insert your answers and this exam into the manila envelope supplied. The exam questions will be returned to you along with your answers after they have been graded.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A - B) + \cos(A + B)$

$2\sin A \sin B = \cos(A - B) - \cos(A + B)$

$2\sin A \cos B = \sin(A + B) + \sin(A - B)$

$2\cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

System of units: MKSA

A linearly-polarized, monochromatic plane-wave propagates along the x-axis, its E-field amplitude being $\boldsymbol{E}(x,t)=E_0\cos\{\omega[t-n(\omega)x/c]\}\hat{y}$. The host medium is a homogeneous, isotropic, non-magnetic (i.e., $\mu=\mu_0$), transparent dielectric, whose frequency-dependent refractive index is specified as $n(\omega)=\sqrt{\varepsilon(\omega)}$.

(3 pts) a) Find the magnetic field $\boldsymbol{H}(x,t)$ of the plane-wave in terms of E_0, c, ω, $n(\omega)$, and the impedance of the free space $Z_0=\sqrt{\mu_0/\varepsilon_0}$.

(3 pts) b) Find the Poynting vector $\boldsymbol{S}(x,t)$ of the above plane-wave, then determine the time-averaged rate-of-flow of optical energy (per unit area per unit time) along the x-axis.

(4 pts) c) Assume a second plane-wave, *identical* with the one above *except* for its frequency ω' differing slightly from ω, is co-propagating with the above plane-wave. Write an expression for the combined E-field of the superposed plane-waves. From this expression, identify the carrier and the envelope of the beat waveform. In terms of c, $\omega_c=\frac{1}{2}(\omega+\omega')$, $\Delta\omega=\omega'-\omega$, $n(\omega_c)$ and $\mathrm{d}n(\omega)/\mathrm{d}\omega$, what is the *phase* and *group* velocity of the combined waveform?

Consider interferometric tests to measure temporal coherence characteristics of a fiber laser, which is spatially coherent, exhibits $\bar{\lambda} = 1050\text{nm}$ and has a power spectrum width of approximately $\Delta\nu = 10 \times 10^9 \text{Hz} = 10\text{GHz}$. In the designs below, include travel and/or observation ranges required and sensitivity of various components. State any assumptions that you make.

a) (4 pts) Design a test of temporal coherence using a Young's double pinhole interferometer (YDPI) or Young's double slit interferometer (YDSI). The object of the test is to verify the width of the power spectrum.

b.) (4 pts) Design a test of temporal coherence using a Twyman-Green interferometer. The object of the test is to verify the width of the power spectrum.

c.) (2 pts) Without changing your designs, if the laser exhibits multiple-longitudinal-mode characteristics with each mode having a width of $\Delta\nu$, qualitatively explain how the experimental results would change.

Koehler illumination is the most common type of illumination system used in optical microscopes.

a. (4 points) Show a diagram of a Koehler illumination system for brightfield microscopy. Be sure to show the source (lamp), lenses, apertures, and object plane as well as critical distances that separate them.

b. (2 points) Explain the aperture size adjustments in the Koehler illuminator (i.e. what apertures and for what purpose are aperture sizes adjusted).

c. (2 points) Explain how misadjustment of the Koehler illuminator condenser aperture size can affect the spatial resolution of the imaging system.

d. (2 points) Describe how the Koehler illumination system is modified for dark-field microscopy.

Suppose we have a radially symmetric function in 3D given by $f(\mathbf{r}) = f_0(r)$.

a) Show that the 3D Fourier transform is also radially symmetric, $F(\rho) = F_0(\rho)$, with

$$F_0(\rho) = 4\pi \int_0^\infty f_0(r)\, sinc(2\rho r)\, r^2 dr$$

b) Now let $F_0^{(1)}(\rho)$ be the 1D Fourier transform of $f_0(r)$, which is extended as an even function to negative values of r, so that $f_0(-r) = f_0(r)$. Show that

$$F_0(\rho) = -\frac{1}{2\pi\rho}\frac{d}{d\rho}F_0^{(1)}(\rho)$$

c) Use this result to find the Fourier transforms of the 3D functions

i) $\exp(-\pi r^2)$

ii) $\exp(i\pi\beta r^2)$

iii) $\delta(r - r_0)$

iv) $sinc(r)$

v) $rect(r)$