**Spring 2017 Written Comprehensive Exam**
**Opti 501, Day 1**

**System of units: MKSA**

Inside a homogeneous, isotropic, non-magnetic, dielectric medium of refractive index $n(\omega)$, a monochromatic, homogeneous plane-wave propagates along the $z$-axis. The plane-wave is linearly-polarized along the $x$-axis, and the medium is transparent, that is, $n(\omega)$ is real and positive.

4 Pts a) Write expressions for the plane-wave's electric and magnetic fields, $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{H}(\boldsymbol{r},t)$, in terms of the $E$-field amplitude $E_0$, the angular frequency $\omega$, the refractive index $n(\omega)$, the speed of light in vacuum $c$, and the impedance of free space $Z_0$.

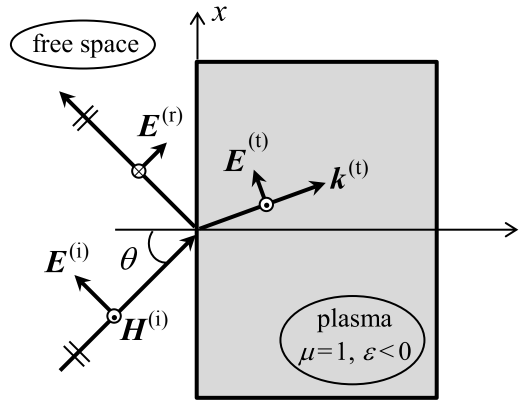
2 Pts b) Express the dielectric function $\varepsilon(\omega)$ and the electric susceptibility $\chi(\omega)$ as functions of the refractive index $n(\omega)$.

4 Pts c) Write an expression for the polarization distribution $\boldsymbol{P}(\boldsymbol{r},t)$ in terms of $E_0$, $\omega$, $c$, $\varepsilon_0$ and $n(\omega)$. What are the distributions of the electric bound-charge and bound-current densities, $\rho_{\text{bound}}(\boldsymbol{r},t)$ and $\boldsymbol{J}_{\text{bound}}(\boldsymbol{r},t)$, in the medium?

**Spring 2017 Written Comprehensive Exam**
**Opti 501, Day 2**

**<u>System of units: MKSA</u>**

A $p$-polarized monochromatic plane-wave arrives from free-space at the flat surface of a plasma at an oblique angle $\theta$, as shown. The optical properties of the plasma are specified by its permittivity $\varepsilon(\omega)$, a real-valued *negative* entity, and by its permeability $\mu(\omega) = 1$.




2 Pts a) Write expressions for the $E$ and $H$ fields of the incident beam as functions of space and time.

2 Pts b) Write expressions for the $E$ and $H$ fields of the reflected beam as functions of space and time.

2 Pts c) Write expressions for the $E$ and $H$ fields of the beam transmitted into the plasma as functions of space and time. Identify the real and imaginary components of the $k$-vector, and relate them to the various parameters of the system.

2 Pts d) Match the boundary conditions at the plasma surface, and obtain expressions for the Fresnel reflection and transmission coefficients $\rho_p$ and $\tau_p$, respectively.

2 Pts e) Show that the reflectivity of the plasma is always 100%, irrespective of the incidence angle $\theta$, or of the exact value of $\varepsilon(\omega)$. Explain the apparent contradiction between a 100% reflectance at the surface and the existence of electromagnetic field energy inside the plasma.

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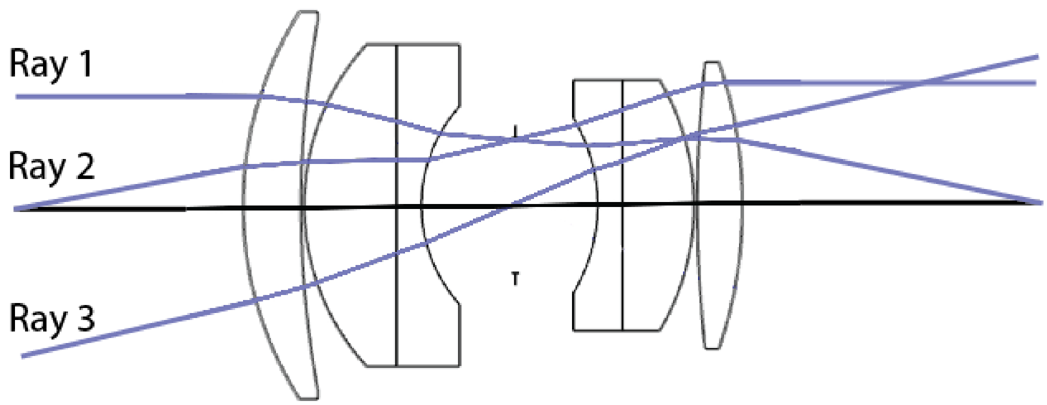
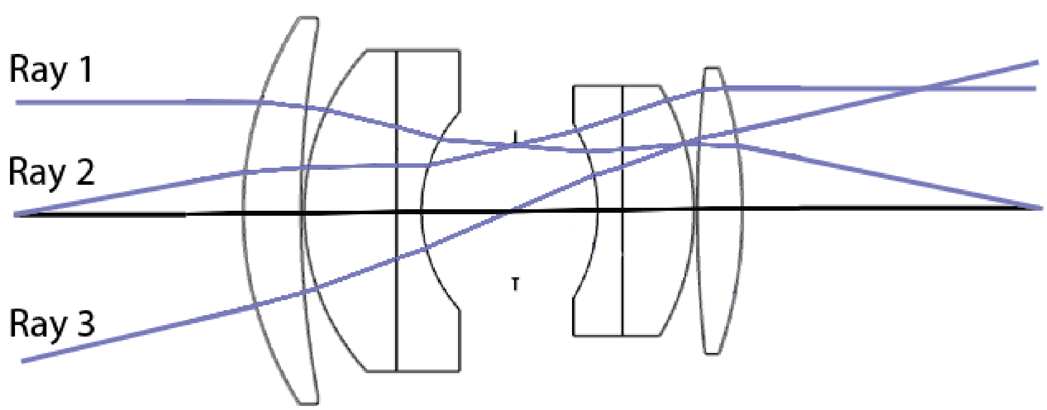
Spring 2017 Comprehensive Exam

OPTI 502 – Day 1

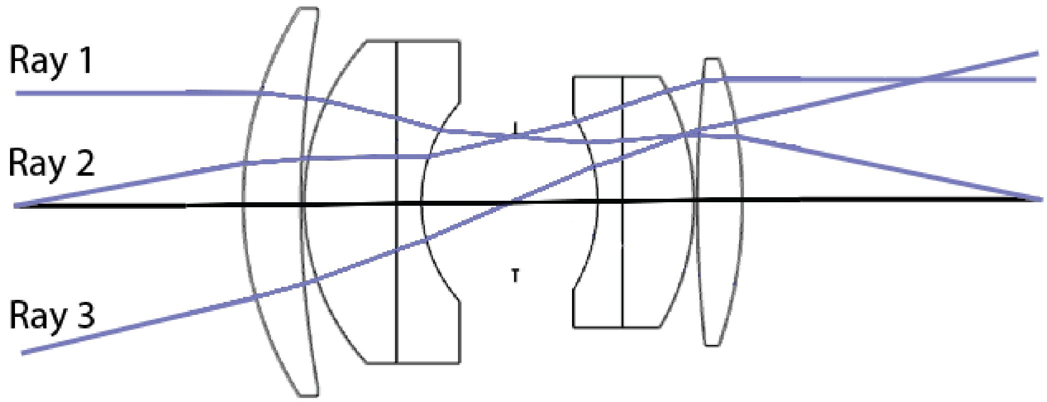
The figure below shows a double Gauss lens with three rays traced through it. Ray 1 is parallel to the optical axis as it enters the lens and passes through the edge of the aperture stop. Ray 2 is parallel to the optical axis as it exits the lens and passes through the edge of the aperture stop. Ray 3 passes through the center of the aperture stop. You will be drawing the cardinal points and pupils on the figures below. Be sure to be neat and illustrate how you determine the locations and diameters.



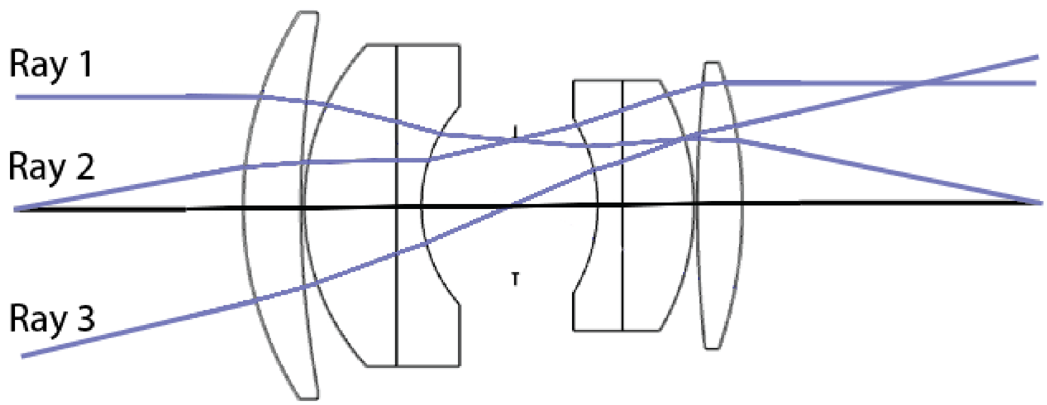

(a) On the figure above, draw the positions of the front and rear focal points and label them F and F', respectively.




(b) On the figure above, draw the front and rear principal planes and label them P and P', respectively. Label the front and rear nodal points N and N', as well.




(c) On the figure above, draw the location and diameter of the entrance pupil. Denote them EP and $D_{EP}$, respectively.




(d) On the figure above, draw the location and diameter of the exit pupil. Denote them XP and $D_{XP}$, respectively.

(e) Why do we calculate the cardinal points?

(f) Why do we calculate the entrance and exit pupil locations and size?

**Spring 2017 Comprehensive Exam**

**OPTI 502, Day 2**

Part A:

This problem is about determining the focal length of a thin lens in air. A thin lens in air is imaging a point source. The lens is working at a magnification $m$, and the total distance between the source and its image is 1000 mm. The lens is then moved along the optical axis a distance of 200 mm so that now it is working at a magnification of $1/m$. Both the source and its image remain at the same positions and separation. In other words, the object-image distance is the same for both lens positions.

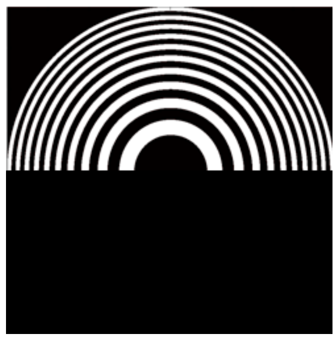
Determine the focal length of this lens. A requirement for this problem is to derive your results or fully explain your reasoning. Write legibly.

Part B:

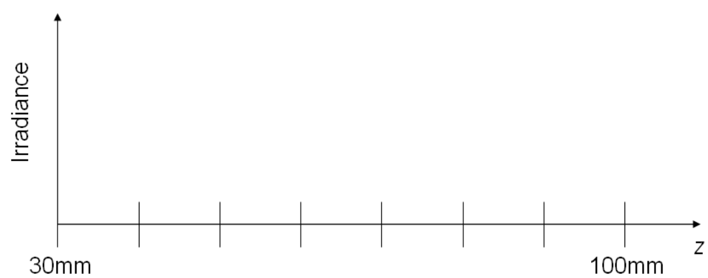
The convex surface of a plano convex lens is measured with a spherometer and the sag is found to be 0.5 mm for a radial distance of 10 mm. Assume that the index of refraction is 1.5. Determine the focal length of this lens. Does the focal length depend on the lens thickness?

**Spring 2017 Written Comprehensive Exam**
**OPTI 505R - Day 1**

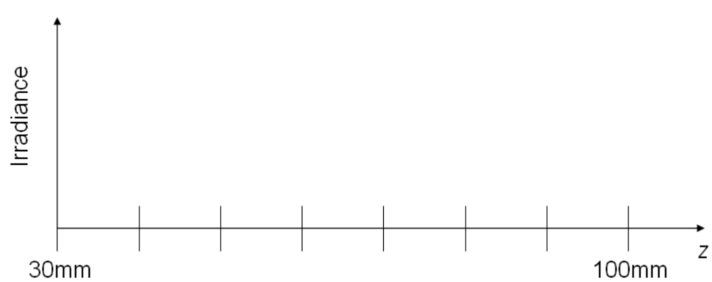
A simple binary amplitude (0 or 1 transmission) Fresnel zone plate is fabricated as shown below, with one-half of the zone plate covered by an opaque mask. White areas indicate unity transmission. The plate and mask are illuminated by an on-axis plane wave with $\lambda$ = 500nm and irradiance $100W/m^2$. The zone plate is designed with ten open zones, and its primary focus (without the half-plane opaque mask) is designed for a primary focus at a distance of 100mm behind the plate. The optical axis is defined normal to the plate at the center of the rings.



**(1) [5pts]** On the graph below, roughly sketch the on-axis irradiance from a distance of 30mm behind the plate to 100mm behind the plate. Calculate precise positions of maxima and minima (if any), and calculate irradiance values at those locations. State any assumptions that you make.




**(2) [5pts]** Replace the opaque half-plane mask with a phase plate that alters the phase of transmitted light relative to the uncovered section of the zone plate by $\pi$ radians in transmission, but does not affect amplitude. On the graph below, sketch the on-axis irradiance from a distance of 30mm behind the plate to 100mm behind the plate. Calculate precise positions of maxima and minima (if any), and calculate irradiance values at those locations. State any assumptions that you make.



**Spring 2017 Written Comprehensive Exam**
**OPTI 505R - Day 2**

Use Jones calculus to show that a half-wave plate converts right handed circularly polarized light into left handed circularly polarized light and the phase of the exiting light can be changed by rotating the half-wave plate.

**Spring 2017 Comprehensive Exam**
**OPTI 507, Day 1**

Consider the model for phonons in a 1-dimensional diatomic chain with harmonic nearest-neighbor interactions. Denote the masses by $M_1$ and $M_2$ and assume the force constants between $M_1$ and $M_2$ and $M_2$ and $M_1$ to be the equal, i.e. $f_1 = f_2 \equiv f$. Write down the equations of motion of the displacements, denoting that of $M_1$ as $u_j(t)$ and that of $M_2$ as $u_{j+1}(t)$. Write down the appropriate "ansatz" (or trial function) for the displacements, but for simplicity only for zero wave vector, i.e. $k = 0$. Find the phonon frequencies, again only for $k = 0$. Name the branch that corresponds to each frequency.

(10 points)

**Spring 2017 Comprehensive Exam**
**OPTI 507, Day 2**

Consider a surface-plasmon polariton (SPP). The SPP dispersion can be written as

$$q^2 = \frac{\omega^2}{c^2}\frac{\varepsilon_1(\omega)\varepsilon_2(\omega)}{\varepsilon_1(\omega)+\varepsilon_2(\omega)}$$

Make a simple plot of the geometry. Indicate the materials with dielectric functions $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$, respectively, and clearly indicate the location and propagation direction of the SPP. Assume now that you have an interface between a non-dispersive dielectric with $\varepsilon_1 = 1.4$ and a metal that can be described by a Drude model without damping, $\varepsilon_\infty = 1$, and $\omega_{pl} = 10^{15}\,\mathrm{s}^{-1}$. Determine the range of frequencies (numerical values) at which SPPs are possible, plot the SPP dispersion (carefully labeling the axes and all special points), and determine the wavelength at frequency $\omega = 0.63\times 10^{15}\,\mathrm{s}^{-1}$.

(10 points)

Spring 2017 Written Comprehensive Exam
OPTI 511, Day 1

Consider a hydrogen atom, and neglect the spins of the electron and the proton. The atom is at rest in the absence of any external fields in the following superposition state at some time $t = 0$ :

$$\Psi(\vec{r}, 0) = \frac{1}{\sqrt{2}}[\psi_{100}(\vec{r}) + \psi_{310}(\vec{r})],$$

where $\psi_{100}(\vec{r})$ and $\psi_{310}(\vec{r})$ are stationary states of hydrogen indicated with the usual $\psi_{nlm}(\vec{r})$ notation and expressed as functions of the position coordinate $\vec{r}$.

(a) (1 point) In this superposition state, what is the expectation value of $L^2$, the operator for the squared magnitude of the total orbital angular momentum of the atom?

(b) (1 point) What is the probability that a single measurement of $L^2$ will yield the value you found in part (a)?

(c) (2 points) Write the time-dependent form of the wavefunction, $\Psi(\vec{r}, t)$ for the superposition state given above.

(d) (6 points) Using the time-dependent superposition determined above, calculate the time-dependent expectation value of the atom's electric dipole moment, $\langle \Psi(t) \,|\, \vec{d} \,|\, \Psi(t) \rangle$, where $\vec{d} = -e\vec{r}$, and $e$ is the fundamental electric charge constant. Indicate the magnitude of the dipole moment (in terms of fundamental constants) and the dipole moment's oscillation frequency (give a numerical value). Neglect the effects of spontaneous emission. See below for helpful expressions.

**Some helpful expressions:**

$\hbar \approx 7 \times 10^{-16}$ eV·s.

Cartesian components of angular matrix elements
$\langle l = 1,\, m = 0 \,|\, \hat{\mathbf{r}} \,|\, l = 0,\, m = 0 \rangle = (0,\, 0,\, \sqrt{\frac{1}{3}})$
$\langle l = 1,\, m = +1 \,|\, \hat{\mathbf{r}} \,|\, l = 0,\, m = 0 \rangle = (-\sqrt{\frac{1}{6}},\, i\sqrt{\frac{1}{6}},\, 0)$
$\langle l = 1,\, m = -1 \,|\, \hat{\mathbf{r}} \,|\, l = 0,\, m = 0 \rangle = (\sqrt{\frac{1}{6}},\, i\sqrt{\frac{1}{6}},\, 0)$

Radial matrix elements for atomic hydrogen in terms of Bohr radius $a_0$
$\langle n = 1,\, l = 0 \,|\, r \,|\, n = 2,\, l = 1 \rangle = 1.29a_0$
$\langle n = 1,\, l = 0 \,|\, r \,|\, n = 3,\, l = 1 \rangle = 0.52a_0$

Spring 2017 Written Comprehensive Exam
OPTI 511, Day 2

An optical cavity for a HeNe laser (633 nm) is being constructed using a flat output coupler ($R_1 = \infty$) and a spherical mirror ($R_2 = 50$ cm). The output coupler has a power reflection coefficient of 99% while the spherical mirror is a 100% reflector. The cavity length is initially set at $L_0 = R_2 = 50$ cm. A micrometer adjustment with a range of $\Delta L = \pm 500$ $\mu$m enables fine adjustment of the total cavity length (given by $L = L_0 + \Delta L$).

(a) (1 point) Calculate the range of cavity lengths $L$ that allow for a stable Gaussian mode based on the criterion $0 \leq g_1 g_2 \leq 1$.

(b) (4 points) If an output beam radius of $w = 50$ $\mu$m is desired, calculate the required micrometer adjustment $\Delta L$ in microns to 2 significant digits, indicating the appropriate sign for $\Delta L$. You should check that your answer is consistent with part (a).

(c) (3 points) If the length of the HeNe gain tube is 20 cm, calculate the small-signal gain coefficient required for lasing.

(d) (1 point) With the laser operating well above threshold, do you expect single frequency or multiple frequency operation? Explain why in a few sentences.

(e) (1 point) If the laser is mode-locked, what will be the period between pulses?