

WRITTEN PRELIM EXAM – FIRST DAY

Fall 2012

September 18, 2012
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied.

The following are some helpful items:

| | |
|--|---|
| $h = 6.625 \times 10^{-34} \text{ J} \cdot \text{s} = 4.134 \times 10^{-15} \text{ eV} \cdot \text{s}$ | $\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$ |
| $e = 1.6 \times 10^{-19} \text{ C}$ | $\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$ |
| $c = 3.0 \times 10^8 \text{ m/s}$ | $\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$ |
| $k_B = 1.38 \times 10^{-23} \text{ J/K}$ | $\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$ |
| $\sigma = 5.67 \times 10^{-8} \text{ W/K}^4 \cdot \text{m}^2$ | $\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$ |
| $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ | $\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$ |
| $\mu_0 = 1.26 \times 10^{-6} \text{ H/m}$ | $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$ |
| $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ | $\nabla \cdot (\nabla \times \mathbf{F}) = 0$ |
| $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ | $\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$ |
| $2 \cos A \cos B = \cos(A - B) + \cos(A + B)$ | $\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$ |
| $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$ | $\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$ |
| $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ | $\nabla \times \nabla\phi = 0$ |
| $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$ | $\oint_S (\mathbf{F} \cdot \mathbf{n}) da = \int_V (\nabla \cdot \mathbf{F}) d^3x$ |
| $\sin 2A = 2 \sin A \cos A$ | $\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} da$ |
| $\cos 2A = 2 \cos^2 A - 1$ | $\oint_S \phi \mathbf{n} da = \int_V \nabla\phi d^3x$ |
| $\cos 2A = 1 - 2 \sin^2 A$ | $\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n}) da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}] d^3x$ |
| $\sinh x = \frac{1}{2}(e^x - e^{-x})$ | $\oint_S (\mathbf{n} \times \mathbf{F}) da = \int_V (\nabla \times \mathbf{F}) d^3x$ |
| $\cosh x = \frac{1}{2}(e^x + e^{-x})$ | |

Fall 2012 Written Comprehensive Exam
Opti 501

System of units: MKSA

A monochromatic plane-wave, having frequency ω and wave-vector \mathbf{k} , propagates in free space. For all practical purposes, one may assume that ω is a real-valued scalar, while \mathbf{k} is a complex-valued vector, that is, $\mathbf{k} = \mathbf{k}' + i\mathbf{k}''$. Let the scalar and vector potentials associated with this plane-wave be written as $\psi(\mathbf{r}, t) = \psi_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$ and $\mathbf{A}(\mathbf{r}, t) = \mathbf{A}_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$, respectively.

- 2 Pts a) Write the differential equation relating the scalar and vector potentials in the Lorenz gauge, then derive the relation among ψ_0 , \mathbf{A}_0 , \mathbf{k} and ω assuming the aforementioned plane-wave satisfies the Lorenz gauge.
- 2 Pts b) Find expressions for the E - and B -fields of the plane-wave in terms of ψ_0 , \mathbf{A}_0 , \mathbf{k} and ω .
- 3 Pts c) Write the differential form of Maxwell's equations, then obtain the constraints on ψ_0 , \mathbf{A}_0 , \mathbf{k} and ω that ensure the above plane-wave is a solution of Maxwell's equations.
- 1 Pt d) Specify the condition(s) under which the plane-wave is evanescent (i.e., inhomogeneous).
- 1 Pt e) Specify the condition(s) under which the plane-wave is homogeneous and linearly polarized.
- 1 Pt f) Specify the condition(s) under which the plane-wave is homogeneous and circularly polarized.

Hint: You might find the following vector identities useful:

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

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Opti 502

Show that the ellipsoidal surface of revolution in Fig. 1 will give perfect image formation with refraction at the focus F' of the ellipse for an object at infinity provided that the eccentricity $e = c/a$ satisfies $e = n/n'$. No paraxial approximations are allowed.

The equation for the ellipse is

$$\frac{z^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad a^2 = b^2 + c^2$$

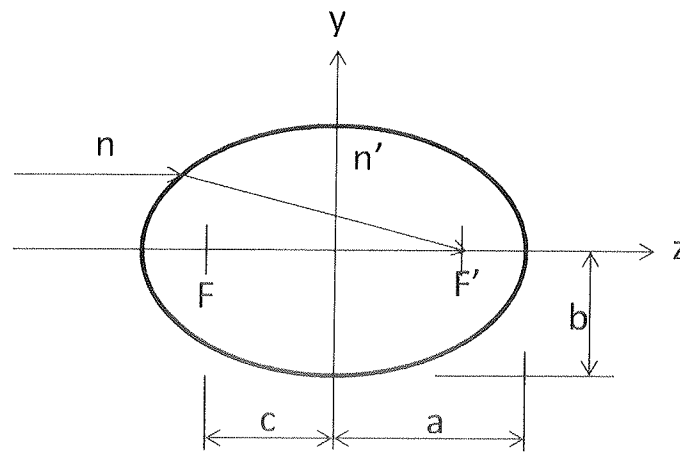
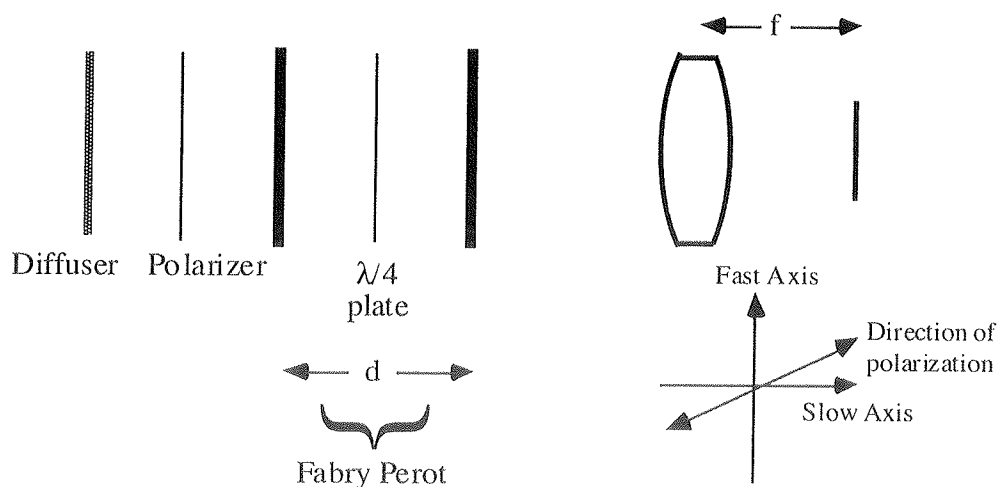


Fig. 1

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OPTI 505R

A quarter-wave plate is placed inside a Fabry Perot cavity as shown below. The mirror separation is d . Both mirrors have an intensity reflectance of R and they introduce no phase change upon reflection and no absorption. A laser of wavelength λ is used to illuminate a diffuser placed before the Fabry Perot. Between the diffuser and Fabry Perot we place a polarizer such that the light entering the Fabry Perot is plane polarized at an angle θ with respect to the slow axis of the quarter-wave plate. Find the irradiance distribution in the focal plane of the lens following the Fabry Perot. You can assume that the quarter-wave plate transmits 100% of all incident energy. Also, for simplicity assume that over the angles of interest the effective optical thickness of the quarter-wave plate is a constant.



Fall 2012 Written Comprehensive Exam Opti 546

There is no need for excessive formulae to answer the six parts of this question, you may simply state equations you feel are relevant.

(a - 1pt) Write down the ray transform that relates the incident ray vector $\begin{pmatrix} x \\ x' \end{pmatrix}_i$ to the final ray vector $\begin{pmatrix} x \\ x' \end{pmatrix}_f$ across a first-order optical system.

(b - 1pt) Prove that for an afocal or telescopic first-order optical system collimated rays at the input are converted into collimated rays at the output.

(c - 2pts) Using a sketch and accompanying words describe what is meant by the term Airy disk as applied to the problem of diffraction of a plane-wave field by a circular aperture.

(d - 2pts) Write down the equation of motion for the Lorentz electron oscillator model describing an atom in interaction with a light field and clearly identify the terms that incorporate the effects of any externally applied electric fields and Coulomb interactions.

(e - 2pts) Write down the general form of the (3×3) matrix representation of the force constant tensor K_{ij} in the principal axes system for the cases of isotropic, uniaxial, and biaxial crystals.

(f - 1pt) In electro-optics a static electric field E_{DC} applied to a medium can be used to alter the refractive indices experienced by two orthogonal linear field polarization states, resulting in a refractive-index difference $\Delta n(E_{DC})$. Describe what distinguishes the electro-optical Pockels and Kerr effects in terms of the variation of $\Delta n(E_{DC})$ with the applied static electric field.

(g - 1pt) Following on from part (f), is it possible to observe the Pockels effect in an isotropic medium?

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OPTI 544

Consider a two-level atom (ground state $|g\rangle$, excited state $|e\rangle$) interacting with a single mode of a quantized electromagnetic field in a cavity. When the cavity mode is exactly resonant with the atomic transition frequency the Hamiltonian has the form

$$H = \hbar\omega|e\rangle\langle e| + \hbar\omega a^\dagger a + \hbar\kappa (a|e\rangle\langle g| + a^\dagger|g\rangle\langle e|).$$

In the absence of atom-field coupling ($\kappa = 0$), this Hamiltonian has eigenstates of the form $|\varphi, n\rangle$, where the atomic state $\varphi = g, e$ and n is the number of photons in the cavity mode.

- (a) Now let $\kappa \neq 0$. Write out a corner of the matrix for H in the basis of uncoupled states corresponding to $n \leq 3$, using an arrangement of these basis states in order of increasing energy. (20%)
- (b) Find the eigenenergies of the coupled atom-field system. (20%)
- (c) Find the eigenstates $|\pm, n\rangle$ corresponding to these eigenenergies, expressed in terms of the uncoupled states. (20%)
- (d) For both $\kappa = 0$ and $\kappa \neq 0$ draw an energy level diagram of the eigenstates of the atom-field system, including all levels with $n \leq 3$. The diagram must clearly indicate the relative magnitude of the level splittings. (20%)
- (e) Assume that the atom, in addition to the cavity mode discussed above, is also coupled to a large number of empty field modes. This leads to spontaneous decay. Use a physical argument to explain which states $|\pm, n\rangle$, $|\pm, n'\rangle$ are connected by spontaneous decay. Indicate the possible decay channels on the energy level diagram. (20%)

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OPTI 511

In the questions below, you will consider a particle of mass m confined in two different time-independent one-dimensional potential wells $V(x)$, and a time-independent three-dimensional well $V(r)$. You might find the following information useful:

$$\int_{-\infty}^{\infty} \exp\left\{-\frac{x^2}{2a^2} + bx\right\} dx = \sqrt{2\pi} a \exp\{b^2 a^2 / 2\} \quad (\text{for } a, b \text{ constant, } a \text{ real, } a \geq 0).$$

- (a) (3 pts) Suppose that $V(x)$ is defined by $V(x) = \begin{cases} 0 & \text{if } 0 < x < L \\ \infty & \text{otherwise} \end{cases}$. Give an expression

for the energy eigenvalues and the normalized energy eigenfunctions associated with this potential, and specify the range of possible quantum numbers that corresponds to your expressions. (This is a problem you should be able to solve if you do not remember the expressions. For the remainder of the problems, however, you need to remember the answers or make good guesses – you should not try to solve the problems from scratch!)

- (b) (1 pt) Suppose that instead of the above potential, $V(x)$ is defined by $V(x) = \frac{1}{2} m \omega^2 x^2$ for all x , where ω is a constant. Give an expression for the energy eigenvalues associated with this potential well, and specify the range of possible quantum numbers that corresponds to your expression.

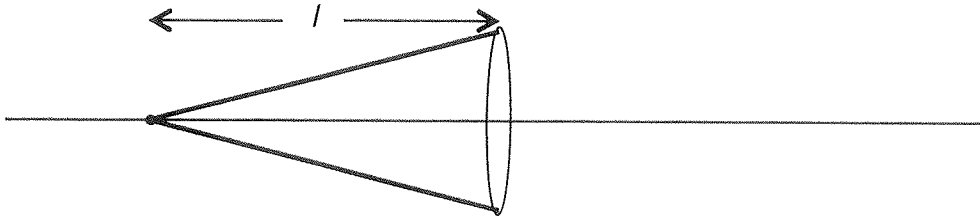
- (c) (4 pts) For $V(x) = \frac{1}{2} m \omega^2 x^2$, give an expression for the ground-state wavefunction. You may use $\sigma = \sqrt{\hbar / m \omega}$, and you may neglect the normalization coefficient. On separate plots sketch the **probability density** distributions for the ground state and first three excited states. What name is given to the full set of energy eigenfunctions? Where else does this set of functions commonly appear in optics?

- (d) (2 pts) Consider the three-dimensional central potential defined by $V(r) = \frac{-e^2}{4\pi\epsilon_0 r}$,

where $e = 1.6 \times 10^{-19}$ C, $\epsilon_0 = 8.85 \times 10^{-12}$ C²/Jm is the permittivity of free space, and the particle's mass is approximately equal to the electron mass. What are the energy eigenvalues (and the range of quantum numbers) in this case? Give numerical values for any constants in your expression.

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OPTI 536

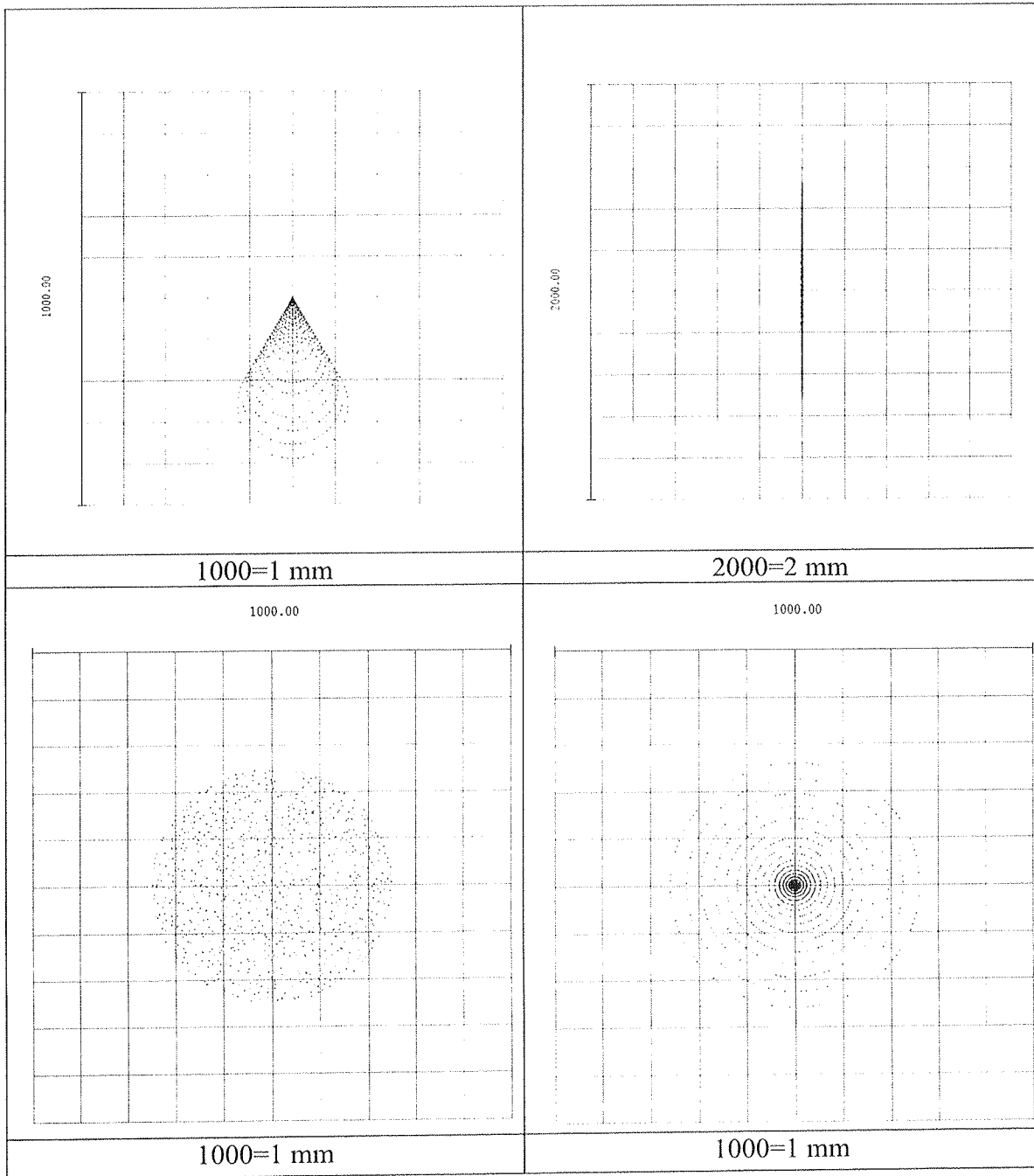
Consider the situation shown in the figure below where there is a quasi-monochromatic point source of wavelength $\lambda=500\text{nm}$ on the optical axis at a distance $l=30\text{cm}$ to the left of a lens that has a positive optical power $\phi=10$ diopters. You can assume this lens is in air.



1. In the limit of paraxial geometrical optics, where is the image of the point source located? Give your answer l' in terms of the parameters l and ϕ , and also provide the numerical result.
2. If the diameter of the lens $d=5\text{cm}$, what is the $f/\#$ of this lens?
3. What is the numerical aperture NA' of this imaging system in image space?
4. Could this lens be used as a magnifying glass, and if so, what would be its magnifying power MP ?
5. Could this lens be used as an objective lens in an infinity-corrected microscope with a tube lens that has a focal length of 200 mm , and if so, what would be the magnification of the objective?
6. If there was another quasi-monochromatic point source located a distance $h=1\text{cm}$ above the first point source on the optical axis, what is the image height h' with respect to the optical axis of the image of the second point?
7. What is the minimum separation Δx between these two points in object space such that the two points can be resolved as being two different point objects in the image? You can consider the two points as being incoherent with respect to one another and the imaging system as diffraction limited.
8. Now consider this as an incoherent imaging system where you have a general quasi-monochromatic incoherent object distribution at the object plane and you want to place a CCD or CMOS detector at the image plane. What would the detector spacing have to be in order for this image to be properly sampled according to the Nyquist sampling criterion? State your answer in terms of the parameters provided in the problem and give a numerical result.
9. Finally, if the lens described above was used as the first lens in a Galilean refracting telescope, what would the focal length of the second lens have to be in order to achieve a total instrument magnifying power of $+10$?

Fall 2012 Written Comprehensive Exam
Opti 509

- 1) List the five monochromatic third-order aberrations. (1 point)
- 2) Write the equation that relates the transverse ray aberrations to the wavefront error W (2 points)
- 3) An optical system has an F-number of $F/10$ and works at wavelength of 0.0005 mm. If the focal length is 100 mm what is the entrance pupil diameter? (1 point)
- 4) Identify which aberration (from question 1) is present in each of the spot diagrams on the next page. (1 point)
- 5) Determine (provide an estimate based on the scale) the amount of aberration in each spot diagram expressed as the wave aberration coefficient W . The optical system works at $F/10$ at a wavelength of 0.0005 mm. The spots are for the edge of the field of view position. Note the scale of the spot diagrams: for the first spot diagram the length of 10 small squares is equal to 1 mm. (5 points)



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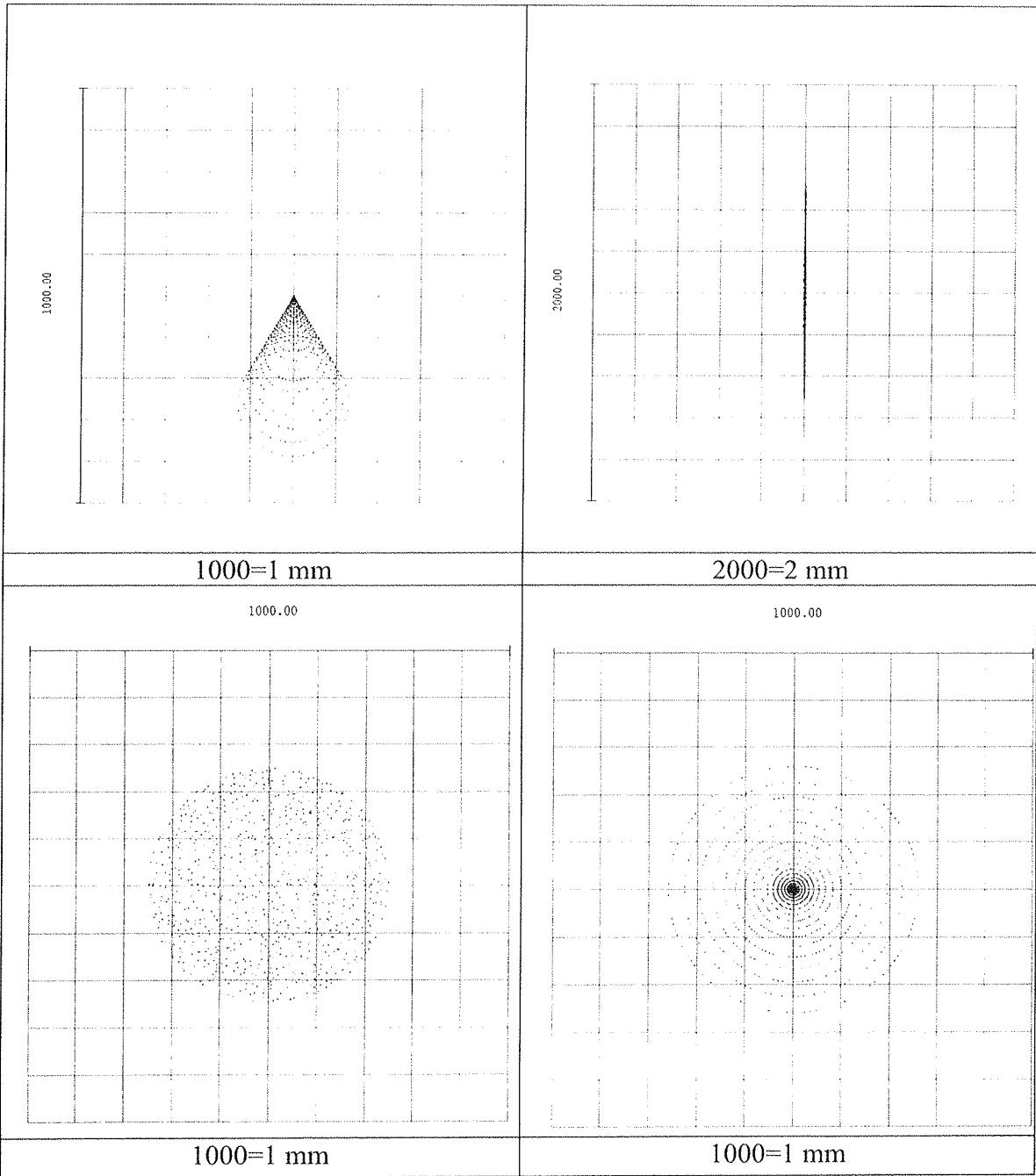
OPTI 510

Problem 1: Semiconductor detector and fiber optics link

- (a) Sketch two band diagrams of silicon illustrating the energy band transitions when a photon is emitted and absorbed in the semiconductor. Do the same for GaAs. Label the axes. Explain why silicon is a good material for making a semiconductor detector and not a semiconductor laser. Your explanation should take into account the energy transitions in the band diagram. (2 points)
- (b) An optical fiber communication link is designed to be an attenuation-limited system and operate at 20Mb/s. The source is a $100\mu\text{W}$ LED operating at 870nm and the fiber attenuation coefficient is 3.5dB/km. Input and output couplers each have a loss of 2dB. The safety margin for the link is 6dB.
1. If the receiver has a sensitivity of 125 photons per bit, what is the receiver sensitivity in dBm? (2 points)
 2. What is the maximum length of the link? (4 points)
 3. Give four changes you can make to increase the maximum length of the link. (2 points)

Fall 2012 Written Comprehensive Exam
Opti 503

- 1) List the five monochromatic third-order aberrations. (1 point)
- 2) Write the equation that relates the transverse ray aberrations to the wavefront error W (2 points)
- 3) An optical system has an F-number of $F/10$ and works at wavelength of 0.0005 mm. If the focal length is 100 mm what is the entrance pupil diameter? (1 point)
- 4) Identify which aberration (from question 1) is present in each of the spot diagrams on the next page. (1 point)
- 5) Determine (provide an estimate based on the scale) the amount of aberration in each spot diagram expressed as the wave aberration coefficient W . The optical system works at $F/10$ at a wavelength of 0.0005 mm. The spots are for the edge of the field of view position. Note the scale of the spot diagrams: for the first spot diagram the length of 10 small squares is equal to 1 mm. (5 points)



Like silicon, germanium (Ge) is an element in Group IV of the periodic table. Some of its properties at room temperature are listed below:

Bandgap energy: $E_g = 0.66 \text{ eV}$

Intrinsic carrier concentration: $n_i = 2 \times 10^{13} \text{ cm}^{-3}$

Electron mobility: $\mu_e = 3900 \text{ cm}^2/(\text{V} \cdot \text{s})$

Hole mobility: $\mu_h = 1900 \text{ cm}^2/(\text{V} \cdot \text{s})$

Refractive index: $n = 4.0$

- (a) Compute the drift velocities for hole and electrons in undoped Ge for an applied electric field of 10 V/cm .
- (b) From the drift velocities, compute the current density J in A/cm^2 and the conductivity σ for undoped Ge at room temperature. If you don't remember the formula, try simple dimensional analysis or think about Ohm's law.
- (c) Give one example each of a donor and an acceptor for Ge.
- (d) Sketch the quantum efficiency and the responsivity for an ideal Ge photodiode as a function of wavelength. Label the axes carefully and state the peak values for each sketch.
- (e) For an application requiring a room-temperature photodiode, when would you prefer Si over Ge? When would you prefer Ge? Why?
- (f) Suppose you cool the Ge photodiode to 77 K . Qualitatively, what parameters of the detector change? Why does its performance improve? Sketch the current-voltage characteristic for the Ge photodiode in the dark and for some level of illumination for both 77 K and 300 K .

Fall 2012 Written Comprehensive Examination
OPTI 537

Answer the following questions related to radiative transport. The radiative transport equation (RTE), or Boltzmann equation, describes the time evolution of a phase-space distribution function w in terms of four processes: absorption, emission, propagation, and scatter

$$\frac{dw}{dt} = \left[\frac{\partial w}{\partial t} \right]_{abs} + \left[\frac{\partial w}{\partial t} \right]_{emiss} + \left[\frac{\partial w}{\partial t} \right]_{prop} + \left[\frac{\partial w}{\partial t} \right]_{scat} \quad (1)$$

(a) What is w a function of and what are the units? What does w describe?

(b) In class, we derived forms for each of the terms in the RTE and wrote an overall spatio-temporal-integro-differential transport equation of the form:

$$\frac{dw}{dt} = -c_m \mu_{total} w + \Xi_{p,E} - c_m \hat{s} \cdot \nabla w + \mathbf{K}w \quad (2)$$

where \mathbf{K} is an integral operator. Please associate each term in equation (1) with the terms in equation (2). Describe and write the units of each of the variables in equation (2).

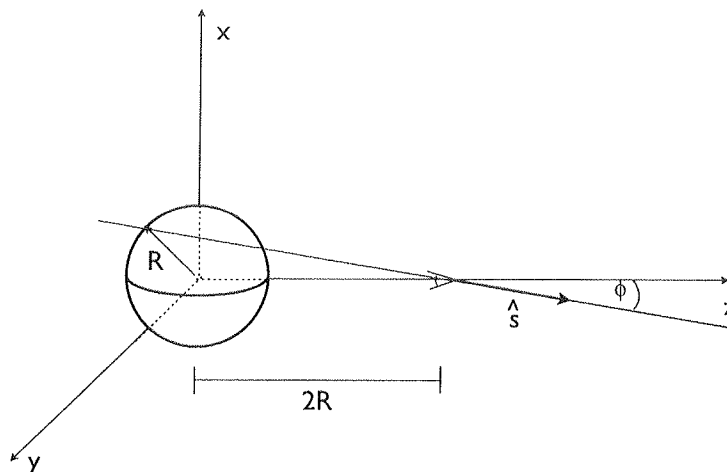
(c) What is meant by the "steady-state solution" of the RTE? What changes in equation (2) when we assume steady state?

(d) Write the steady-state equation for a non-absorbing and non-scattering medium.

(e) Now assume that we have an incoherent, uniform spherical source of radius R centered at the origin that isotropically emitting photons of energy E at a rate of A photons per second. These photons are propagating in a non-absorbing and non-scattering medium. We are observing photons at a distance of $2R$ away from the origin along the z axis (see Figure below). What is the steady-state phase-space distribution function at this location for photons traveling in the z direction ($\phi = 0$)?

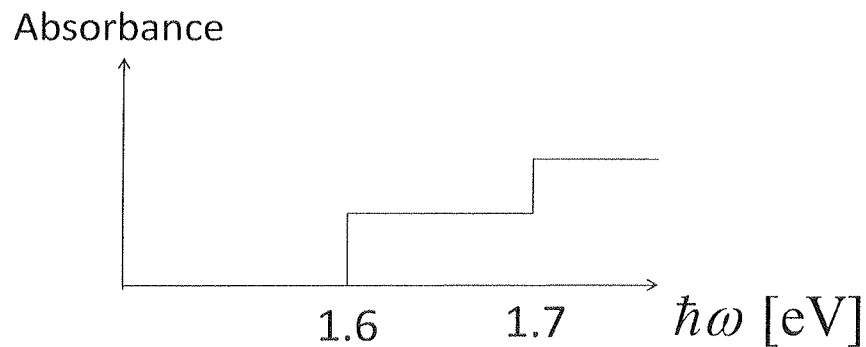
(f) Building on part (e). What is the steady-state phase-space distribution function at the observation point for photons traveling at an angle of $\phi = 15^\circ$? You may leave your answer in terms of ϕ if you would like.

(g) Building on part (e). What is the steady-state phase-space distribution function at the observation point for photons traveling at an angle of $\phi = 45^\circ$?



**Fall 2012 Comprehensive Exam
OPTI 507**

Assume you have measured an interband (i.e. valence-to-conduction band) absorption spectrum of an infinite-barrier semiconductor quantum well. For simplicity, we assume there are no Coulomb interaction or excitonic effects. The spectrum is shown in the figure. Determine the thickness of the quantum well, L_z .



Instructions. If you remember the formula for the confinement energies, you can use it without proving it. If you don't remember that formula, you can get the confinement energies easily from the fact that the envelope functions of the lowest two subbands are $\xi_1(z) \sim \cos(\pi z / L_z)$ and $\xi_2(z) \sim \sin(2\pi z / L_z)$. You must take into account the selection rules for the quantum wells irradiated in normal incidence (i.e. not every valence band subband can make an optical transition to every conduction band subband).

Parameters: Assume the electron effective mass to be $m_e = 0.06m_0$ and that of the hole to be $m_h = 0.1m_0$ (where m_0 is the electron mass in vacuum). Also, use $\hbar^2 / m_0 = 7.62 \times 10^{-16} \text{ eV cm}^2$.

(10 points)

WRITTEN PRELIM EXAM – SECOND DAY

Fall 2012

September 19, 2012
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied.

The following are some helpful items:

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$$c = 3.0 \times 10^8 \text{ m/s}$$

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/K}^4 \cdot \text{m}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 1.26 \times 10^{-6} \text{ H/m}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$$

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$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\sinh x = \frac{1}{2} (e^x - e^{-x})$$

$$\cosh x = \frac{1}{2} (e^x + e^{-x})$$

$$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$$

$$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$$

$$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$$

$$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$$

$$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$$

$$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$$

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0$$

$$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$$

$$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$$

$$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$$

$$\nabla \times \nabla\phi = 0$$

$$\oint_S (\mathbf{F} \cdot \mathbf{n}) da = \int_V (\nabla \cdot \mathbf{F}) d^3x$$

$$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} da$$

$$\oint_S \phi \mathbf{n} da = \int_V \nabla\phi d^3x$$

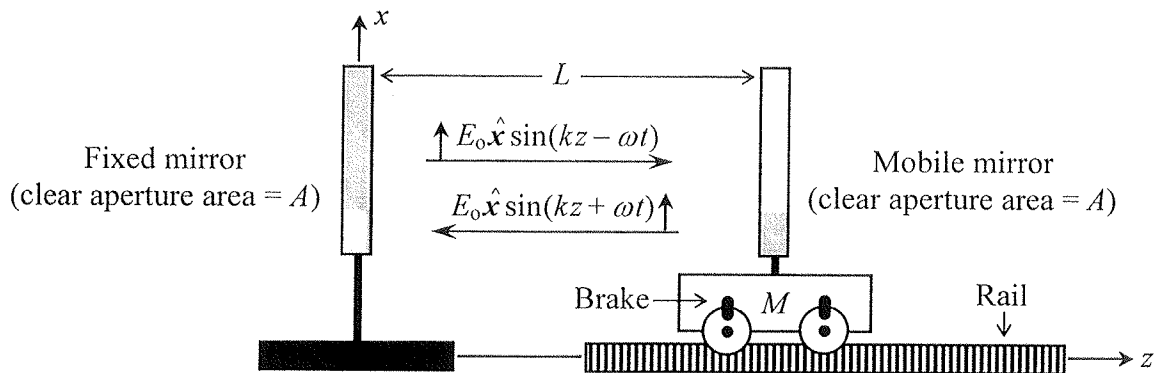
$$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n}) da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}] d^3x$$

$$\oint_S (\mathbf{n} \times \mathbf{F}) da = \int_V (\nabla \times \mathbf{F}) d^3x$$

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Opti 501

System of units: MKSA

A pair of monochromatic, linearly-polarized, counter-propagating plane waves of frequency ω is trapped in free space between two perfectly electrically conducting flat mirrors, as shown in the figure. The distance between the mirrors is $L = N\lambda_0/2$, where $\lambda_0 = 2\pi c/\omega$ is the vacuum wavelength, and N is an arbitrary integer. The E -fields of the two plane-waves are given by $E_0\hat{x}\sin(kz \pm \omega t)$, where $k = \omega/c$. The cross-sectional area A of the two beams may be assumed to be large and uniform throughout the cavity.



- 2 Pts a) Write expressions for the total E - and H -fields as functions of z and t within the cavity.
- 2 Pts b) Find the total electromagnetic energy contained within the volume AL of the cavity. Considering that each photon is known to have an energy $\hbar\omega$, where \hbar is Planck's reduced constant, how many photons are trapped inside the cavity?
- 3 Pts c) Find the induced current-density $\mathbf{J}_s(t)$ at the surface of each mirror, then use this current-density to determine the Lorentz force exerted by the trapped radiation on each mirror.

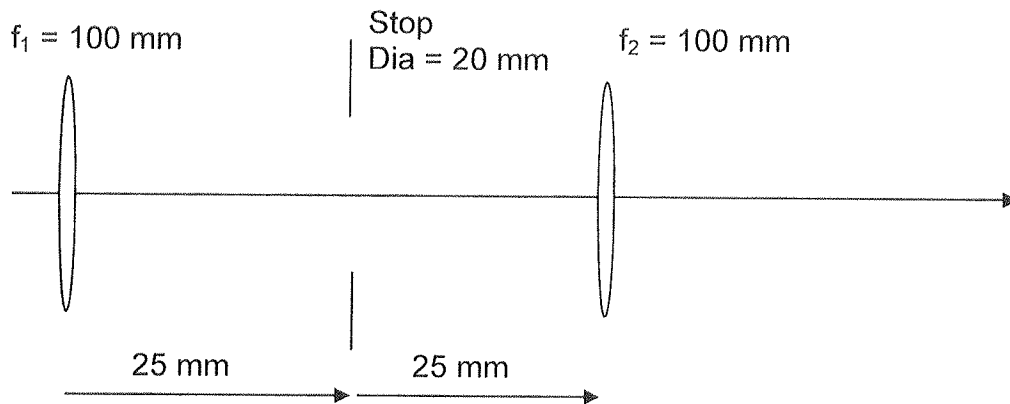
At $t = t_0$ the brakes preventing the motion of the mirror located at $z = L$ are released. Thereafter, radiation pressure pushes this mirror forward along the rail, with negligible losses due to friction. A short time later, at $t = t_0 + \Delta t$, where $\Delta t = 2L/c$, the mirror will be at $z = L + \Delta z$ and will have acquired a velocity $V\hat{z}$. Assume the mass M of the mirror is sufficiently large that relativistic effects may be ignored.

- 2 Pt d) Find the kinetic energy $\frac{1}{2}MV^2$ of the mirror by calculating the mechanical work done by the radiation pressure in moving the mirror from $z = L$ to $z = L + \Delta z$.
- 1 Pt e) Find the electromagnetic momentum transferred to the mirror during the interval $\Delta t = 2L/c$.

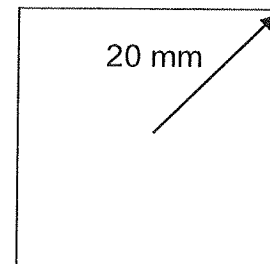
Hint: $\sin(a) + \sin(b) = 2\sin[\frac{1}{2}(a+b)]\cos[\frac{1}{2}(a-b)]$ and $\sin(a) - \sin(b) = 2\sin[\frac{1}{2}(a-b)]\cos[\frac{1}{2}(a+b)]$.

Fall 2012 Written Comprehensive Exam
Opti 502

An optical system comprised of two thin lenses in air places the system stop between the two elements. The object is at infinity.



This lens is used with a sensor that has a corner-to-corner dimension of 40 mm . What are the required element diameters for this system to be unvignetted over this field of view?



A raytrace sheet is attached on the next page and may be used for the solution.

Fall 2012 Comprehensive Exam
OPTI 505

A normally incident plane wave of wavelength λ illuminates a diffusing screen at $z = 0$ from one side, and on the other side of the screen at $z = 0^+$ is a diffracting screen with transmission

$$t(\rho) = \frac{1}{2} \left[1 + \operatorname{sgn} \left(\cos \left(\pi \frac{9\rho^2}{2a^2} \right) \right) \right] \operatorname{cyl} \left(\frac{\rho}{2a} \right).$$

We will be observing the irradiance along the axis for $z > 0$.

- (2 pts) Approximately how far from the diffusing screen must we be to use the Fresnel diffraction approximation?
- (4 pts) Provide an expression for the on-axis diffracted irradiance as a function of z . This expression should be as simplified as you can make it.
- (4 pts) Provide the locations $z = z_n$, where n is a positive integer, where the axial diffracted irradiance has a local maximum.

Helpful formulas:

$$\operatorname{sgn}(u) = \begin{cases} -1 & u < 0 \\ 0 & u = 0, \\ 1 & u > 0 \end{cases}, \quad \operatorname{cyl}(u) = \begin{cases} 1 & u < 1/2 \\ 0 & u > 1/2 \end{cases}$$

In the Fresnel Region:
$$U_o(x_o, y_o; z) = \frac{e^{jkz}}{j\lambda z} \iint U_s^+(x_s, y_s; 0) e^{j\frac{\pi}{\lambda z} [(x_o - x_s)^2 + (y_o - y_s)^2]} dx_s dy_s$$

Fall 2012 Written Comprehensive Exam Opti 546

Consider Young's double-slit experiment for a pair of monochromatic point sources located at $(x_0, z_0) = (\pm d/2, 0)$. The point sources propagate to the output plane (x_1, z_1) through a first-order optical system described by a ray transfer matrix with elements A, B, C, D , the optical propagator in one transverse dimension being given by

$$K(x_1, z_1; x_0, z_0) = \sqrt{\frac{1}{iB\lambda}} e^{ik_0L}, \quad L = L_0 + \frac{1}{2B} [Ax_0^2 - 2x_0x_1 + Dx_1^2], \quad (1)$$

with L_0 the on-axis optical path length.

(a - 1pt) Consider the case that an incident plane-wave field is passed through a pair of slits to produce the point sources. What condition is required of the transverse coherence length ℓ_T of the incident field so that interference fringes of near unity contrast may be expected?

(b - 1pt) We hereafter consider the case that the two point sources are mutually coherent and of equal strength. Write down an expression for the field $\mathcal{E}(x_1, z_1)$ at the output plane, to within an overall constant, in terms of the optical propagator in Eq. (1).

(c - 3pts) Show that the intensity profile at the output plane $I(x_1, z_1)$ displays interference fringes of the form

$$I(x_1, z_1) \propto \cos^2\left(\frac{k_0 dx_1}{2B}\right),$$

and obtain an expression for the fringe spacing Λ between successive peaks of the intensity profile.

(d - 3pts) By using the physical interpretation of the argument of the exponential appearing in the optical propagator in Eq. (1), obtain an expression for the optical path length (OPL) difference $\Delta L = (L_+ - L_-)$ between the OPL L_+ evaluated along the geometric optical path between $(x_0, z_0) = (+d/2, 0)$ and (x_1, z_1) , and the OPL L_- evaluated along the geometric optical path between $(x_0, z_0) = (-d/2, 0)$ and (x_1, z_1) .

(e - 2pts) Show that considerations based on the OPL difference ΔL leads to the same fringe spacing Λ found in part (a).

Consider a large ensemble of hydrogen atoms all initially in the ground state ψ_{100} . At time $t = 0$ a monochromatic and linearly polarized laser field of magnitude $E = \frac{1}{2}E_0 e^{-i\omega t} + c.c.$ is turned on and interacts with all the atoms. The laser frequency ω is slightly detuned from the $n = 1$ to $n = 3$ transition, with detuning $\Delta = \omega - \omega_0$, where $|\Delta| \ll \omega_0$. In this problem, assume the electric dipole and rotating wave approximations are valid, and that the atoms are motionless (ie no Doppler broadening).

(a) What is the approximate wavelength of the laser?

(b) Write down the general expression that gives the probability for finding the atom in the excited state as a function of time in the absence of spontaneous emission. Be sure to clearly define any symbols used in your expression.

(c) Given a laser detuning of $\Delta = 2\pi \times 100$ MHz and an electric field strength of $E_0 = \left(\frac{3\pi\hbar}{\epsilon a_0}\right) \times 10^8 \approx 11.6 \times 10^3 \left[\frac{J}{C \cdot m}\right]$, calculate the maximum possible fraction of atoms that can be excited to level $n = 3$ (ignoring spontaneous emission). Provide a numeric answer. See the bottom of this page for information you may find useful.

(d) Now assume the entire ensemble of atoms is in the energy eigenstate $\psi_{3,1,0}$. Taking into account spontaneous emission at optical frequencies, what are the possible states into which the atom can decay?

(e) Using the following expression for the Einstein A coefficient and the appropriate matrix elements of hydrogen, determine the *total* lifetime of this state (provide a numeric answer).

$$A = \frac{\omega_0^3 |\vec{p}|^2}{3\pi\epsilon_0 \hbar c^3} = \left|\frac{\delta E}{E_1}\right|^3 \times \frac{|\vec{p}|^2}{c^2 a_0^2} \times 2.35 \times 10^9 \text{ s}^{-1}, \text{ where } \delta E \text{ is the energy difference between 2 states.}$$

$$\langle l = 0, m = 0 | \hat{r} | l = 1, m = 0 \rangle = (0, 0, \sqrt{\frac{1}{3}})$$

$$\langle l = 0, m = 0 | \hat{r} | l = 1, m = 1 \rangle = \left(-\sqrt{\frac{1}{6}}, -i\sqrt{\frac{1}{6}}, 0\right)$$

$$\langle l = 0, m = 0 | \hat{r} | l = 1, m = -1 \rangle = \left(\sqrt{\frac{1}{6}}, -i\sqrt{\frac{1}{6}}, 0\right)$$

$$\langle n = 1, l = 0 | r | n = 2, l = 1 \rangle = 1.29a_0$$

$$\langle n = 1, l = 0 | r | n = 3, l = 1 \rangle = 0.517a_0$$

$$\langle n = 2, l = 0 | r | n = 3, l = 1 \rangle = 3.07a_0$$

$$\langle n = 2, l = 1 | r | n = 3, l = 0 \rangle = 0.95a_0$$

Fall 2012 Written Comprehensive Exam
OPTI 544

The so-called coherent states of the electromagnetic field are described by quantum states that, in the Heisenberg picture, have the form

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

- (a) Briefly explain in words why coherent states are of special importance in quantum optics. [1 pt]
- (b) In the Heisenberg picture the creation and annihilation operators $a(t)$ and $a^+(t)$ are time dependent. Write down expressions for $a(t)$, $a^+(t)$ in terms of $a(0)$, $a^+(0)$ and explain the meaning of any parameters in your expressions that have not already been used above. (Hint: if you don't remember how to write $a(t)$, consider the motion of a point in the phase space of the associated classical problem.) [1 pt]
- (c) Calculate expectation values for the quadrature operators $\hat{X}(t) = [a(t) + a^+(t)]/2$, $\hat{Y}(t) = i[a(t) - a^+(t)]/2$ for the state $|\alpha\rangle$. [2 pt]
- (d) Give the mean number of photons in the state $|\alpha\rangle$. You may simply state the result, or calculate it if you do not remember it. [1 pt]
- (e) Give the standard deviation for the number of photons in the state $|\alpha\rangle$. You may simply state the result, or calculate it if you do not remember it. [1 pt]
- (f) Calculate the probability distribution $P(n)$ as a function of photon number n for the coherent state $|\alpha\rangle$. [2 pt]
- (g) Plot $P(n)$ for a coherent state with a mean photon number of 9. What is the probability that if you measure the number of photons in this state, you will obtain 9 for your result? (Give a number.) [1 pt]
- (h) For a coherent state with a mean photon number of 9, what is the probability of finding 0 photons? (Give a number.) [1 pt]

Fall 2012 Written Comprehensive Exam
OPTI 509

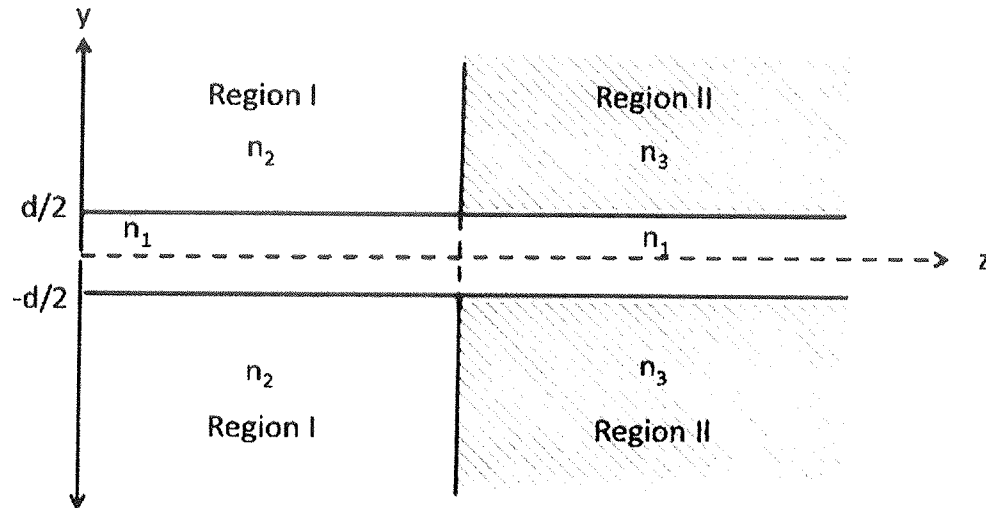
You are putting together a lab experiment where the students will measure the radiometric properties of various sources. You need to pick the equipment to include in the lab and the goal is to maximize the signal coming from the detector. If two different configurations produce the same signal, cost should be the deciding factor. You may choose from the following parts:

| Detector | Active area | Cost | Responsivity (A/W) |
|------------------------|--------------------|------|--------------------|
| A | 0.5mm diameter | \$35 | 0.42 |
| B | 1mm by 1mm square | \$41 | 0.28 |
| | | | |
| Lens focal length (mm) | type | cost | diameter (mm) |
| 50 | uncoated thin lens | \$22 | 12.7 |
| 100 | uncoated thin lens | \$17 | 12.7 |
| 200 | uncoated thin lens | \$12 | 12.7 |

To prevent stray light, a black paper tube 12.7 mm in diameter will be placed over the detector with a minimum length of 50 mm. If you wish to make it longer, the extra paper is free, but you may not make it shorter.

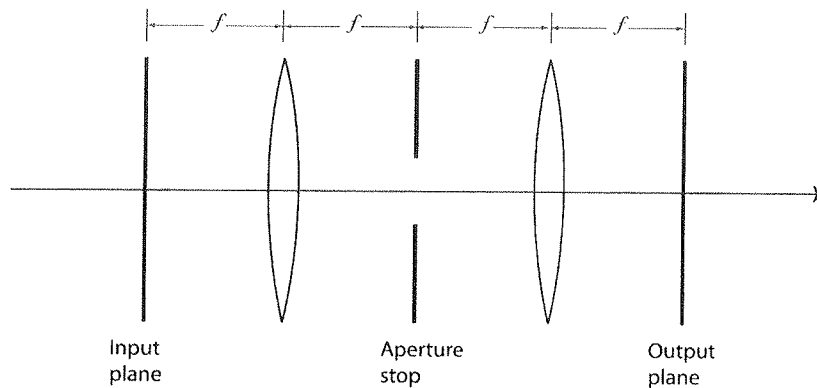
- A) The first source is a television that is 700 by 400 mm that is located 1,000 mm from the system that you are putting together (i.e. the closest thing to the television is 1,000 mm from it). Sketch the system that you would use. Make sure that your sketch has enough detail so that someone would know what components to order and how to position them. What would the total cost of your system be?
- B) For the system that you sketched in part A, what would the signal from the detector be if the television is a Lambertian source with a radiance of $0.33 \text{ W/m}^2\text{sr}$.
- C) Since you might not be able to get the television, the backup plan is to measure a light emitting diode (LED). The distance from the LED to the nearest element of your system is still 1,000 mm. Because you don't have any measured data on the LED, you can only assume that it is an isotropic point source. If you only have to measure the signal from the LED, sketch the system that you would use. Make sure that your sketch has enough detail so that someone would know what components to order and how to position them. What would the total cost of your system be?
- D) For the LED measurement system you sketched in part C, what would the signal from the detector be if the LED has an intensity of 3mW/sr ?

Dielectric slab waveguide transition



Consider the slab waveguide structure above. There is a dielectric waveguide in Region I where the cladding has a refractive index n_2 and a different dielectric slab waveguide in Region II where the cladding has a refractive $n_3 (\neq n_2)$. The core material is common to both regions and has refractive index n_1 , where $n_1 > n_2$ and $n_1 > n_3$. The cladding materials in Regions I and II can be considered to extend to infinity along the $\pm y$ axis. The waveguides in both Region I and Region II can be taken to extend to infinity along the $\pm x$ axis, perpendicular to the plane of the drawing. The waveguide in Region I can be assumed to extend to infinity along the $-z$ axis, while the waveguide in Region II can be assumed to extend to infinity along the $+z$ axis. TE polarized light of free space wavelength $\lambda_0 = 1550\text{nm}$ propagates in the positive z direction starting in Region I. The waveguide is single-mode in both Region I and Region II. If $n_1 = 1.46$, $n_2 = 1.00$ (Region I), and $n_3 = 1.45$ (Region II),

- Give the numerical aperture (NA) of the waveguide in both Region I and Region II (2 points).
- What is the self-consistency condition for the single TE mode of the dielectric waveguide in Region I in terms of the material wavelength $\lambda (= \lambda_0/n_1)$, thickness d , mode angle θ^I , and ϕ^I , which is the phase shift on reflection at the dielectric interface in Region I? (2 points)
- For the given values of the refractive indices will the mode angle in Region II be larger or smaller than the mode angle in Region I? (1 point)
- Calculate the maximum thickness allowed for single mode operation in Region I (3 points)
- Qualitatively sketch the field distributions for the TE mode in the two regions, given that they are both single-mode (2 points).



This question relates to the so-called 4f imaging system depicted above. The horizontal axis is the z axis and the lenses and the aperture stop are circular and centered on this axis. The lenses are ideal and have the same focal length f and same diameter D_{lens} . The diameter of the aperture stop is D_{ap} .

The parts are equally weighted. If you must make additional assumptions to answer any of the parts, be sure to state them explicitly. For the requested sketches, label the axes carefully.

- (a) Demonstrate by a simple ray trace that an object placed in the input plane will be imaged to the output plane.
- (b) What is the magnification between input plane and output plane?
- (c) Suppose that the object in the input plane is a transparency with amplitude transmittance $t_{obj}(\mathbf{r})$, where \mathbf{r} is a two-dimensional vector with components (x, y) . You may assume that the size of the transparency is small compared to the diameter of the lenses. Suppose that this transparency is illuminated with a monochromatic plane wave traveling in the z direction. Write an expression for the complex electric field just to the right of the transparency.
- (d) Is the system as described so far coherent or incoherent? Is it linear and shift-invariant (LSIV)? Why or why not?
- (e) Give an integral expression for the relation between object and image and sketch the point-spread function. (No derivation is needed, but explain all symbols used)
- (f) Now consider what happens if a rapidly rotating ground glass is placed between the illuminating plane wave and the object transparency. You may assume that the ground glass is thin so both it and the transparency are in the input plane. With this setup, is the system coherent or incoherent? Is the system LSIV now? Why or why not?
- (g) Give an integral expression for the relation between object and image with the rotating ground glass in place. Again, explain all symbols used.
- (h) Sketch the point-spread function of the system with the ground glass in place.
- (i) Sketch the transfer function of the system with the ground glass in place.
- (j) What is the highest spatial frequency transmitted by the system, with and without the ground glass?

Fall 2012 Written Comprehensive Exam
OPTI 506

You are putting together a lab experiment where the students will measure the radiometric properties of various sources. You need to pick the equipment to include in the lab and the goal is to maximize the signal coming from the detector. If two different configurations produce the same signal, cost should be the deciding factor. You may choose from the following parts:

| Detector | Active area | Cost | Responsivity (A/W) |
|------------------------|--------------------|------|--------------------|
| A | 0.5mm diameter | \$35 | 0.42 |
| B | 1mm by 1mm square | \$41 | 0.28 |
| | | | |
| Lens focal length (mm) | type | cost | diameter (mm) |
| 50 | uncoated thin lens | \$22 | 12.7 |
| 100 | uncoated thin lens | \$17 | 12.7 |
| 200 | uncoated thin lens | \$12 | 12.7 |

To prevent stray light, a black paper tube 12.7 mm in diameter will be placed over the detector with a minimum length of 50 mm. If you wish to make it longer, the extra paper is free, but you may not make it shorter.

- A) The first source is a television that is 700 by 400 mm that is located 1,000 mm from the system that you are putting together (i.e. the closest thing to the television is 1,000 mm from it). Sketch the system that you would use. Make sure that your sketch has enough detail so that someone would know what components to order and how to position them. What would the total cost of your system be?
- B) For the system that you sketched in part A, what would the signal from the detector be if the television is a Lambertian source with a radiance of $0.33 \text{ W/m}^2\text{sr}$.
- C) Since you might not be able to get the television, the backup plan is to measure a light emitting diode (LED). The distance from the LED to the nearest element of your system is still 1,000 mm. Because you don't have any measured data on the LED, you can only assume that it is an isotropic point source. If you only have to measure the signal from the LED, sketch the system that you would use. Make sure that your sketch has enough detail so that someone would know what components to order and how to position them. What would the total cost of your system be?
- D) For the LED measurement system you sketched in part C, what would the signal from the detector be if the LED has an intensity of 3mW/sr ?

Fall 2012 Written Comprehensive Exam
OPTI 503

The spectrum emitted by a blackbody with a temperature of 2000 Kelvin is filtered separately by two band-pass color filters to obtain two narrow-band beams. The filters are centered at 600nm and 500nm, and both filters have a bandwidth of 5nm. The filters are assumed to have 100% transmission within the band-pass window and zero transmission outside of the filter window. The spectral radiance of a blackbody is given by

$$L(\lambda) = \frac{C_1}{\pi\lambda^5} \frac{1}{e^{C_2/\lambda T} - 1} \quad (\text{watt}/(\text{m}^2 \cdot \text{sr} \cdot \mu\text{m}))$$

$$\text{where } C_1 = 2\pi^5 hc^2 = 3.741451 \times 10^8 \text{ (W}\mu\text{m}^4/\text{m}^2), \quad C_2 = \frac{hc}{k} = 1.4387 \times 10^4 \text{ (}\mu\text{m} \cdot \text{K)},$$

and T is the absolute temperature in Kelvin. The luminous efficiencies and corresponding xyz color matching functions are given in the Table 1.

Answer the following questions:

- (2 points) Compute the spectral radiance of both of the filtered beams.
- (4 points) Compute the luminance L_v , XYZ tristimulus values, and x-y chromaticity coordinates for both of the filtered beams.
- (4 points) When these two filtered beams are combined to produce a new color, compute the total luminance, the XYZ tristimulus values, chromaticity coordinates of the resulting color.

Table 1 Selected data for luminous efficiencies and color matching functions

| Wavelength (λ) (nm) | Luminous efficiency $v(\lambda)$ | x-bar (λ) | y-bar (λ) | z-bar (λ) |
|----------------------------------|-------------------------------------|---------------------|---------------------|---------------------|
| 495 | 0.259 | 0.0147 | 0.2586 | 0.3533 |
| 500 | 0.323 | 0.0049 | 0.323 | 0.272 |
| 505 | 0.407 | 0.0024 | 0.4073 | 0.2123 |
| ... | | | | |
| 595 | 0.695 | 1.0567 | 0.6949 | 0.001 |
| 600 | 0.631 | 1.0622 | 0.631 | 0.0008 |
| 605 | 0.567 | 1.0456 | 0.5668 | 0.0006 |
| ... | | | | |
| | | | | |

Fall 2012 Comprehensive Exam
OPTI 507

Consider the dielectric response of NaCl due to phonons, described by the dielectric function

$$\varepsilon = \varepsilon_{\infty} + \frac{\varepsilon_0 - \varepsilon_{\infty}}{1 - (\omega^2 + i\gamma\omega) / \Omega_T^2}$$

Assuming, for simplicity, that you can neglect damping, derive the Lyddane-Sachs-Teller relation and determine Ω_L (in NaCl, $\varepsilon_0 = 5.62$, $\varepsilon_{\infty} = 2.25$, and $\Omega_T = 3.08 \times 10^{13} \text{ s}^{-1}$).

(10 points)

Fall 2012 Written Comprehensive Examination
OPTI-537

Answer the following questions related to semiconductor detectors. All parts weighted as indicated.

(a) (15%) What is the definition of a Bravais lattice? What is a primitive lattice vector? Write an expression for a translation vector that spans the equivalent locations in a real-space 3D Bravais lattice. How many distinct Bravais lattices are there?

(b) (15%) Write the 3D time-independent Schrödinger equation for a single electron in a Bravais lattice. Show where periodicity is expressed in the Hamiltonian and explain what assumptions are necessary to invoke the time-independent form of the Schrödinger equation to describe the electronic structure and related properties of crystals.

(c) (15%) Consider a translation operator $T_{\mathbf{R}}$ such that $T_{\mathbf{R}}[\psi(\mathbf{r})] = \psi(\mathbf{r} + \mathbf{R})$. If \mathbf{R} is a translation vector in the Bravais lattice that describes the periodic crystal, work out whether or not $T_{\mathbf{R}}$ commutes with the one-electron Hamiltonian of question (b) above.

(d) (15%) If $\psi(\mathbf{r})$ is an eigenfunction of $T_{\mathbf{R}}$, use the properties of the 3D FT to work out the associated eigenvalue. What have you just derived?

(e) (10%) Sketch a one dimensional band diagram for the nearly-free electron model. (hint: x-axis units are momentum, y-axis units are energy) in the reduced-zone scheme. If the direct lattice constant is 5 Angstroms, what is the wavelength of the electron at the positive boundary of the 1st Brillouin zone.

(f) (15%) What is the definition of the Fermi level (energy)? Write the expression for the Fermi-Dirac distribution and state what it means.

(g) (15%) Draw a band diagram of the unbiased PN junction. Show valence and conduction bands, donor and acceptor states, label which side is N and which is P, indicate majority and minority carriers, and show the Fermi level at room temperature. Indicate the depletion region, describe how it forms, and state its importance for photodetection.