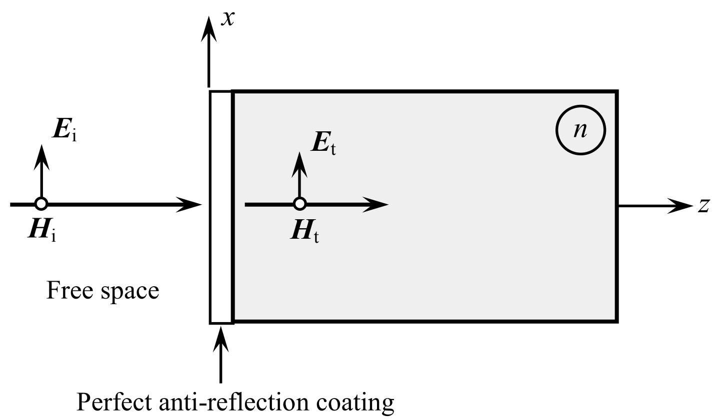
A semi-infinite slab of transparent, non-magnetic glass (refractive index $= n$) has a *perfect* anti-reflection coating applied to its entrance facet. A monochromatic, linearly-polarized plane-wave arrives at the slab at normal incidence, as shown. The incidence medium is free space, the vacuum wavelength of the light is λ_0, and the incident E-field is along the x-axis.

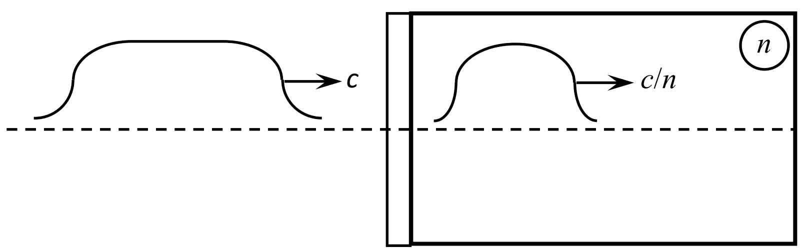


a) What is the relation between the incident E- and H-fields, E_i and H_i, in terms of the impedance of free-space, $Z_0 = \sqrt{\mu_0/\varepsilon_0}$?

b) What is the relation between the fields E_t, H_t transmitted into the slab in terms of Z_0 and n?

c) Without making any assumptions about the structure of the anti-reflection coating, simply knowing that the optical energy of the beam passes entirely from free space into the slab, determine the relation between the incident and transmitted E-fields E_i and E_t.

d) Assume now that, instead of a plane-wave, the incident beam is a pulse of light having the same central wavelength λ_0 as before. Moreover, the front-facet coating is effective as a perfect anti-reflection coating for the entire pulse, and the semi-infinite slab is free from dispersion, so that, inside the slab, the pulse propagates with velocity c/n, as shown. What are the E- and H-field energies inside the slab? Is the total E-field energy of the pulse equal to its total H-field energy? Is the pulse energy conserved before and after incidence?

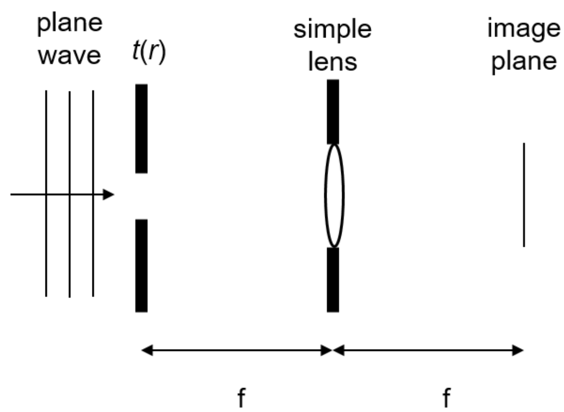


Hint: You may find the vector identity $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \cdot \boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A} \cdot \boldsymbol{B})\boldsymbol{C}$ useful.

A monocular telescope (i.e. for a single eye) is fixed in a wall, has an exit pupil diameter of 4 mm, and images a full field of view of ±2 degrees in object space. The monocular has an entrance pupil diameter of 20 mm. The exit pupil is too close to the wall for the user to place their eye at the exit pupil of the monocular. However, the user has a box of achromatic doublets with a focal length of 100 mm each, and a diameter of 20 mm.

a) Using at least two doublets (modeled as thin lenses in air) design an optical system to reimage the exit pupil of the monocular telescope with a magnification of -1. This pupil re-imager will allow the users to place their eye at the relayed exit pupil which must be 4 mm in diameter. Provide the distances along the optical axis of the doublets with respect to the exit pupil of the monocular, provide the distance from the last doublet to the overall system exit pupil (which must be a positive distance).
b) Provide a clear and concise drawing of your design showing the marginal and chief rays, and all the pupils.
c) Determine the Lagrange invariant of the system
d) Determine the unvignetted field of view in object space that is provided by your design.
e) What should be the diameter of the doublets for the system to be unvignetted over the full ±2 degree field of view of the telescope?

A simple lens of focal length $f = 50$mm is illuminated by an on-axis laser beam passing through a small hole, where $t(r) = \text{circ}(r/D)$ and $D = 1$mm, where r is the radial distance from the center of the hole and circ(r/D) = 0 for $r > 0.5$. The laser beam has a wavelength of 620nm. State any assumptions that you make. {NOTE: For parts c, d, e, f and g, do not substitute values for the variables. Keep the equation in symbolic form. You do not need to evaluate any Fourier transforms for part c, d, e and f. Instead, use the format ***F*** [ ] and ***F***$^{-1}$ [ ]. }



a.) (2 pts) What is the Fresnel number of the irradiance pattern at the lens?

b.) (2 pts) Sketch the irradiance profile at the lens, including approximate dimensions across a diameter.

c.) (1 pt) Derive an expression for the angular spectrum of the light just before the lens.

d.) (1 pt) Derive an expression for the complex electric field incident on the lens, using your result from (c).

e.) (1 pt) Assume that the lens transmission function is given by $t(\rho_{lens}) = \exp\left(-j\pi \frac{\rho_{lens}^2}{\lambda f}\right)$, and derive an expression for the complex electric field transmitted through the lens.

f.) (1 pt) Use Fresnel diffraction to propagate the field in (e) to the image plane.

g.) (1 pt) Find the image irradiance by first simplifying the expression in (f). You will need to evaluate some Fourier transforms, but keep the expression in symbolic form.

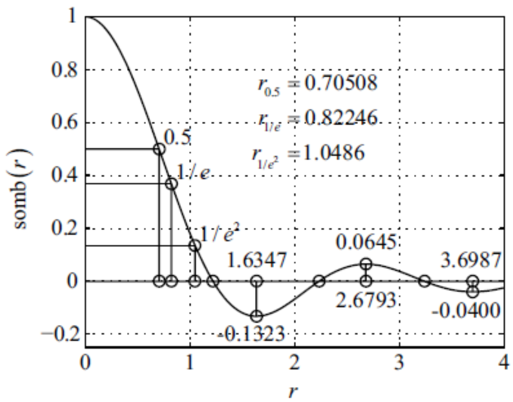
h.) (1 pt) Sketch the profile of the irradiance pattern in the image plane. Label the horizontal axis with appropriate distance values.

HELPFUL INFORMATION:

$$\mathbf{F}_\eta \mathbf{F}_\xi [h_z^H(x_0, y_0; z_0)] = \exp(jk\gamma z_0) = H_z(\gamma; z_0)$$

$$U_0(\boldsymbol{r}_0) = -\frac{j\exp(jkz_0)}{\lambda z_0}\exp\left[\frac{jk}{2z_0}\left(x_0^2 + y_0^2\right)\right]\mathbf{F}_\eta \mathbf{F}_\xi\left\{U_s^+(x_s, y_s)\exp\left[\frac{jk}{2z_0}\left(x_s^2 + y_s^2\right)\right]\right\}$$

$$\mathbf{B}_{\rho_\xi = \frac{\rho_0}{\lambda z_0}}\left[\mathrm{circ}\left(\frac{\rho_s}{2a}\right)\right] = \pi a^2 \mathrm{somb}\left(\frac{2a\rho_0}{\lambda z_0}\right)$$



Fall 2019 Qualifying Exam
OPTI 511/Optical Physics, Question 1

This problem involves transitions between the energy eigenstates of the hydrogen atom that occur by spontaneous emission of a single photon. You are to assume that the electric dipole approximation is valid. You should also ignore any effects due to electron and nuclear spins (i.e., ignore hydrogen's fine and hyperfine structure). The energy eigenstates are to be labeled in the usual way as $|n,\, l,\, m\rangle$, where n is the principal quantum number, l is the quantum number for the electron's orbital angular momentum, and m is the quantum number for the electron's z-component of orbital angular momentum. [10 pts total.]

Planck's constant: $h \;=\; 6.63 \times 10^{-34}$ J·s $\;=\; 4.14 \times 10^{-15}$ eV·s

(a. - 1pt.) State the selection rules for the single-photon transitions that can occur in hydrogen.

(b. - 1pt.) Suppose that a hydrogen atom in its ground state absorbs a photon from a laser beam that is propagating along the z axis. The atom makes a transition to a state with $m = 1$. What was the specific polarization of the photon that was absorbed?

(c. - 2pts.) Consider a gas of hydrogen atoms. All atoms are initially prepared in the $|4, 0, 0\rangle$ state. List *all* of the transitions that can occur as the atoms decay into various states by spontaneous emission. Make sure you also list the decays that can occur from each of the states in your list.

(d. - 2pts.) Answer the same question as in part **(c)**, except in this part assume that all of the atoms are initially prepared in the $|4, 1, 0\rangle$ state.

(e. - 2pts.) Considering all of the transitions listed in parts **(c)** and **(d)**,

- **(e1.)** what wavelength is emitted in the case of part **(c)** that is *not* emitted in the case of part **(d)**?
- **(e2.)** what wavelength is emitted in the case of part **(d)** that is *not* emitted in the case of part **(c)**?

(f. - 2pts.) Consider again a gas of hydrogen atoms. All of the atoms are either prepared in the $|3, 2, 0\rangle$ state, or they are all prepared in the $|3, 2, 2\rangle$ state. Suppose that it is your job to make a measurement that will enable you to determine which of these two states the atoms are initially prepared in. You will do this by examining the light emitted from the gas as the atoms decay by spontaneous emission. Describe what specific characteristic(s) of the light will be measured in order to determine the initial state. State the different possible outcomes of the measurement, and what conclusions you can make from each of the different possible outcomes.

A linearly-polarized, monochromatic plane-wave propagates along the x-axis, with its E-field amplitude given as $\boldsymbol{E}(x,t) = E_0 \cos\{\omega[t - n(\omega)x/c]\}\,\hat{\boldsymbol{y}}$. The host medium is a homogeneous, linear, isotropic, non-magnetic (i.e., $\mu(\omega) = 1$), transparent dielectric, whose frequency-dependent refractive index is specified as $n(\omega) = \sqrt{\varepsilon(\omega)}$.

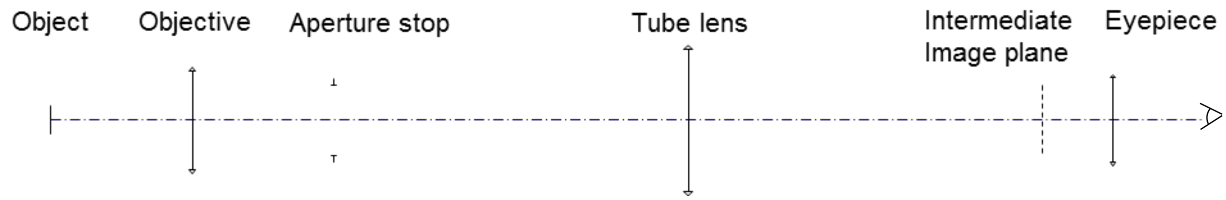
a) Find the magnetic field $\boldsymbol{H}(x,t)$ of the plane-wave in terms of $E_0, c, \omega, n(\omega)$, and the impedance of free space $Z_0 = \sqrt{\mu_0/\varepsilon_0}$.

b) Find the Poynting vector $\boldsymbol{S}(x,t)$ of the above plane-wave, then determine the time-averaged rate-of-flow of optical energy (per unit area per unit time) along the x-axis.

c) Assume a second plane-wave, *identical* with the one above *except* for its frequency ω' differing slightly from ω, is co-propagating with the above plane-wave. Write an expression for the combined E-field of the superposed plane-waves. From this expression, identify the carrier and the envelope of the beat waveform. In terms of $c, \omega_c = ½(\omega + \omega'), \Delta\omega = \omega' - \omega$, $n(\omega_c)$ and $\mathrm{d}n(\omega)/\mathrm{d}\omega$, what are the *phase* and *group* velocities of the combined waveform?

Hint: You may find the trigonometric identity $\cos a + \cos b = 2\cos[½(a+b)]\cos[½(a-b)]$ useful.

A research-grade microscope consists of an infinite conjugate objective lens, a tube lens with a focal length of 200 mm, and a 10x eyepiece. When used with this microscope system, the objective lens has a magnification of 10X and a numerical aperture (NA) of 0.1. The maximum object size is +/- 1 mm. The aperture stop of the system is located so that the system is telecentric in object space. Consider all of the lenses to be thin lenses in air. All calculations will be based on first-order optics.



* Figure is not to scale

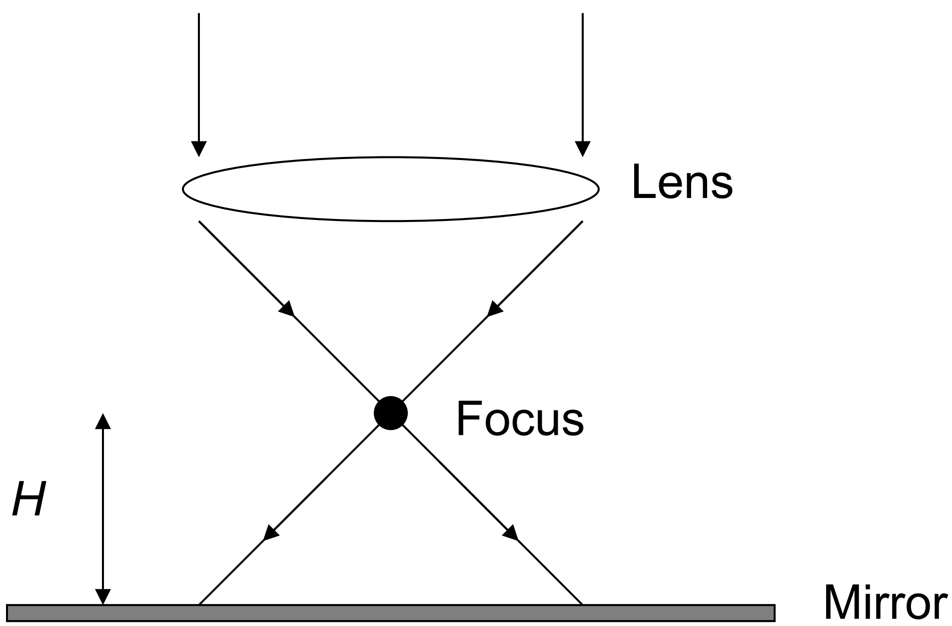
1. Sketch the chief ray and the marginal ray from the object to the eye pupil. (1 pt)
2. What is the focal length of the objective lens and the overall visual magnification of the microscope? (1 pt)
3. Determine the location and diameter of the aperture stop for the microscope. (2 pts)
4. What is the size of the intermediate image? (1 pt)
5. If a 25.4mm beamsplitter cube is placed between the objective lens and the tube lens, what is the shift of the intermediate image plane? What is the shift of the intermediate image plane if the beamsplitter cube is placed between the tube lens and the intermediate image plane? The index of refraction of the beamsplitter cube is 1.5. (2 pts)
6. We want to convert this microscope to a digital microscope with a CMOS sensor which has 2,000x2,000 pixels (pixel size is: 5x5 µm). The sensor is required to be located 150 mm to the right of existing tube lens, and the sensor must record the image of the entire object. This requires that an additional lens element is added to the right of the tube lens. Determine the focal length of this additional thin lens and its separation from the tube lens. (3 pts)

A lens is used to focus a $\lambda = 0.5\mu m$ laser beam so that the focus point is $H = 1.0\ \mu m$ above a perfect electrical conductor (PEC) mirror. The full cone angle of the focus cone is 90^o. State any assumptions that you make.

a.) (6 pts) Make a sketch showing the center of bright fringes in the interference pattern generated between the incident laser beam and the reflected beam between the focus and the mirror. Bright fringe centers should be indicated with a line. Dark fringe centers should be indicated with a dashed line. The sketch should include fringe positions and fringe shapes. It is not necessary to shade with grayscale. Do not include any interference that may occur above the focus toward the lens. Ignore any effects due to diffraction from the mirror surface.

b.) (2 pts) Label the fringe lines with fringe order numbers.

c.) (2 pts) How many bright fringes are observed?



Fall 2019 Qualifying Exam
OPTI 511R, Question 2

A particle of mass m is trapped in a 1-dimensional potential well of the form $V(x) = \frac{1}{2}m\omega^2 x^2$, where ω is an angular frequency. The particle is in a superpostion of the lowest two energy eigenstates. The particle's wavefunction is:

$$\psi(x) = C[\psi_0(x) + \psi_1(x)]$$

Here, C is a real and positive constant.

(a) [**10 points**] What is the value of C?

(b) [**10 points**] What are the energy eigenvalues for this potential well?

(c) [**20 points**] What is the average energy of the particle in the given superposition state?

(d) [**20 points**] Write down the time-dependent form of the wavefunction, $\Psi(x, t)$, in terms of its energy eigenstates and ω.

(e) [**40 points**] Calculate the expectation value of the position operator, $\langle \hat{x} \rangle$(t). Express your answer as a real function of time in terms of m and ω. Make use of the raising and lowering operators, $\hat{a}_+$ and $\hat{a}_-$, so you do not have to solve any integrals.

$$\hat{a}_+\psi_n = \sqrt{n+1}\,\psi_{n+1}$$
$$\hat{a}_-\psi_n = \sqrt{n}\,\psi_{n-1}$$

$$\hat{p} = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$$
$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$$